

## Announcements

1. Summarize results on gravitational force law and satellite motion
2. Fluid mechanics

Satellites orbiting earth (approximately circular orbits):

$$T = 2\pi \sqrt{\frac{R_E^3}{GM_E}} (1 + h/R_E)^{3/2} = 5058(1 + h/R_E)^{3/2} s$$

$$R_E \sim 6370 \text{ km}$$

Examples:

Satellite	h (km)	T (hours)	v (mi/h)
Geosynchronous	35790	~24	6900
NOAA polar orbitor	800	~1.7	16700
Hubble	600	~1.6	16900
Inter. space station*	390	~1.5	17200

\*Link: <http://liftoff.msfc.nasa.gov/temp/StationLoc.html>

## Sample question:

Suppose that the space shuttle ( $m=10^5\text{kg}$ ) was initially in the same orbit as the International space station ( $h_i=390\text{km}$ ) and the engines are fired to give it exactly the amount of energy  $\Delta W$  to raise it to the same orbit as the Hubble space telescope ( $h_f=600\text{km}$ ). What is the energy  $\Delta W$ ?

You can show that the energy of a satellite of mass  $m$  in a circular orbit of height  $h$  above the Earth's surface is given by:

$$E_{mech} = K + U = -\frac{GM_E m}{2(R_E + h)}$$

$$\Delta W = -\frac{GM_E m}{2R_E} \left( \frac{1}{\left(1 + \frac{h_f}{R_E}\right)} - \frac{1}{\left(1 + \frac{h_i}{R_E}\right)} \right) = 89,000 J$$

## The physics of fluids.

- Fluids include liquids (usually “incompressible”) and gases (highly “compressible”).
- Fluids obey Newton’s equations of motion, but because they move within their containers, the application of Newton’s laws to fluids introduces some new forms.

➤ Pressure:  $P = \text{force/area}$        $1 \text{ (N/m}^2\text{)} = 1 \text{ Pascal}$

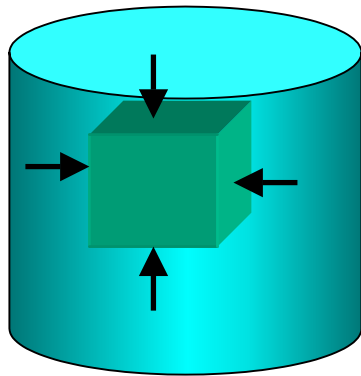
➤ Density:  $\rho = \text{mass/volume}$        $1 \text{ kg/m}^3 = 0.001 \text{ gm/ml}$

## Pressure exerted by air at sea-level

$$1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$$

Example: What is the force exerted by 1 atm of air pressure on a circular area of radius 0.08m?

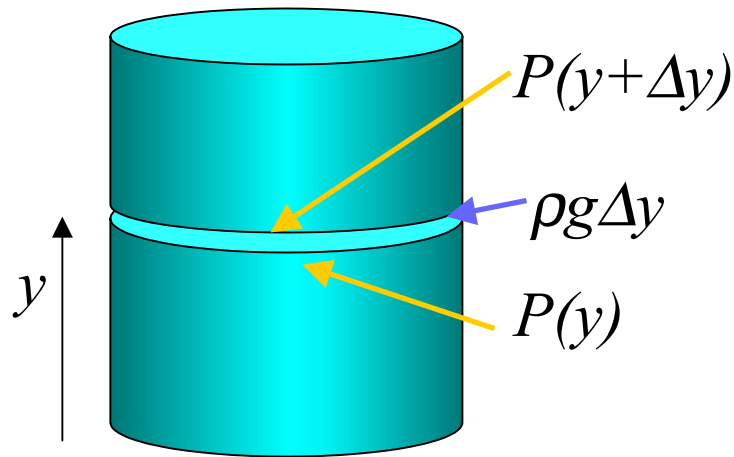
$$\begin{aligned} F &= PA = 1.013 \times 10^5 \text{ Pa} \times \pi(0.08\text{m})^2 \\ &= 2040 \text{ N} \end{aligned}$$



Pressure exerted by a fluid acts in all directions.

Density:  $\rho$  = mass/volume

Effects of the weight of a fluid on pressure.



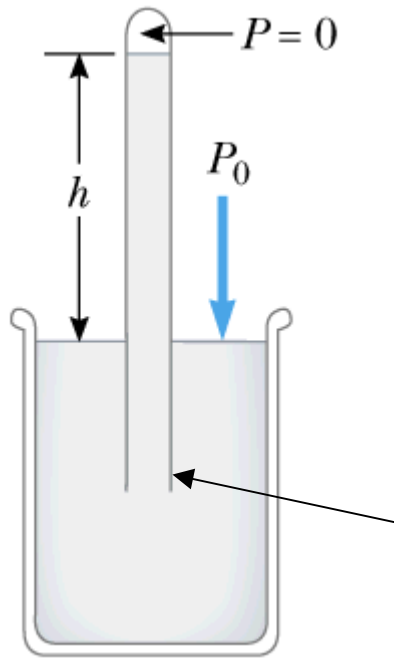
$$P(y) = P(y + \Delta y) + \rho g \Delta y$$
$$\Rightarrow \frac{dP}{dy} = -\rho g$$

Note: In this formulation  $+y$  is defined to be in the **up** direction.

For an “incompressible” fluid (such as water):

$$\frac{dP}{dy} = -\rho g \quad \Rightarrow \quad P = P_0 - \rho g(y - y_0)$$

Example:



$$h = y - y_0 = \frac{P_0}{\rho g}$$

$$= \frac{1.013 \times 10^5 \text{ Pa}}{13.595 \times 10^3 \text{ kg/m}^3 \cdot 9.8 \text{ m/s}^2}$$
$$= 0.76 \text{ m}$$

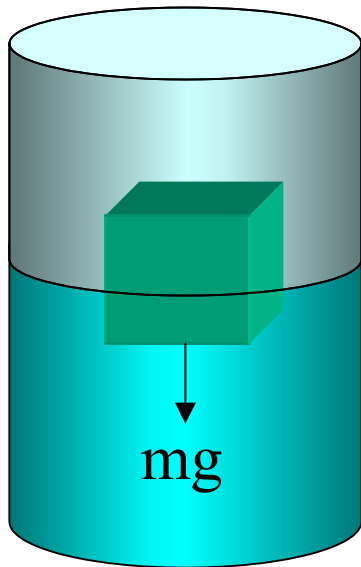
$$\rho = 13.595 \times 10^3 \text{ kg/m}^3$$

Buoyant force for fluid acting on a solid:

$$F_B = \rho_{\text{fluid}} V_{\text{displaced}} g$$

$$P(y + \Delta y) = P(y) - \rho g \Delta y$$

$$F_B = \{P(y) - P(y + \Delta y)\} A = \rho g \Delta y A = \rho g V$$



$$F_B - mg = 0$$

$$\rho_{\text{fluid}} V_{\text{submerged}} g - \rho_{\text{solid}} V_{\text{solid}} g = 0$$

$$\frac{V_{\text{submerged}}}{V_{\text{solid}}} = \frac{\rho_{\text{solid}}}{\rho_{\text{fluid}}}$$



## Peer instruction question

Suppose that a ball floats in water ( $\rho_w=1000\text{kg/m}^3$ ) with 90% of it submerged. What will happen if we pour oil ( $\rho_o=800\text{kg/m}^3$ ) on top of the ball and water?

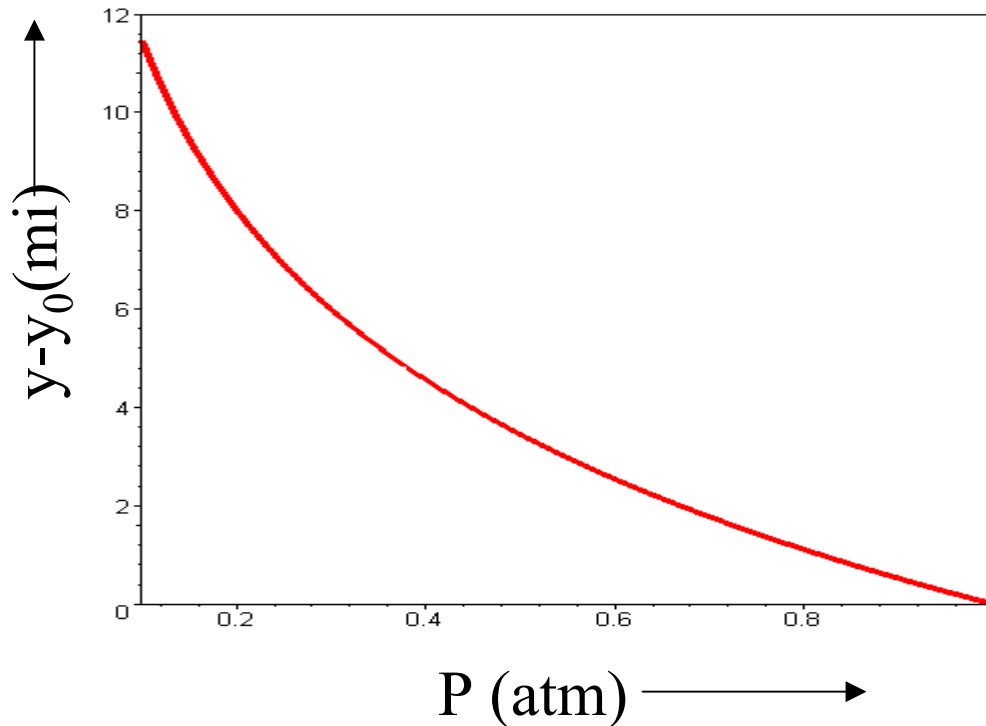
- (A) The ball will sink in the water so that more than 90% of it is submerged in the water.
- (B) The ball will be raised toward the oil so that less than 90% of it is submerged in the water.
- (C) The ball will not be effected by the oil.
- (D) Not enough information is given.

Effects of the weight of a “compressible” fluid on pressure.

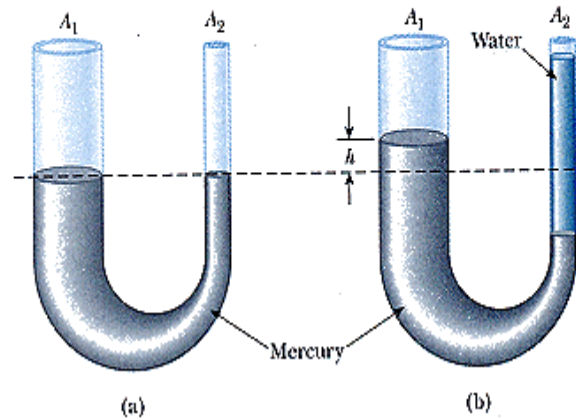
$$\frac{dP}{dy} = -\rho g$$

Solution :

For a gas,  $\rho = P \frac{\rho_0}{P_0}$        $P(y) = P_0 e^{-\frac{\rho_0 g}{P_0}(y-y_0)} \approx P_0 e^{-\frac{y-y_0}{8000m}} \approx P_0 e^{-\frac{y-y_0}{5mi}}$



Mercury is poured into a U-tube as in Figure P15.18a. The left arm of the tube has a cross-sectional area  $A_1$  of  $93.0 \text{ cm}^2$ , and the right arm has a cross-sectional area  $A_2$  of  $5.50 \text{ cm}^2$ . One hundred grams of water are then poured into the right arm, as in Figure P15.18b.



- (a) Determine the length of the water column in the right arm of the U-tube.
- (b) Given that the density of mercury is  $13.6 \text{ g/cm}^3$ , what distance,  $h$ , does the mercury rise in the left arm?