Announcements

1. Third hour exam on Wednesday Nov. 6, 2002
   Focusing on Chapters 13-15 (Simple harmonic motion,
      gravitational force law, and fluid mechanics)
   5 problems (may choose 1 out of 3 problems for #5)
   Problem review sessions: Monday 6 PM, Tuesday 5:30 PM
   Bring to the exam:
      Clear head
      Calculator
      Equation sheet (8 ½” x 11”)
2. On Friday – start Chapter 19 -- temperature, heat, thermodynamics
3. Wave motion (Chapters 16-18 will be covered after Thanksgiving.

Hooke's law

$$F_s = -k(x-x_0)$$

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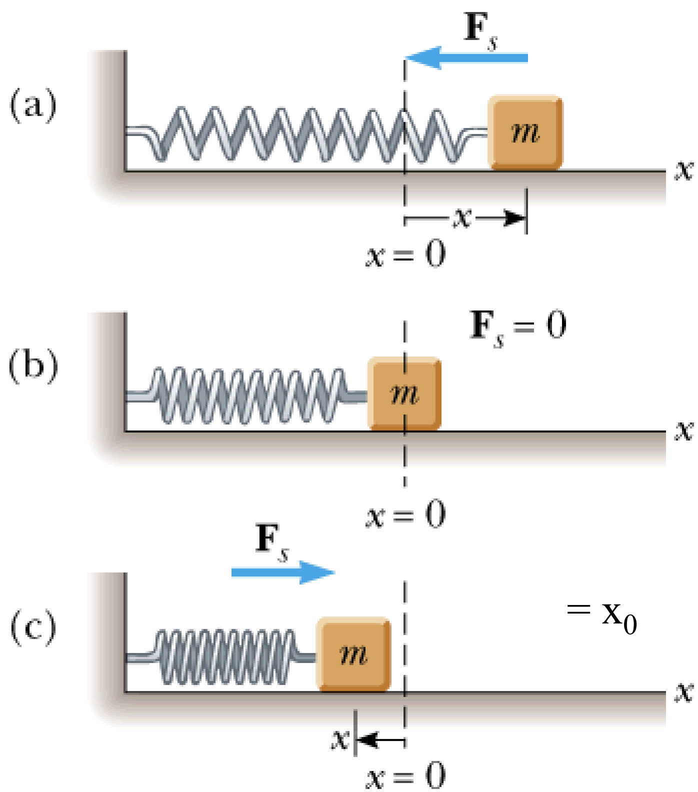
Figure 13.1



$$F_s = -k(x-x_0) = m\frac{d^2(x-x_0)}{dt^2}$$

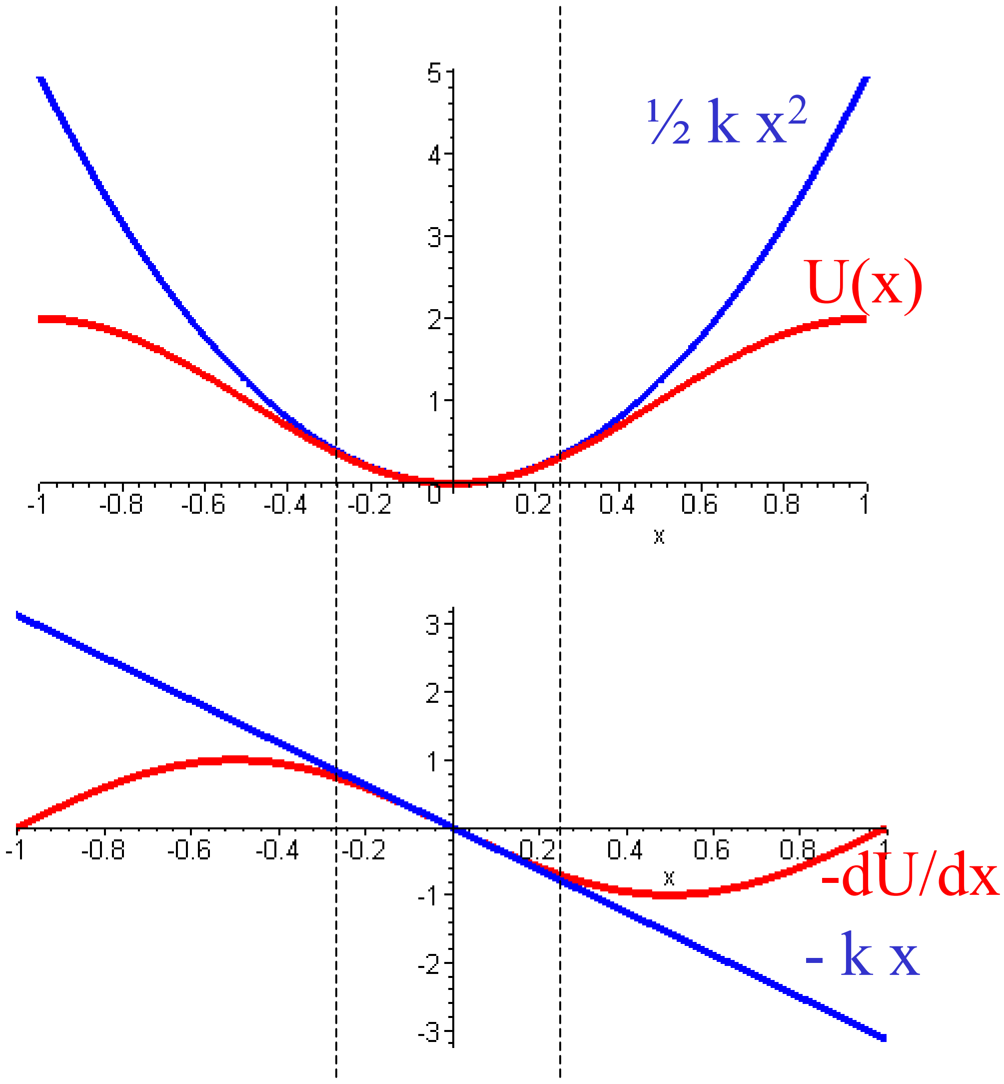
$$\frac{d^2(x-x_0)}{dt^2} = -\frac{k}{m}(x-x_0)$$

Solutions:

$$x(t) = x_0 + A\cos(\omega t + \varphi)$$

$$x(t) = x_0 + A\cos(\omega t) + B\sin(\omega t)$$

were: $\omega = \sqrt{k/m}$



½ k x2
U(x)
-dU/dx
- k x

Simple harmonic motion:

$$F = -k(x - x_0) = m\frac{d^2(x - x_0)}{dt^2}$$

$$\frac{d^2(x - x_0)}{dt^2} = -\frac{k}{m}(x - x_0)$$

Conveniently evaluated in *radians*

$$x(t) = x_0 + A\cos(\omega t + \varphi); \quad \omega = \sqrt{\frac{k}{m}}$$

Constants

Note that:

$$v(t) = \frac{dx}{dt} = -A\omega\sin(\omega t + \varphi)$$

$$a(t) = \frac{dv}{dt} = -A\omega^2\cos(\omega t + \varphi)$$

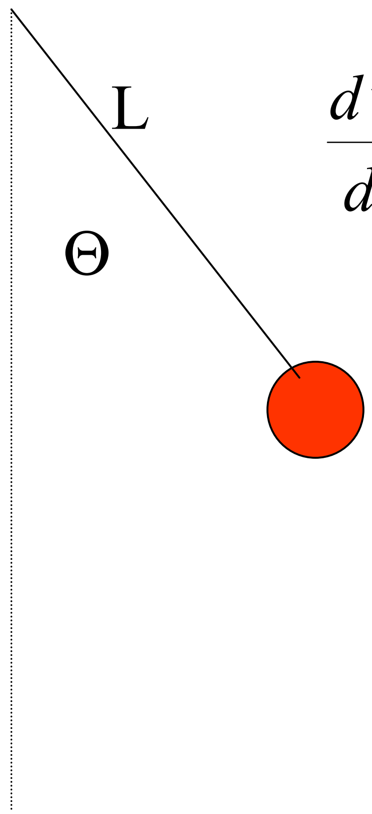
$$E = K + U$$

$$= \tfrac{1}{2} m\omega^2 A^2$$

$$= \tfrac{1}{2} kA^2$$

Simple harmonic motion for a pendulum:

$$\tau = mgL\sin\Theta = -I\alpha = -I\,\frac{d^2\Theta}{dt^2}$$



$$\frac{d^2\Theta}{dt^2} = -\frac{mgL}{I}\sin\Theta = -\frac{g}{L}\sin\Theta \quad (\text{since } I = mL^2)$$

Approximation for small Θ:

$$\sin\Theta \approx \Theta$$

$$\Rightarrow \frac{d^2\Theta}{dt^2} = -\frac{g}{L}\Theta$$

Solution :

$$\Theta(t) = A\cos(\omega t + \varphi); \quad \omega = \sqrt{\frac{\mathrm{g}}{\mathrm{L}}}$$

Motion with both Hooke's law and oscillatory "driving" force:

Suppose $F=-kx+F_0 \sin(\Omega t)$

According to Newton:

$$-kx + F_0 \sin(\Omega t) = m\frac{d^2x}{dt^2}$$

Differential equation ("inhomogeneous"):

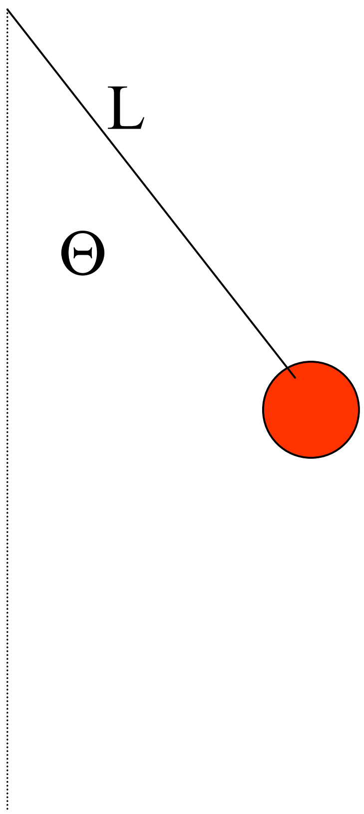
$$\frac{d^2x}{dt^2} = -\frac{k}{m}x + \frac{F_0}{m}\sin(\Omega t)$$

Solution:

$$x(t) = \frac{F_0/m}{k/m - \Omega^2}\sin(\Omega t) + A\cos(\omega t + \varphi)$$

Pendulum example:

Suppose L=2m, what is the period of the pendulum?

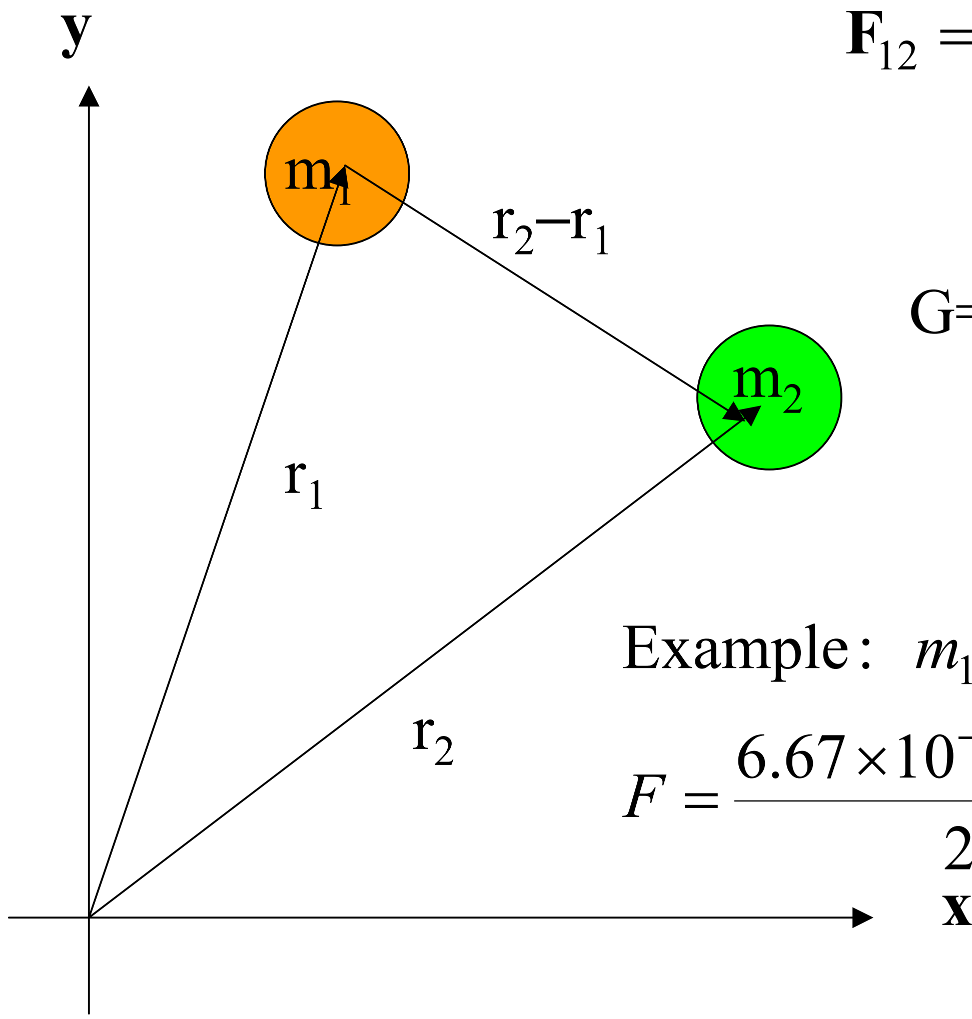


$$\omega = \sqrt{\frac{g}{L}} = \sqrt{\frac{9.8\text{m/s}^2}{2\text{m}}} = 2.2135\ \text{rad/s} = \frac{2\pi}{T}$$

$$T = \frac{2\pi}{\omega} = 2.84\ \text{s}$$

$$\Theta(t) = A\cos(\omega t + \varphi); \quad \omega = \sqrt{\frac{g}{L}}$$

Newton's law of gravitation: m_2 *attracts* m_1 according to:



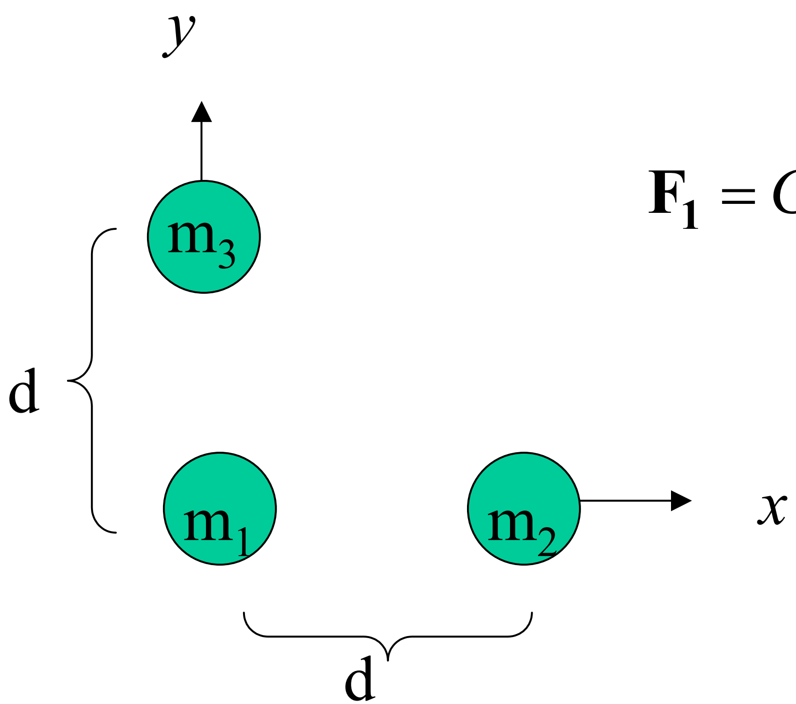
$$\mathbf{F}_{12} = \frac{G m_1 m_2 \hat{\mathbf{r}}_{12}}{r_{12}^2}$$

G=6.67 x 10^{-11} N m^2/kg^2

$$\text{Example: } \quad m_1 = m_2 = 70\,\text{kg}; \quad r_{12} = 2\,\text{m}:$$

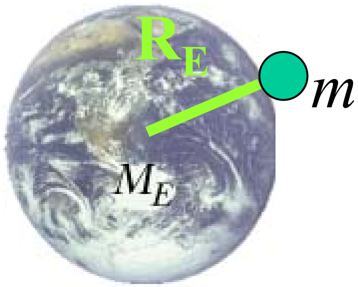
$$F = \frac{6.67\times 10^{-11}\cdot 70\cdot 70}{2^2} N = 8.17\times 10^{-8} N$$

Vector nature of Gravitational law:



$$\mathbf{F_1} = G\frac{m_1}{d^2}\left(m_2\mathbf{i} + m_3\mathbf{j}\right)$$

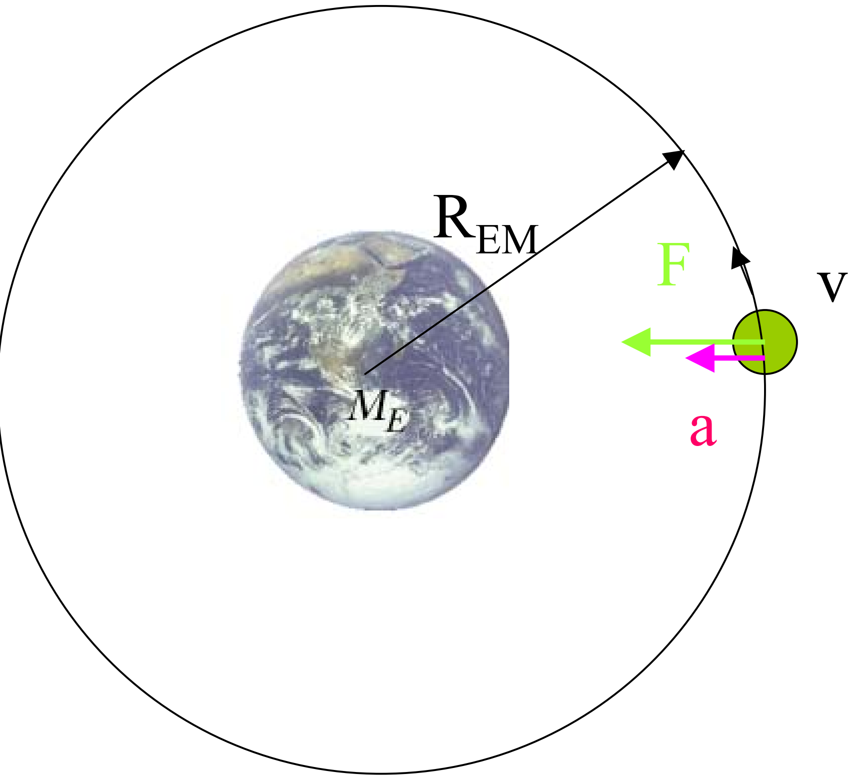
Gravitational force of the Earth



$$F = \frac{GM_E m}{R_E^2}$$

$$\Rightarrow g = \frac{GM_E}{R_E^2} = \frac{6.67\times 10^{-11}\cdot 5.98\times 10^{24}}{(6.37\times 10^{6})^2}\,\mathrm{m/s^2} = 9.8\mathrm{m/s^2}$$

Stable circular orbit of two gravitationally attracted objects (such as the moon and the Earth)



$$a = \frac{v^2}{R_{EM}} = \frac{GM_E}{R_{EM}^2}$$

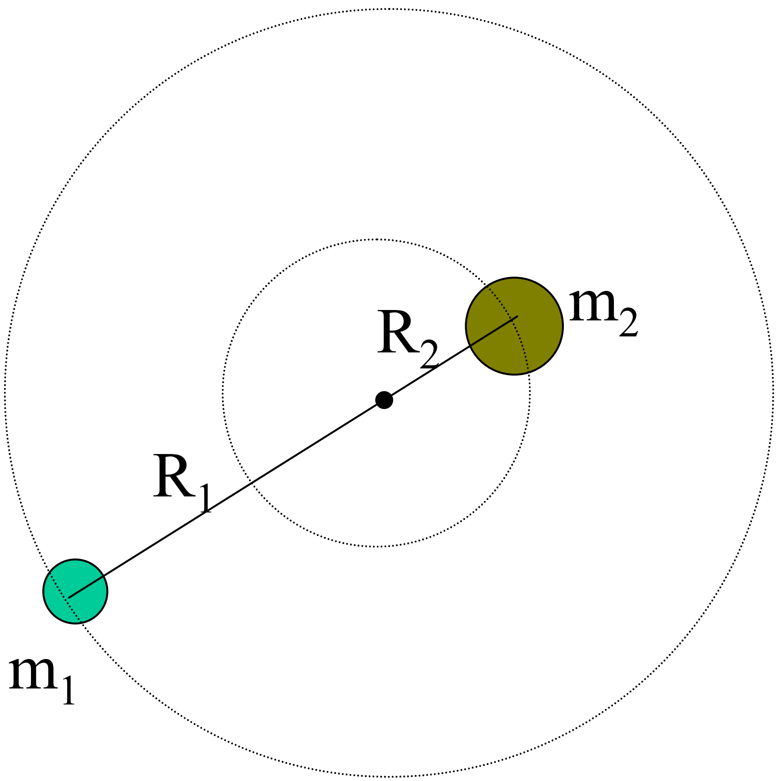
$$v = \omega R_{EM} = \frac{2\pi}{T} R_{EM}$$

$$T = 2\pi \sqrt{\frac{R_{EM}^3}{GM_E}}$$

$$= 2\pi \sqrt{\frac{(3.84 \times 10^8)^3}{6.67 \times 10^{-11} \cdot 5.98 \times 10^{24}}}$$

$$= 2367353.953 \text{ s} = 27.4 \text{ days}$$

Analysis of two masses in stable circular orbits about their center of mass:



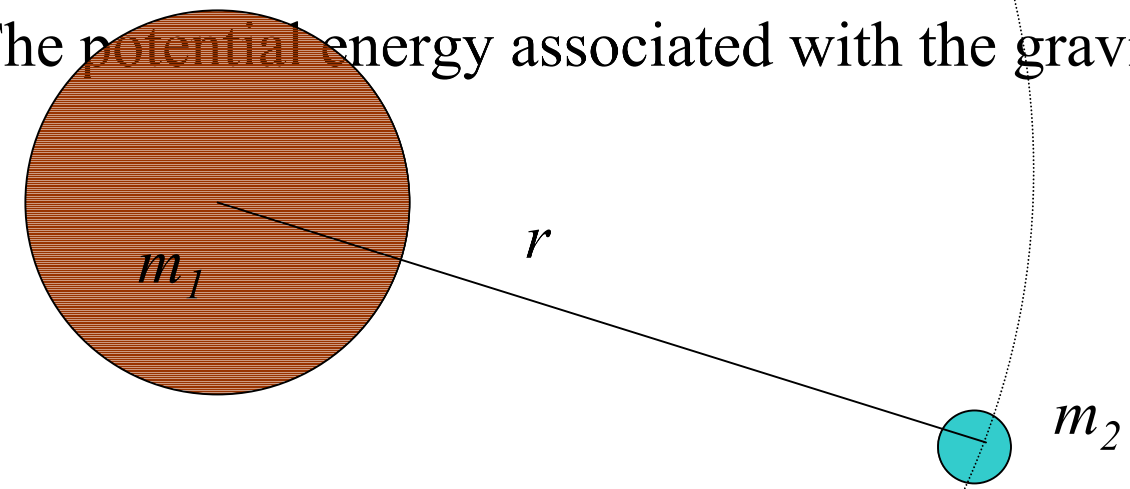
Radial forces on m$_1$:

$$F_{r1} = \frac{Gm_1m_2}{(R_1 + R_2)^2} = m_1 a_{r1} = m_1 \frac{v_1^2}{R_1}$$

$$v_1 = \frac{2\pi R_1}{T_1}$$

$$T_1 = 2\pi \sqrt{\frac{R_1 (R_1 + R_2)^2}{Gm_2}}$$

The potential energy associated with the gravitational force.



For circular orbit :

$$v_r \equiv 0 \text{ and } \frac{v_\theta^2}{r} = \frac{Gm_1}{r^2}$$

$$E_{\text{circular orbit}} = -\frac{Gm_1 m_2}{2r}$$

$$U(r) = -\int_{r_{ref}}^{r} \frac{Gm_1 m_2}{r^2} dr = -\int_{\infty}^{r} \frac{Gm_1 m_2}{r^2} dr = -\frac{Gm_1 m_2}{r}$$

Total mechanical energy of m_2 (assuming $m_2 << m_1$):

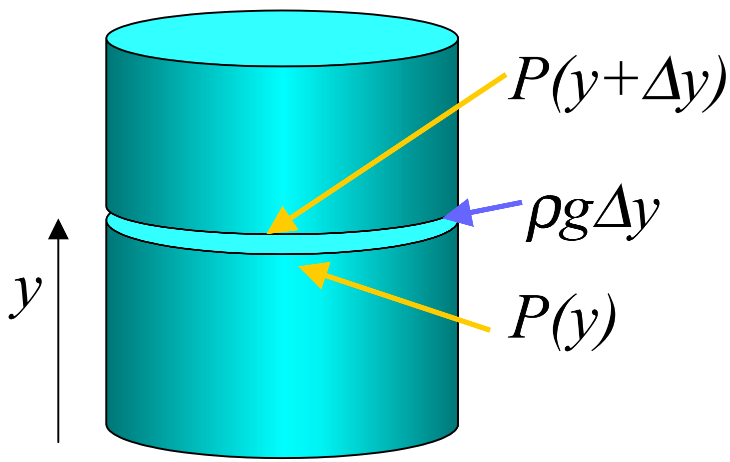
$$E = K + U = \tfrac{1}{2} m_2 v_r^2 + \tfrac{1}{2} m_2 v_\theta^2 - \frac{Gm_1 m_2}{r}$$

The physics of fluids.

- Fluids include liquids (usually “incompressible) and gases (highly “compressible”).
- Fluids obey Newton’s equations of motion, but because they move within their containers, the application of Newton’s laws to fluids introduces some new forms.
    - Pressure: P=force/area 1 (N/m^2) = 1 Pascal
    - Density: ρ =mass/volume 1 kg/m^3 = 0.001 gm/ml

Density: ρ =mass/volume

Effects of the weight of a fluid on pressure.



$$P(y) = P(y + \Delta y) + \rho g \Delta y$$

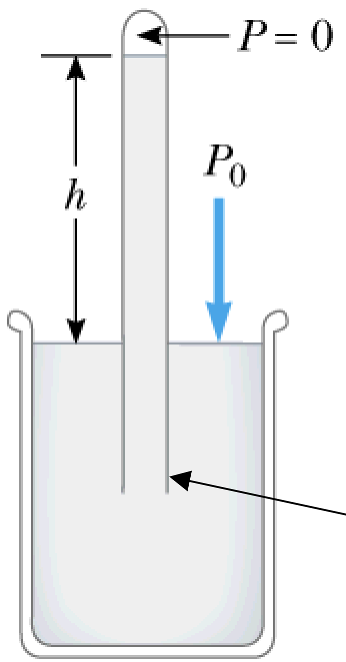
$$\Rightarrow \frac{dP}{dy} = -\rho g$$

Note: In this formulation +y is defined to be in the up direction.

For an "incompressible" fluid (such as water):

$$\frac{dP}{dy} = -\rho g \quad \Rightarrow P = P_0 - \rho g(y - y_0)$$

Example:



$$h = y - y_0 = \frac{P_0}{\rho g}$$

$$= \frac{1.013\times10^5 \text{ Pa}}{13.595\times10^3 \text{ kg/m}^3 \cdot 9.8 \text{ m/s}^2}$$

$$= 0.76 \text{ m}$$

$\rho = 13.595 \times 10^3$ kg/m^3

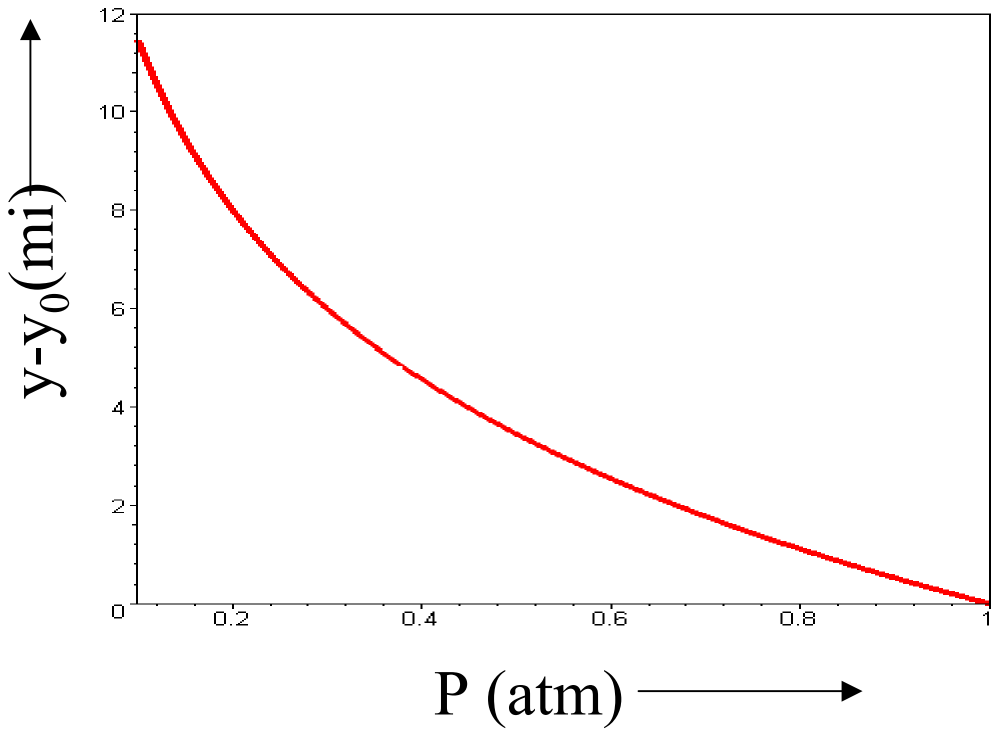
Effects of the weight of a "compressible" fluid on pressure.

$$\frac{dP}{dy} = -\rho g$$

Solution :

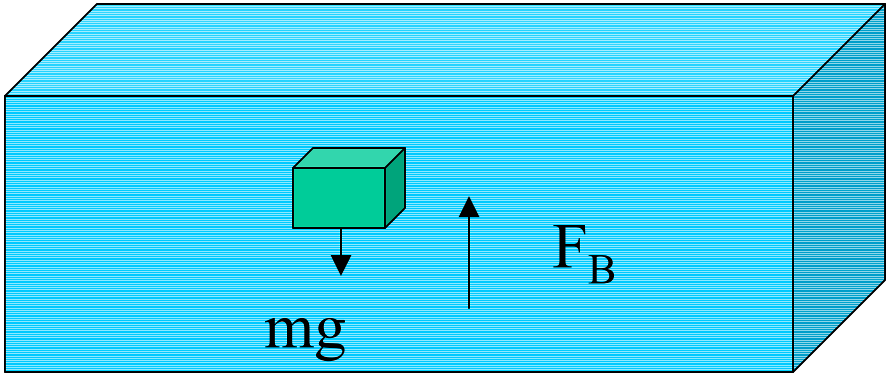
For a gas, $\rho = P\frac{\rho_0}{P_0}$

$$P(y) = P_0 e^{-\frac{\rho_0 g}{P_0}(y-y_0)} \approx P_0 e^{-\frac{y-y_0}{8000m}} \approx P_0 e^{-\frac{y-y_0}{5mi}}$$



Buoyant force for fluid acting on a solid:

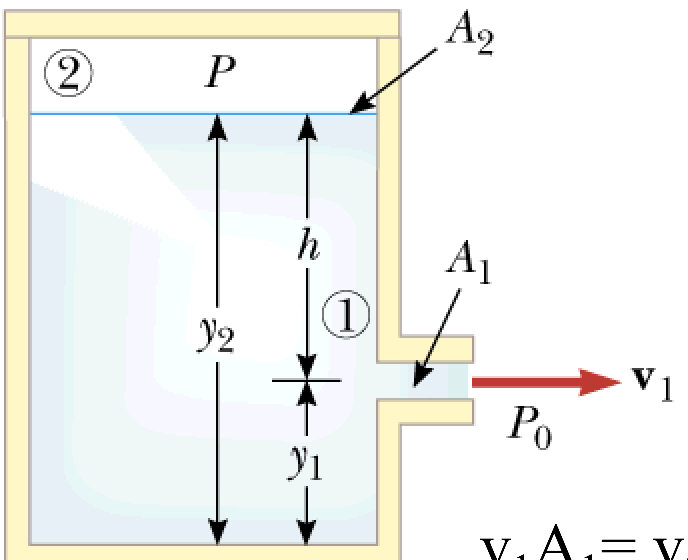
$F_B = \rho_{fluid} V_{displaced} g$



Bernoulli's equation:

$$P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g h_2 = P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g h_1$$

Examples:



$$P + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 =$$

$$P_0 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1$$

$$v_1 = \sqrt{\frac{2\left(gh - (P_0 - P)/\rho\right)}{1 - \left(A_1/A_2\right)^2}}$$

$$v_1 A_1 = v_2 A_2$$

Streamline flow of air around an airplane wing:

$$P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g h_2 = P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g h_1$$

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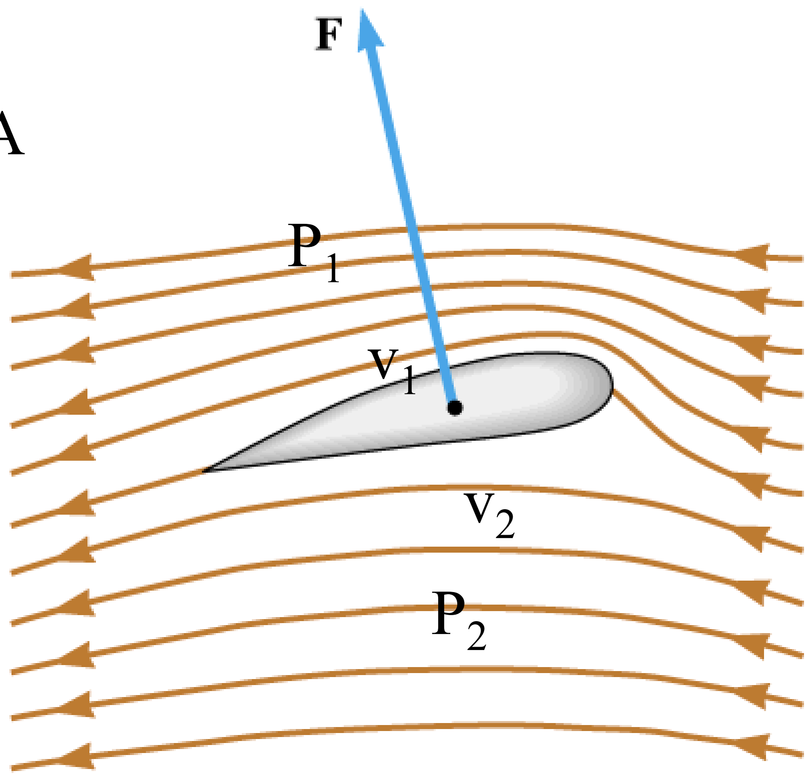
Figure 15.24

$$h_1 \approx h_2$$

$$v_1 > v_2$$

$$F_{lift} = (P_2 - P_1)A$$

$$P_2 = P_1 + \tfrac{1}{2}\rho\left(v_1^2 - v_2^2\right)$$



Example:

$v_1 = 270$ m/s

$v_2 = 260$ m/s

$\rho = 0.6$ kg/m^3

$A = 40$ m^2

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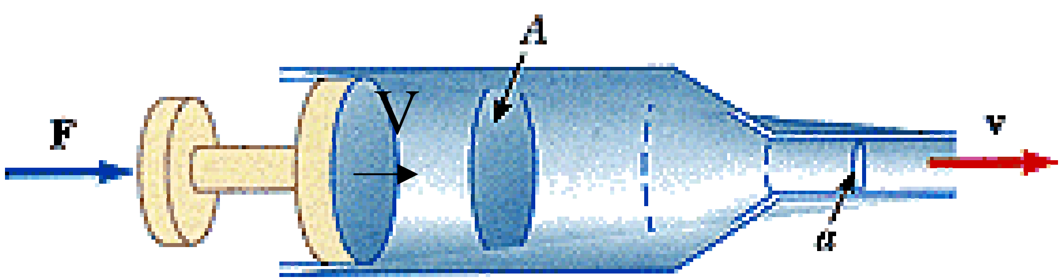
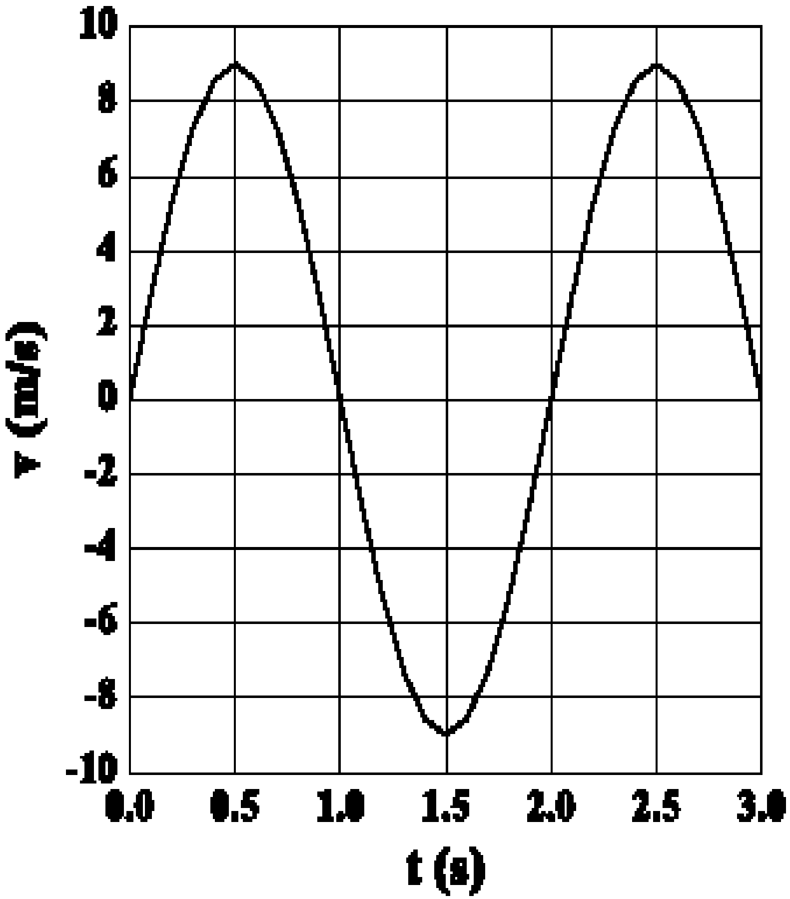
$F_{lift} = 63{,}600$ N

Homework problem: A hypodermic syringe contains a medicine with the density of water. The barrel of the syringe has a cross-sectional area $A=2.5\times10^{-5}m^2$, and the needle has a cross-sectional area $a= 1.0\times10^{-8}m^2$. In the absence of a force on the plunger, the pressure everywhere is 1 atm. A force F of magnitude 2 N acts on the plunger, making the medicine squirt horizontally from the needle. Determine the speed of the medicine as leave the needle's tip.



$$P_0+F/A+1/2\ \rho\ V^2 = P_0+1/2\ \rho\ v^2$$

$$v = \sqrt{\frac{2F/(\rho A)}{\left(1-\left[{}^{a}\!/\!{}_{A}\right]^2\right)}}$$

1. 

The figure on the left shows a graph of the velocity $v(t)$ versus time t of a particle in simple harmonic motion. Find (a) the maximum magnitude of the velocity, (b) the frequency of the motion (using either units of radians/sec or cycles/sec), and (c) the maximum magnitude of the displacement of the particle.