Announcements

1. HW Assignments #19-24 have been posted.
2. Physics lecture and workshop this week on teaching techniques for PHY 113 & 114 (http://www.wfu.edu/physics/seminars/chabay.html)
3. Topics for today:

   Simple harmonic motion for a spring

   Simple harmonic motion for a pendulum

   Notion of resonance

Simple harmonic motion:

$$F = -kx = m\frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

Conveniently evaluated in *radians*

$$x(t) = A\cos(\omega t + \varphi); \quad \omega = \sqrt{\frac{k}{m}}$$

Constants

Note that:

$$v(t) = \frac{dx}{dt} = -A\omega\sin(\omega t + \varphi)$$

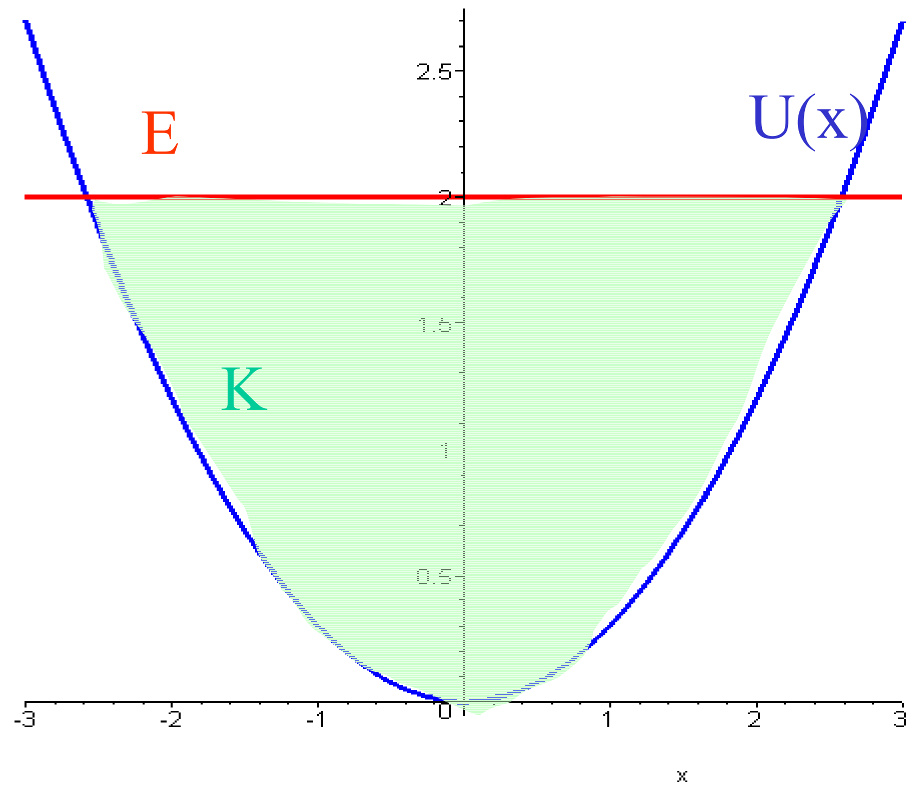
$$a(t) = \frac{dv}{dt} = -A\omega^2\cos(\omega t + \varphi)$$

Mechanical energy associated with simple harmonic motion

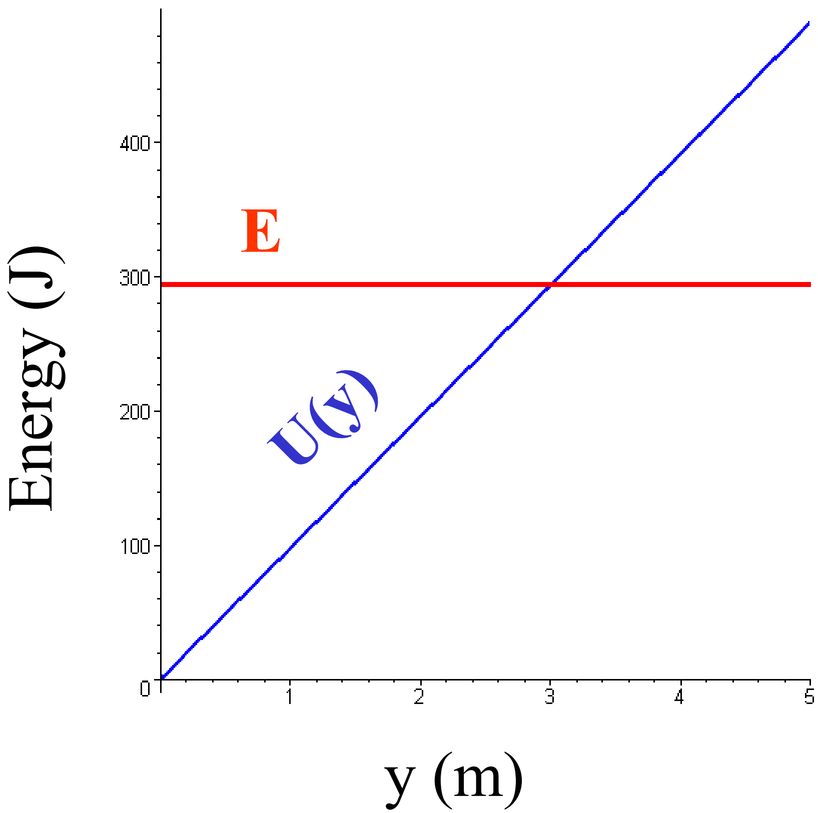
$$E = K + U = ½ m v^2 + ½ k x^2$$

$$E = \tfrac{1}{2} m\{A\omega \sin(\omega t + \varphi)\}^2 + \tfrac{1}{2} k\{A \cos(\omega t + \varphi)\}^2$$

$$= \tfrac{1}{2} m\omega^2 A^2 = \tfrac{1}{2} kA^2$$



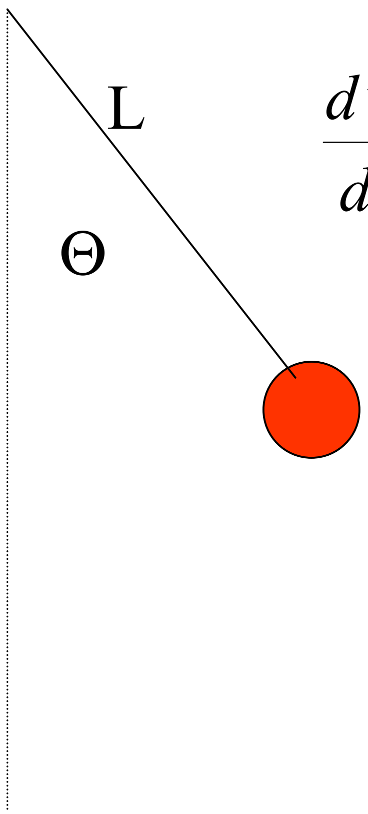
Peer instruction question



The diagram on the left shows a graph of potential energy versus height y for a system whose total energy is E=294J. What is the largest value of y that this system can have?

(a) 0m (b) 3m (c) 5m

Simple harmonic motion for a pendulum:



$$\tau = mgL\sin\Theta = -I\alpha = -I\,\frac{d^2\Theta}{dt^2}$$

$$\frac{d^2\Theta}{dt^2} = -\frac{mgL}{I}\sin\Theta = -\frac{g}{L}\sin\Theta \quad (\text{since } I = mL^2)$$

Approximation for small Θ:

$$\sin\Theta \approx \Theta$$

$$\Rightarrow \frac{d^2\Theta}{dt^2} = -\frac{g}{L}\Theta$$

Solution :

$$\Theta(t) = A\cos(\omega t + \varphi); \quad \omega = \sqrt{\frac{\mathrm{g}}{\mathrm{L}}}$$

The notion of resonance:

Suppose $F=-kx+F_0 \sin(\Omega t)$

According to Newton:

$$-kx + F_0 \sin(\Omega t) = m\frac{d^2x}{dt^2}$$

Differential equation ("inhomogeneous"):

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x + \frac{F_0}{m}\sin(\Omega t)$$

Solution:

$$x(t) = \frac{F_0/m}{k/m - \Omega^2}\sin(\Omega t) \equiv \frac{F_0/m}{\omega^2 - \Omega^2}\sin(\Omega t)$$

Physics of a "driven" harmonic oscillator:

$$-kx + F_0 \sin(\Omega t) = m \frac{d^2 x}{dt^2}$$

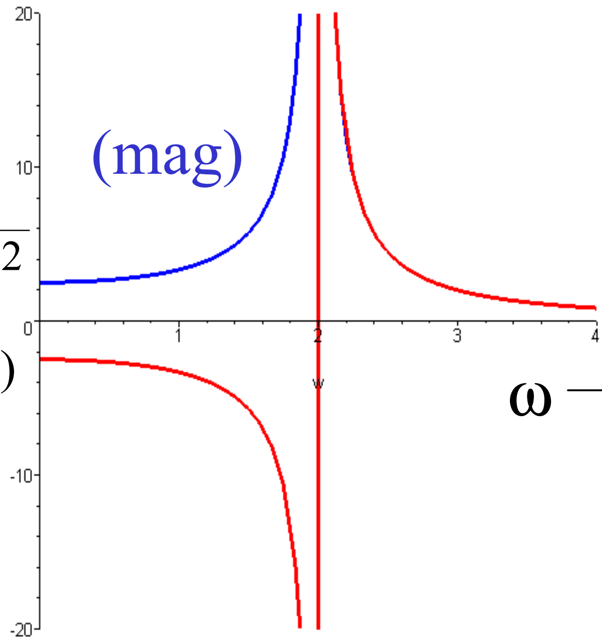
"driving" frequency

$$x(t) = \frac{F_0 / m}{k/m - \Omega^2} \sin(\Omega t) \equiv \frac{F_0 / m}{\omega^2 - \Omega^2} \sin(\Omega t)$$

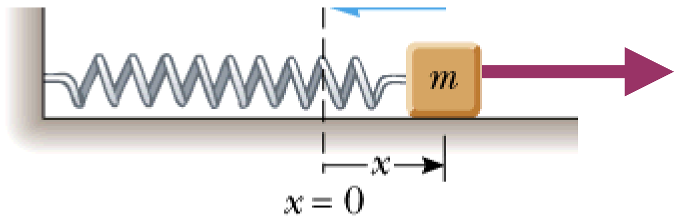
"natural" frequency

$$\frac{F_0 / m}{\omega^2 - \Omega^2}$$

(Ω=2rad/s)



ω



F(t)=1 N sin (3t)

Examples:

Suppose a mass m=0.2 kg is attached to a spring with k=1.81N/m and an oscillating driving force as shown above. Find the steady-state displacement x(t).

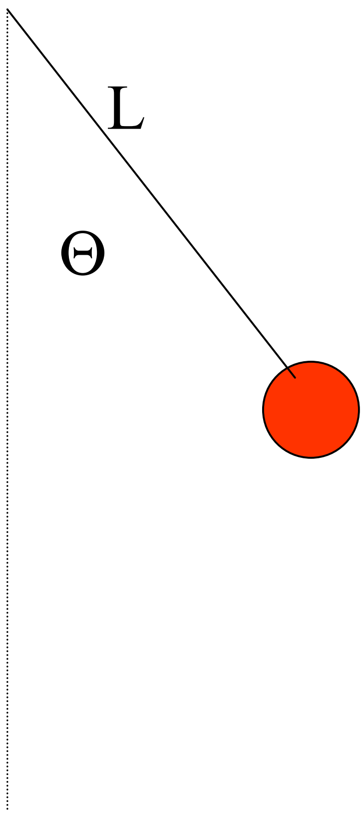
$$x(t)=\frac{F_0/m}{k/m-\Omega^2}\sin(\Omega t)=\frac{1/0.2}{1.81/0.2-3^2}\sin(3t)\ \mathrm{m}=100\sin(3t)\ \mathrm{m}$$

Note: If k=1.90 N/m then:

$$x(t)=\frac{F_0/m}{k/m-\Omega^2}\sin(\Omega t)=\frac{1/0.2}{1.90/0.2-3^2}\sin(3t)\ \mathrm{m}=10\sin(3t)\ \mathrm{m}$$

Pendulum example:

Suppose L=2m, what is the period of the pendulum?



$$\omega = \sqrt{\frac{g}{L}} = \sqrt{\frac{9.8\text{m/s}^2}{2\text{m}}} = 2.2135\ \text{rad/s} = \frac{2\pi}{T}$$

$$T = \frac{2\pi}{\omega} = 2.84\ \text{s}$$

$$\Theta(t) = A\cos(\omega t + \varphi); \quad \omega = \sqrt{\frac{g}{L}}$$