Announcements

1. Correction to Problem Set #20 – problem 13.30(c) (answer was incorrect)
2. Today's topic – Universal law of gravity

   Form of gravitational force law

   Gravitational acceleration near surface of earth

   Satellite motion

13.30 A simple pendulum has a length L. (a) What is the period of simple harmonic motion for this pendulum if it is hanging in an elevator that is accelerating upward at an acceleration a **j**? (b) What is its period if the elevator is accelerating downward at an acceleration –a **j**? (c) What is the period of simple harmonic motion for this pendulum if it is placed in a truck that is accelerating horizontally at an acceleration a **i**?

(a)

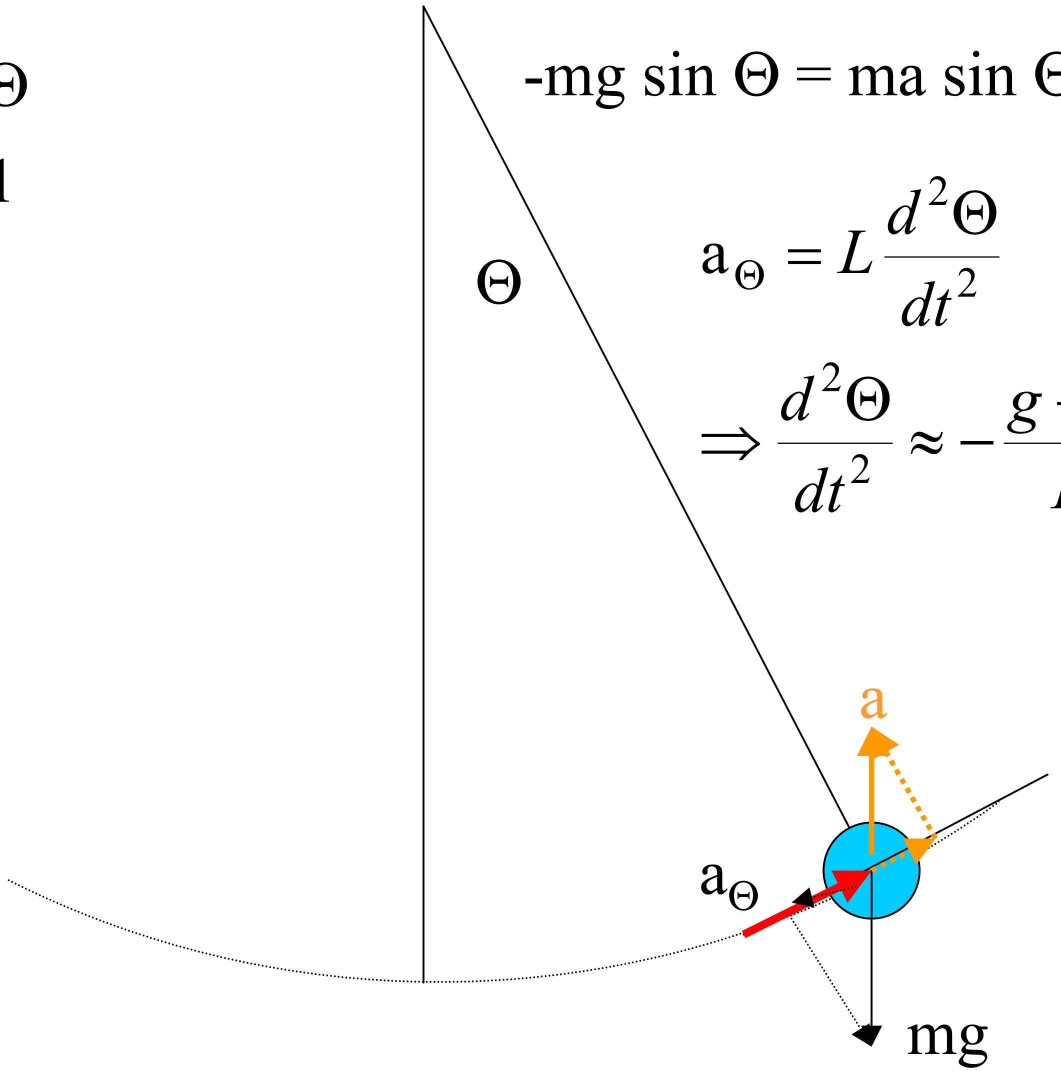
$$\sin\Theta \approx \Theta$$

$$\cos\Theta \approx 1$$

$$-mg \sin\Theta = ma \sin\Theta + ma_\Theta$$

$$a_\Theta = L\frac{d^2\Theta}{dt^2}$$

$$\Rightarrow \frac{d^2\Theta}{dt^2} \approx -\frac{g+a}{L}\Theta$$



(c)

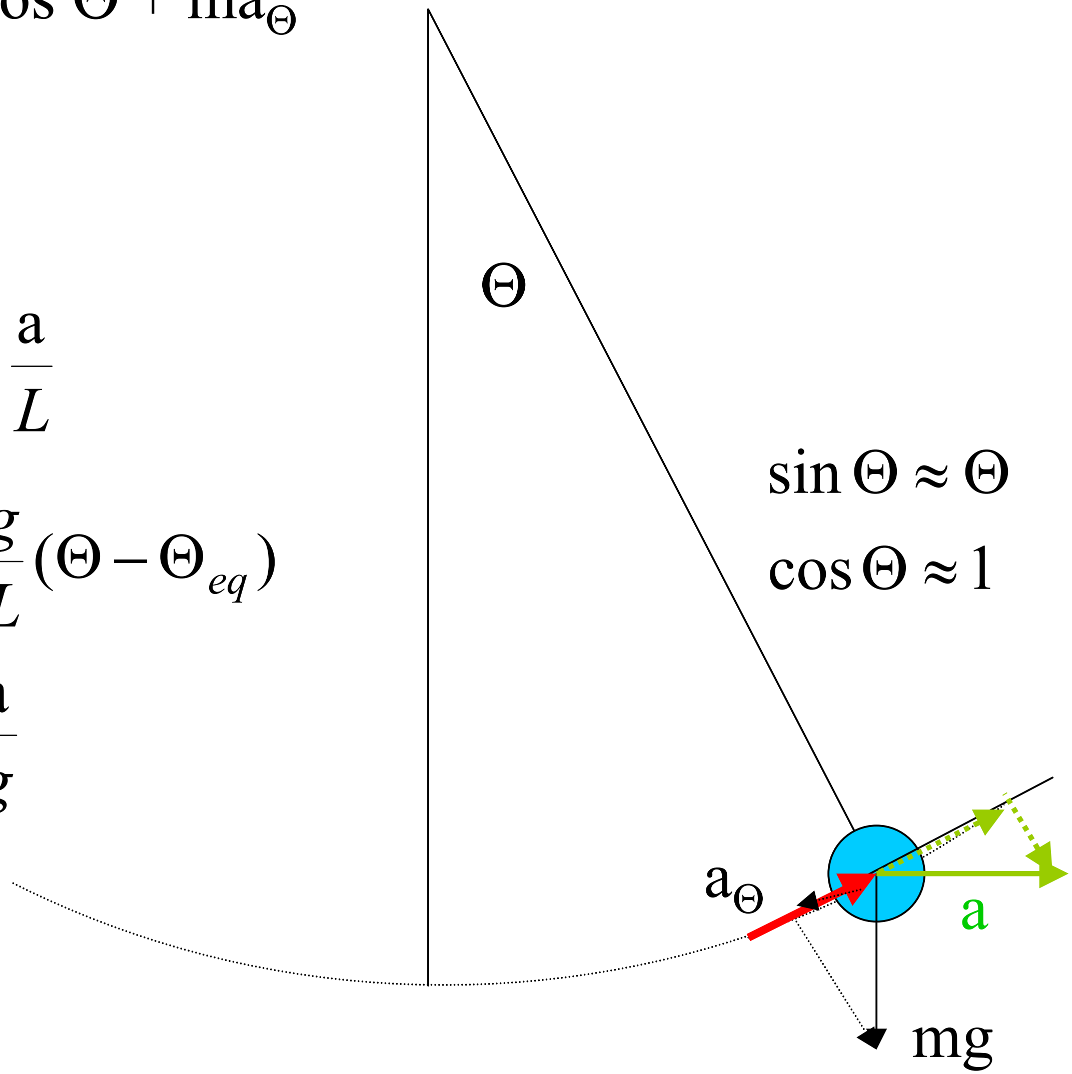
-mg sin Θ = ma cos Θ + ma_Θ

$$a_\Theta = L\frac{d^2\Theta}{dt^2}$$

$$\Rightarrow \frac{d^2\Theta}{dt^2} \approx -\frac{g}{L}\Theta - \frac{\mathrm{a}}{L}$$

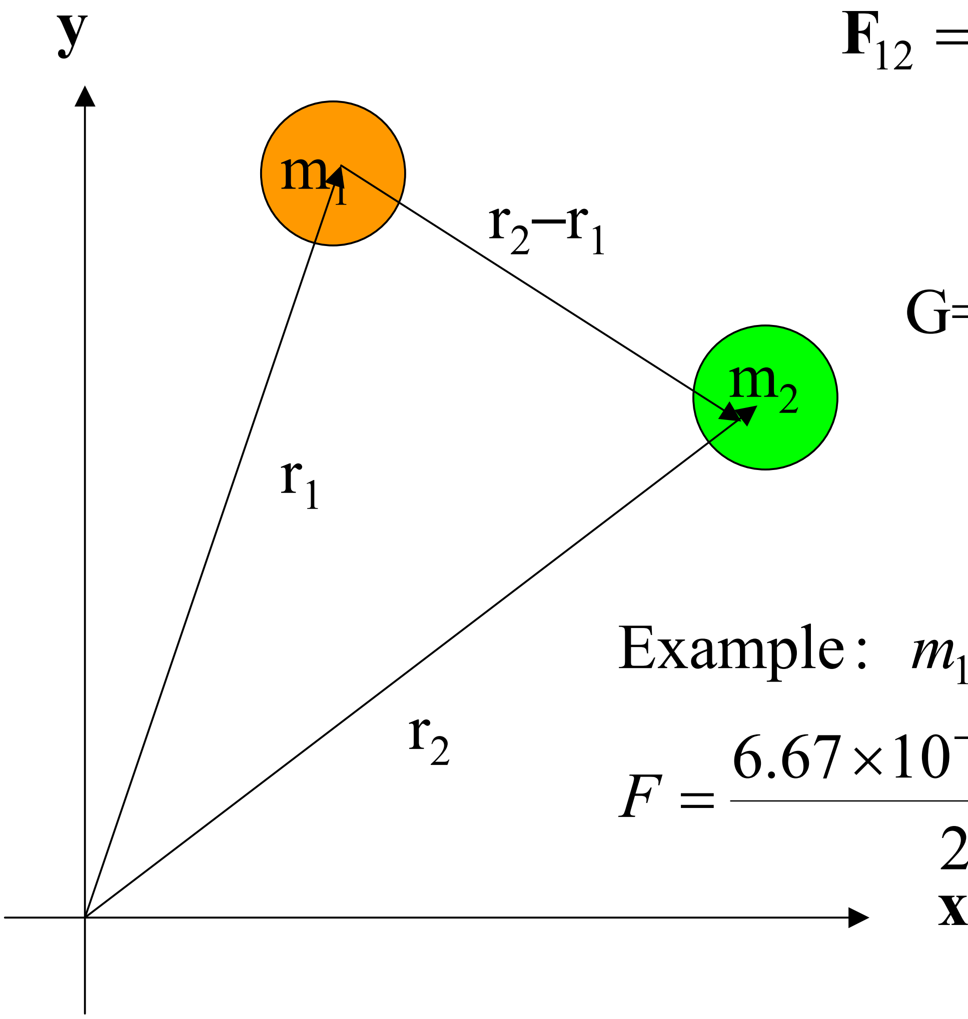
$$\frac{d^2(\Theta-\Theta_{eq})}{dt^2} \approx -\frac{g}{L}(\Theta-\Theta_{eq})$$

$$\text{where} \quad \Theta_{eq} = -\frac{\mathrm{a}}{g}$$



Newton's law of gravitation: m_2 *attracts* m_1 according to:

$$\mathbf{F}_{12} = \frac{G m_1 m_2 \hat{\mathbf{r}}_{12}}{r_{12}^2}$$

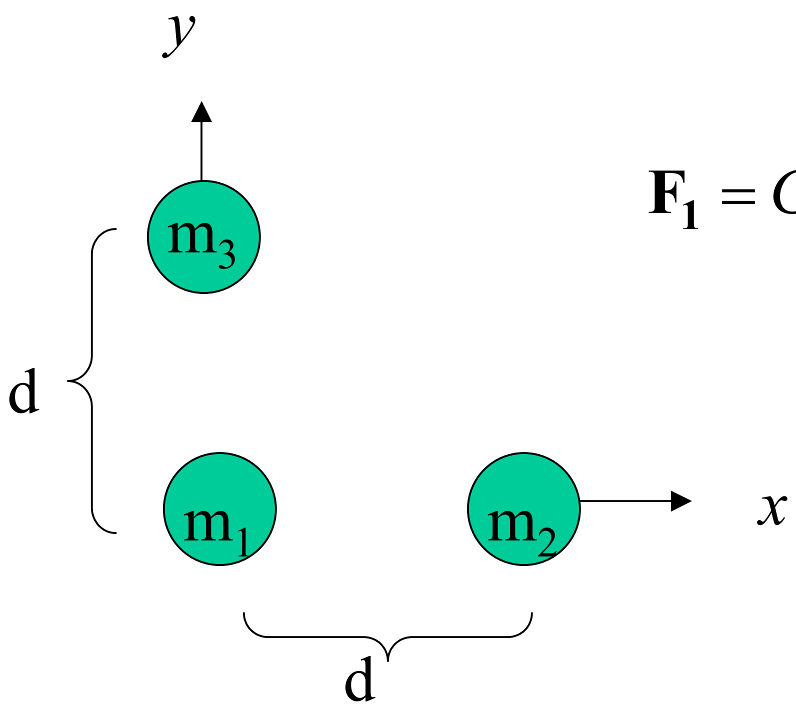


G=6.67 x 10^{-11} N m^2/kg^2

$$\text{Example: } \; m_1 = m_2 = 70\,\text{kg}; \quad r_{12} = 2\,\text{m}:$$

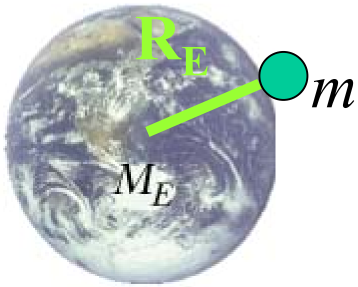
$$F = \frac{6.67 \times 10^{-11} \cdot 70 \cdot 70}{2^2} N = 8.17 \times 10^{-8} N$$

Vector nature of Gravitational law:



$$\mathbf{F_1} = G\frac{m_1}{d^2}\left(m_2\mathbf{i} + m_3\mathbf{j}\right)$$

Gravitational force of the Earth



$$F = \frac{GM_E m}{R_E^2}$$

$$\Rightarrow g = \frac{GM_E}{R_E^2} = \frac{6.67\times10^{-11}\cdot 5.98\times10^{24}}{(6.37\times10^{6})^2}\,\mathrm{m/s^2} = 9.8\mathrm{m/s^2}$$

Question:

Suppose you are flying in an airplane at an altitude of 35000ft~11km above the Earth's surface. What is the acceleration due to Earth's gravity?

$$F = \frac{GM_E m}{(R_E + h)^2} = ma$$

$$a = \frac{GM_E}{(R_E + h)^2} = \frac{6.67 \times 10^{-11} \cdot 5.98 \times 10^{24}}{((6.37 + 0.011) \times 10^6)^2}\,\mathrm{m/s^2} = 9.796\mathrm{m/s^2}$$

$$a/g = 0.997$$

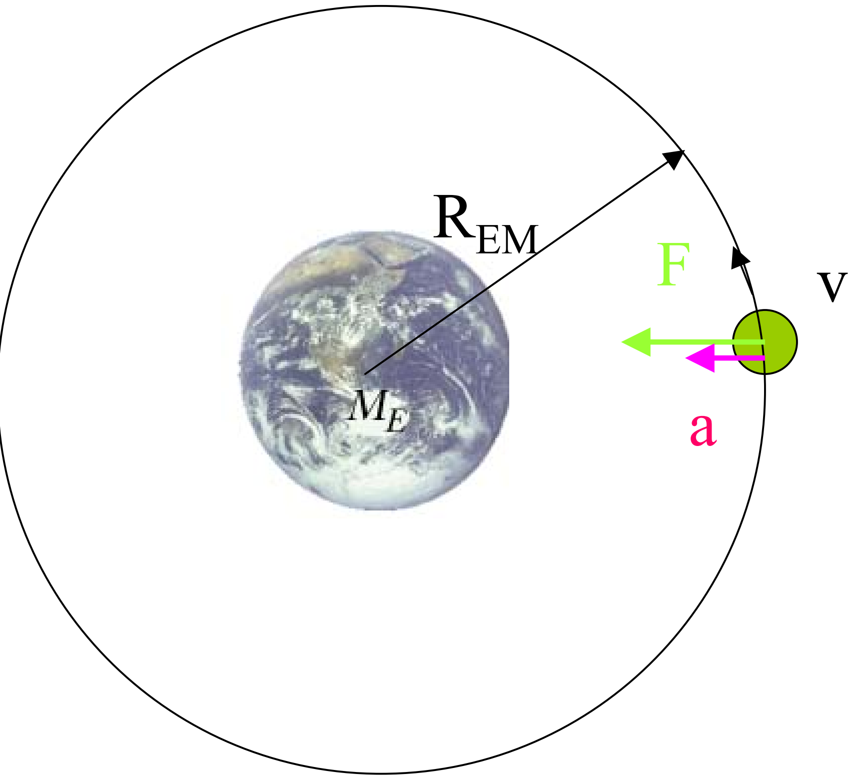
Attraction of moon to the Earth:

$$F = \frac{GM_E M_M}{R_{EM}^2} = \frac{6.67\times10^{-11}\cdot5.98\times10^{24}\cdot7.36\times10^{22}}{(3.84\times10^8)^2}\,\mathrm{N} = 1.99\times10^{20}\,\mathrm{N}$$

Acceleration of moon toward the Earth:

$F = M_M a$ ➔ $a = 1.99x20^{20}$ N/7.36x10^{22}kg =0.0027 m/s^2

Stable circular orbit of two gravitationally attracted objects (such as the moon and the Earth)



$$a = \frac{v^2}{R_{EM}} = \frac{GM_E}{R_{EM}^2}$$

$$v = \omega R_{EM} = \frac{2\pi}{T} R_{EM}$$

$$T = 2\pi\sqrt{\frac{R_{EM}^3}{GM_E}}$$

$$= 2\pi\sqrt{\frac{(3.84\times10^8)^3}{6.67\times10^{-11}\cdot5.98\times10^{24}}}$$

$$= 2367353.953\ \mathrm{s} = 27.4\ \mathrm{days}$$

Peer instruction question

In the previous discussion, we saw how the moon orbits the Earth in a stable circular orbit because of the radial gravitational attraction of the moon and Newton's second law: F=ma, where a is the centripetal acceleration of the moon in its circular orbit. Is this the same mechanism which stabilizes airplane travel? Assume that a typical cruising altitude of an airplane is 11 km above the Earth's surface and that the Earth's radius is 6370 km.

(a) Yes (b) No

More details

If we examine the circular orbit more carefully, we find that the correct analysis is that the stable circular orbit of two gravitationally attracted masses is about their center of mass.