Announcements

1. Posted practice tests for 3rd hour exam.

   http://www.wfu.edu/~natalie/f02phy113/extrapractice

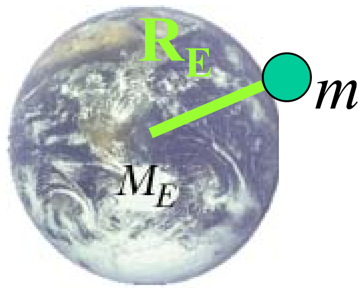
2. Today's lecture – gravitational forces

   gravitational potentials

   satellite motion

Newton's law of gravitation: $$\mathbf{F}_{12} = \frac{Gm_1 m_2 \hat{\mathbf{r}}_{12}}{r_{12}^2}$$

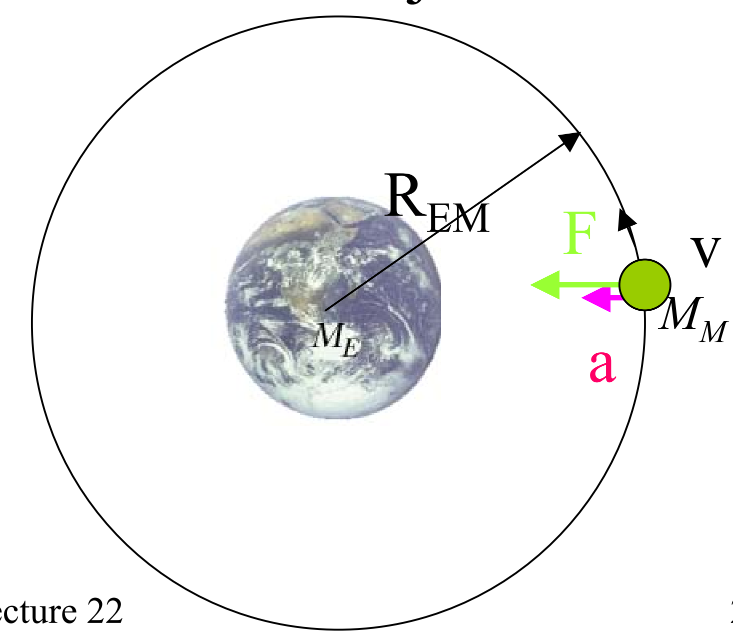
Earth's gravity:



$$F = \frac{GM_E m}{R_E^2}$$

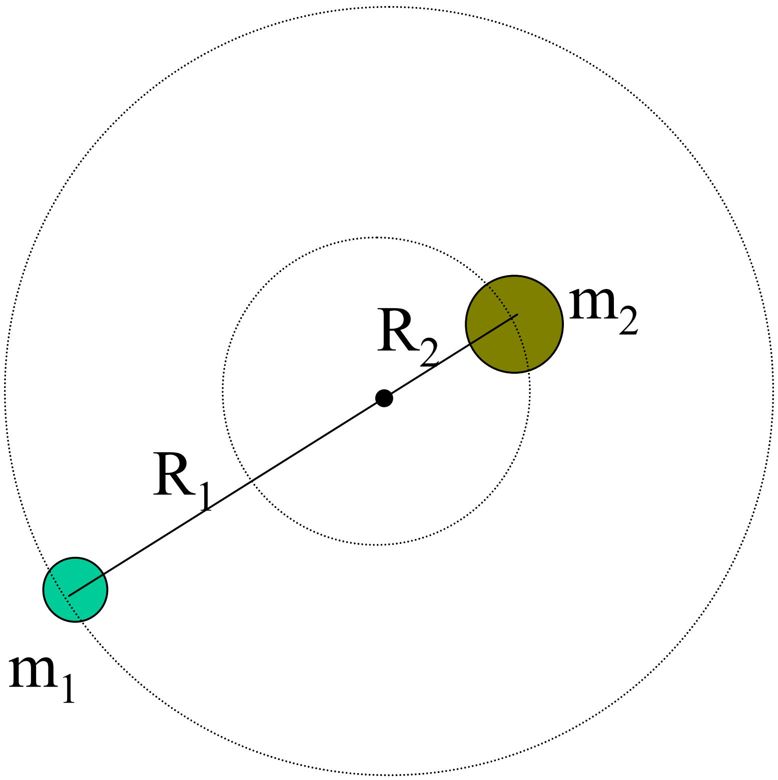
$$\Rightarrow g = \frac{GM_E}{R_E^2} = \frac{6.67\times10^{-11}\cdot5.98\times10^{24}}{(6.37\times10^{6})^2}\,\text{m/s}^2 = 9.8\text{m/s}^2$$

Stable circular orbits of gravitational attracted objects:



What is the physical basis for stable circular orbits?

1. Newton's second law? $\mathbf{F} = m\mathbf{a} = \frac{d\mathbf{p}}{dt}$
2. Conservation of mechanical energy? $E = K + U = \text{(const)}$
3. Conservation of linear momentum? $\mathbf{p} = \text{(const)}$
4. Torqued motion? $\tau = I\,\alpha$?
5. Conservation of angular momentum? $\mathbf{L} = \text{(const)}$



Radial forces on m_1:

$$F_{r1} = \frac{G m_1 m_2}{(R_1 + R_2)^2} = m_1 a_{r1} = m_1 \frac{v_1^2}{R_1}$$

$$v_1 = \frac{2\pi R_1}{T_1}$$

$$T_1 = 2\pi \sqrt{\frac{R_1 (R_1 + R_2)^2}{G m_2}}$$

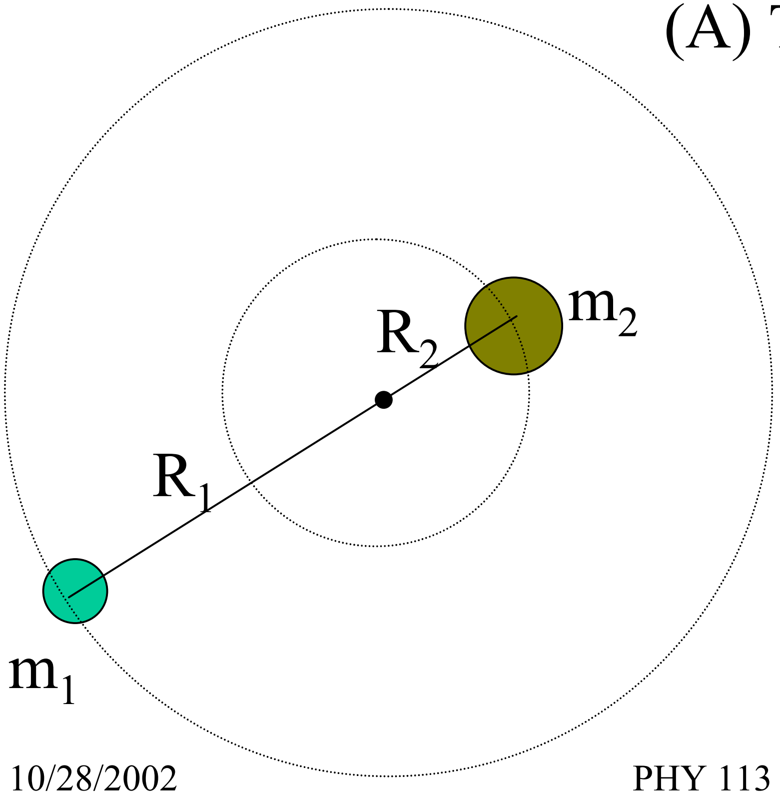
T_2 ?

Tangential forces ?

Peer instruction question

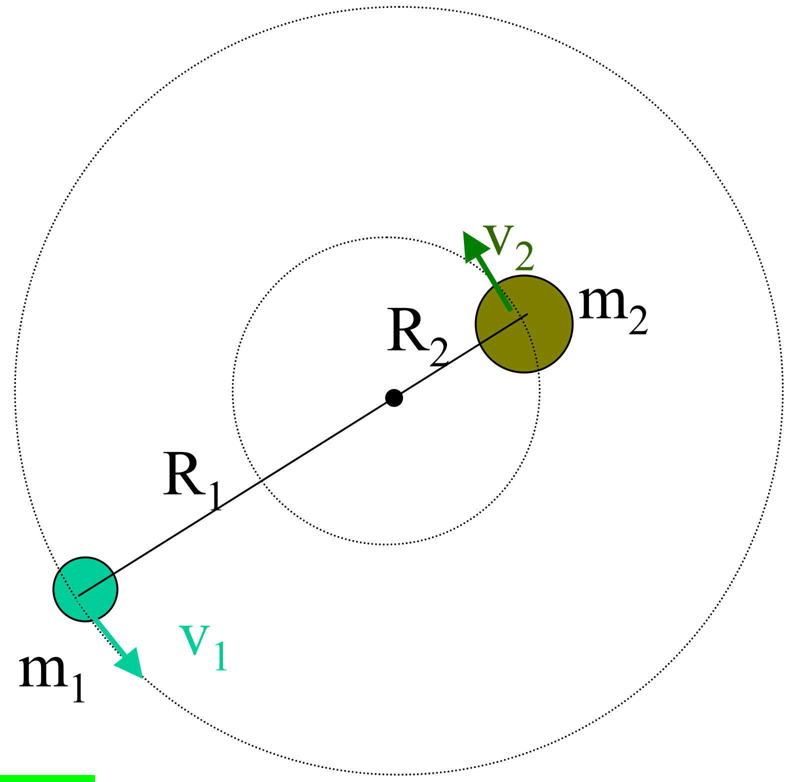
What is the relationship between the periods T_1 and T_2 of the two gravitationally attracted objects rotating about their center of mass? (Assume that $m_1 < m_2$.)

(A) $T_1=T_2$ (B) $T_1<T_2$ (C) $T_1>T_2$



$$\boldsymbol{\tau} = \frac{d\mathbf{L}}{dt} = 0$$

$$\Rightarrow \mathbf{L} = (\text{const})$$



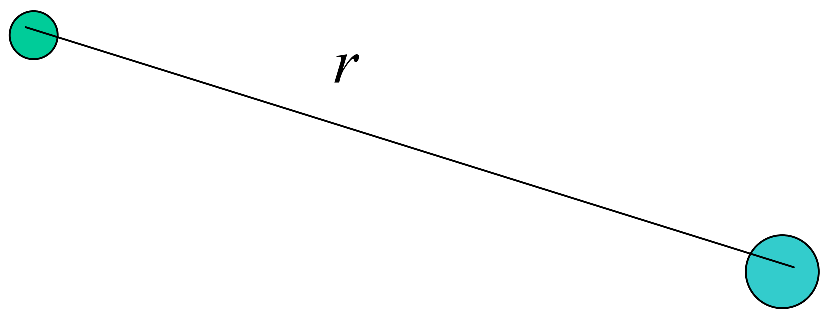
$L_1 = m_1 v_1 R_1$

$L_2 = m_2 v_2 R_2$

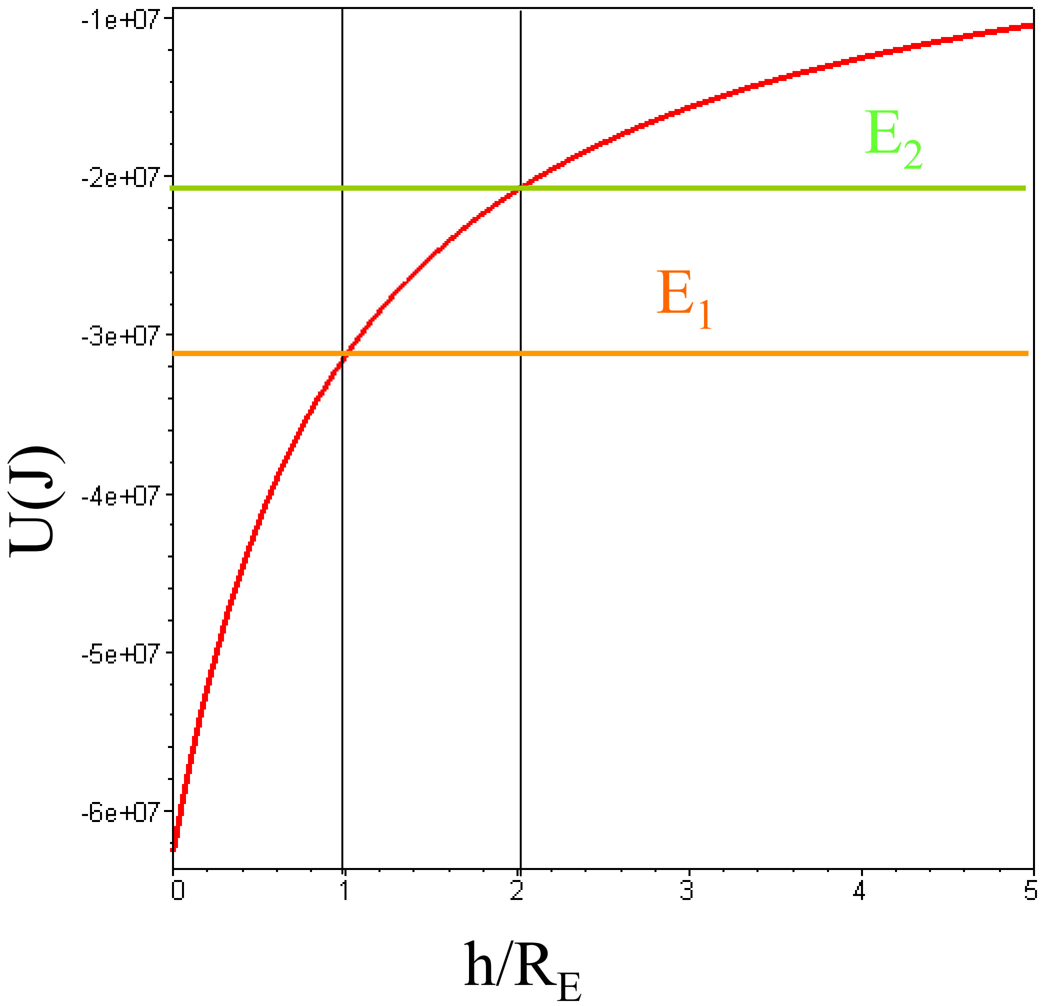
Question:

How are the magnitudes of L_1 and L_2 related?

The potential energy associated with the gravitational force.



$$U(r) = -\int_{r_{ref}}^{r} \frac{Gm_1m_2}{r^2}dr = -\int_{\infty}^{r} \frac{Gm_1m_2}{r^2}dr = -\frac{Gm_1m_2}{r}$$



Total mechanical energy for circular orbits:

(assume M >> m)

$$E = \tfrac{1}{2} m v^2 + U(r)$$

$$\frac{mv^2}{r} = \frac{GMm}{r^2} \quad \Rightarrow v^2 = \frac{GM}{r}$$

$$U(r) = -\frac{GMm}{r}$$

$$E = -\frac{GMm}{2r}$$

Peer instruction question

What is wrong with the previous analysis?

A. Nothing is wrong. (The description of circular motion due to gravitational attraction is complete.)

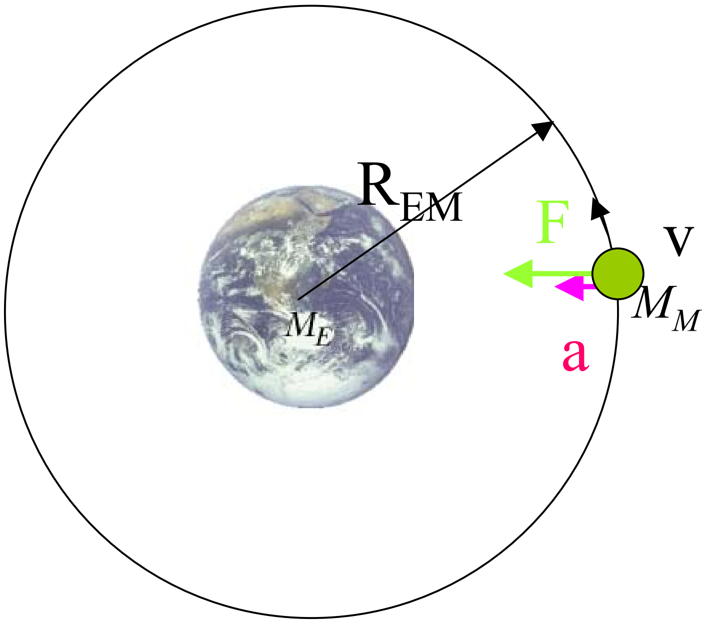
B. E depends on r and therefore must not be constant.

C. E can only be constant if r is constant, but it is not obvious why r is constant.

D. Conservation of angular moment will come to the rescue.

Angular momentum: **L** = **r** x m**v**

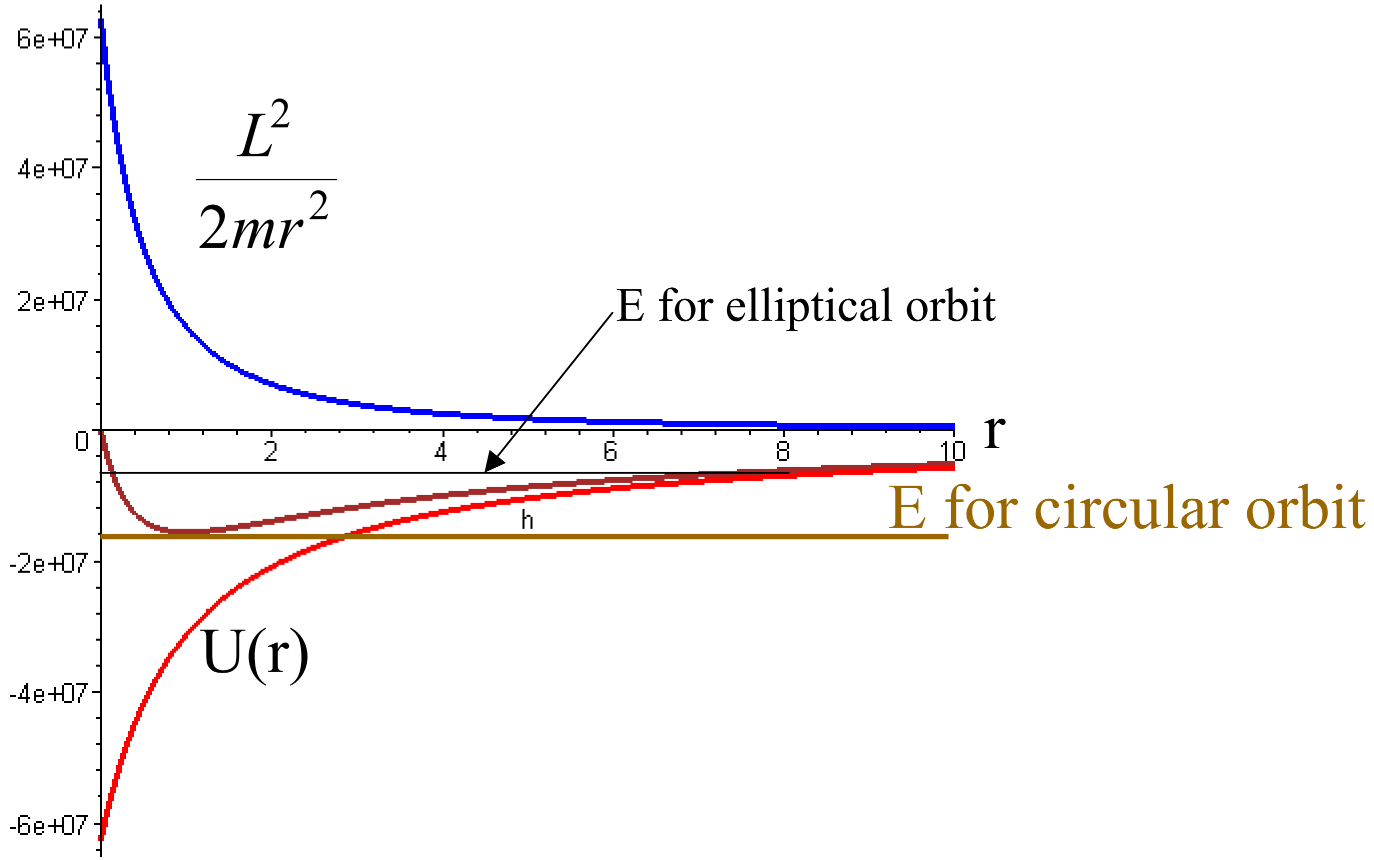
$$L = R_{ME} M_M v$$



$$v = \frac{L}{M_M R_{EM}}$$

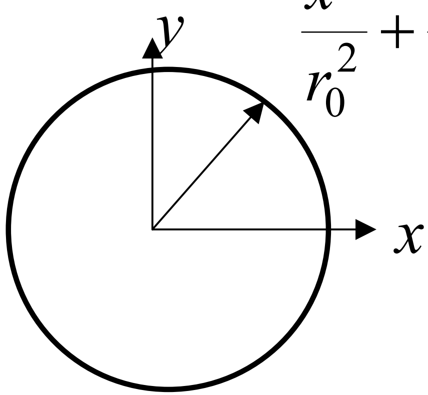
$$K = \tfrac{1}{2} M_M v^2$$

$$= \frac{L^2}{2 M_M R_{EM}^2}$$


6e+07
4e+07
2e+07
0
-2e+07
-4e+07
-6e+07
2
4
6
8
10
r
h
$\frac{L^2}{2mr^2}$
E for elliptical orbit
E for circular orbit
U(r)

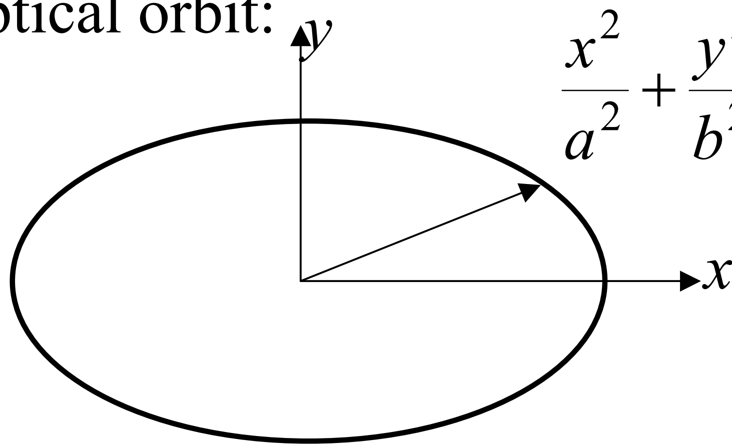
Circular orbit:

$$\frac{x^2}{r_0^2}+\frac{y^2}{r_0^2}=1$$



Elliptical orbit:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$



Satellites orbiting earth (approximate circular orbit):

$$T = 2\pi\sqrt{\frac{R_E^3}{GM_E}}\left(1+h/R_E\right)^{3/2} = 5058\left(1+h/R_E\right)^{3/2} s$$

$R_E \sim 6370$ km

Examples:

Satellite	h (km)	T (hours)
Geosynchronous	35790	~24
NOAA (polar orbitor)	800	~1.7
Hubble	600	~1.6