Announcements

1. Second student presentation session – Sunday, 11/17 – 2 PM in Olin 101
2. Practice exams for 4th hour test posted

   www.wfu.edu/~natalie/f02phy113/extrapractice
3. Today's topic – kinetic theory of gasses

2. [SB5 20.P.32.(47870)] A sample of an ideal gas goes through the process shown in Figure P20.32. From *A* to *B*, the process is adiabatic; from *B* to *C*, it is isobaric, with 98 kJ of energy flowing into the system by heat. From *C* to *D*, the process is isothermal; and from *D* to *A*, it is isobaric, with 158 kJ of energy flowing out of the system by heat. Determine the difference in internal energy, $E_{\text{int, B}} - E_{\text{int, A}}$.

[0.1428571] ✗ [52.9] kJ

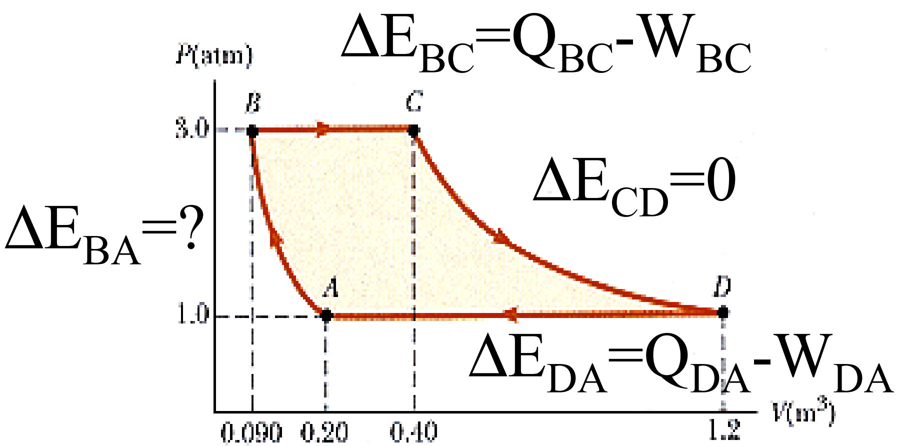


Figure P20.32.

First law of thermodynamics

$$\Delta E_{int} = Q - W$$

$$W = \int_{V_i}^{V_f} P dV$$

For an "ideal gas" we can write an explicit relation for E_{int}.

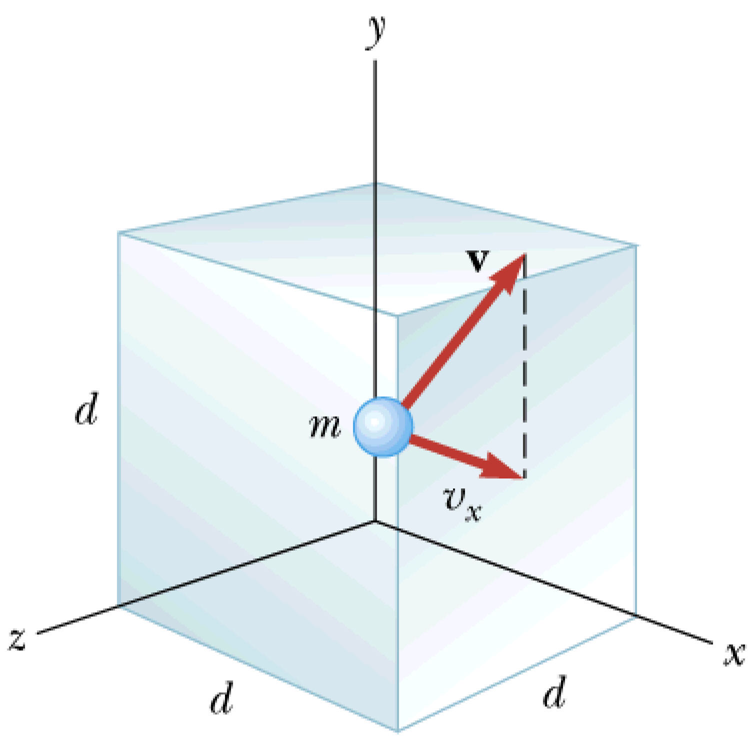
What we will show:

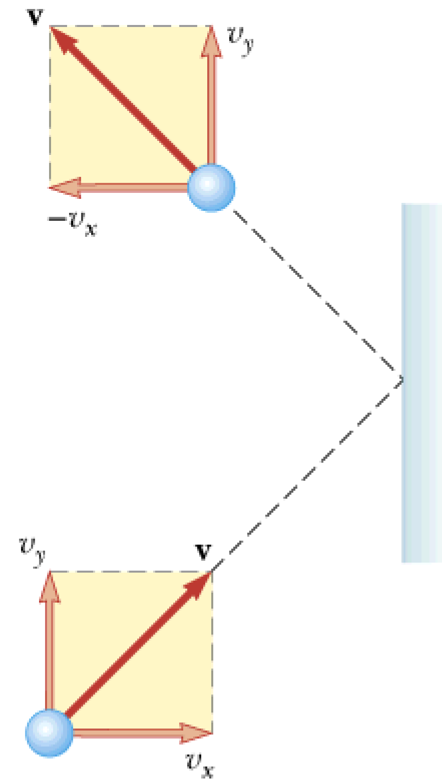
$$\boxed{E_{int}^{(\text{ideal gas})} = \frac{n}{\gamma\text{-}1} RT = \frac{N}{\gamma\text{-}1} k_B T}$$

γ is a parameter which depends on the type of gas (monoatomic, diatomic, etc.) which can be measured as the ratio of two heat capacities: $\gamma = C_P/C_V$.

Ideal gas model:

Each atom is represented as a tiny hard sphere of mass *m* with velocity **v**. Collisions and forces between atoms are neglected. Collisions with the walls of the container are assumed to be elastic.





What we can show is the pressure exerted by the atoms by their collisions with the walls of the container is given by:

$$P = \frac{2}{3}\frac{N}{V}\tfrac{1}{2}m\left\langle v^2\right\rangle_{avg} = \frac{2}{3}\frac{N}{V}\left\langle K\right\rangle_{avg}$$

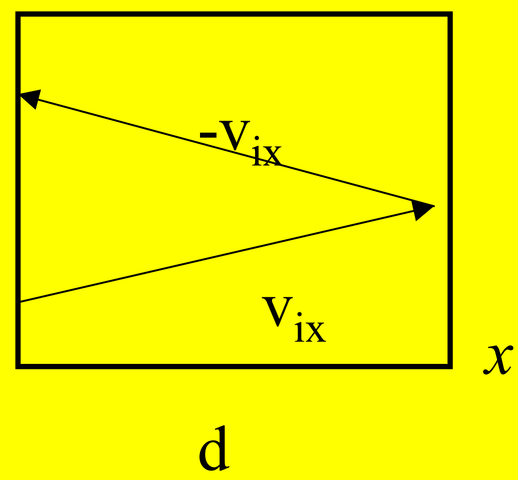
Proof:

Force exerted on wall perpendicular to x-axis by an atom which collides with it:

$$F_{ix} = -\frac{\Delta p_{ix}}{\Delta t} = \frac{2mv_{ix}}{\Delta t} \qquad \Delta t \approx 2d / v_{ix}$$

$$\Rightarrow F_{ix} \approx \frac{2mv_{ix}}{2d / v_{ix}} = \frac{mv_{ix}^2}{d}$$

$$P = \sum_i \frac{F_{ix}}{A} = \sum_i \frac{mv_{ix}^2}{V} = \frac{N}{V}m\left\langle v_x^2\right\rangle$$



Ideal gas law continued:

Recall that $PV = nRT = N\frac{R}{N_A}T = Nk_BT$

$$\Rightarrow \frac{2}{3}N\left\{\frac{1}{2}m\langle v^2\rangle\right\} = Nk_BT$$

Therefore: $\left\{\frac{1}{2}m\langle v^2\rangle\right\} = \frac{3}{2}k_BT$

$$\Rightarrow v_{rms} = \sqrt{\langle v^2\rangle} = \sqrt{\frac{3k_BT}{m}}$$

Also: $\left\{\frac{1}{2}m\langle v^2\rangle\right\} = \frac{3}{2}k_BT$

$$\Rightarrow E_{\text{int}} = N\left\{\frac{1}{2}m\langle v^2\rangle\right\} = N\frac{3}{2}k_BT$$

Big leap!

Internal energy of an ideal gas:

$$E_{\text{int}} = N\left\{\frac{1}{2}m\langle v^2\rangle\right\} = N\frac{3}{2}k_B T \Rightarrow \frac{N}{\gamma\text{-}1}k_B T = \frac{n}{\gamma\text{-}1}RT$$

derived for monoatomic ideal gas

more general relation for polyatomic ideal gas

Gas	**γ (theory)**	**γ (exp)**
He	5/3	1.67
N_2	7/5	1.41
H_2O	4/3	1.30

Determination of Q for various processes in an ideal gas:

$$E_{\text{int}} = \frac{n}{\gamma\text{-}1} RT$$

$$\Delta E_{\text{int}} = \frac{n}{\gamma\text{-}1} R\Delta T = Q - W$$

Example: Isovolumetric process – (V=constant ➔ W=0)

$$\Delta E_{\text{int}\,i\to f} = \frac{n}{\gamma\text{-}1} R\Delta T_{i\to f} = Q_{i\to f}$$

In terms of "heat capacity": $Q_{i\to f} = \frac{n}{\gamma\text{-}1} R\Delta T_{i\to f} \equiv nC_V \Delta T_{i\to f}$

$$C_V = \frac{R}{\gamma\text{-}1}$$

Example: Isobaric process (P=constant):

$$\Delta E_{\text{int}\, i\to f} = \frac{n}{\gamma\text{-}1} R\Delta T_{i\to f} = Q_{i\to f} - W_{i\to f}$$

In terms of "heat capacity":

$$Q_{i\to f} = \frac{n}{\gamma\text{-}1} R\Delta T_{i\to f} + P_i\left(V_f - V_i\right) = \frac{n}{\gamma\text{-}1} R\Delta T_{i\to f} + nR\Delta T_{i\to f} \equiv nC_P\Delta T_{i\to f}$$

$$C_P = \frac{R}{\gamma\text{-}1} + R = \frac{\gamma R}{\gamma\text{-}1}$$

Note: $\gamma = C_P/C_V$

Example:

Isothermal process (T=0)

$$E_{\text{int}} = \frac{n}{\gamma\text{-}1} RT$$

$$\Delta E_{\text{int}} = \frac{n}{\gamma\text{-}1} R\Delta T = Q - W$$

➔ $\Delta T=0$ ➔ $\Delta E_{\text{int}} = 0$ ➔ $Q=W$

$$W = \int_{V_i}^{V_f} P dV = nRT \int_{V_i}^{V_f} \frac{dV}{V} = nRT \ln\left(\frac{V_f}{V_i}\right)$$

Example:

Adiabatic process (Q=0)

$$\Delta E_{\text{int}} = -W$$

$$\frac{n}{\gamma\text{-}1} R\Delta T = -P\Delta V$$

$$PV = nRT$$

$$\Delta PV + P\Delta V = nR\Delta T$$

$$nR\Delta T = -(\gamma\text{-}1)P\Delta V = \Delta PV + P\Delta V$$

$$-\gamma \frac{\Delta V}{V} = \frac{\Delta P}{P}$$

$$\Rightarrow -\ln\left(\frac{V_f^\gamma}{V_i^\gamma}\right) = \ln\left(\frac{P_f}{P_i}\right) \quad \Rightarrow P_i V_i^\gamma = P_f V_f^\gamma$$

Peer instruction question

Suppose that an ideal gas expands adiabatically. Does the temperature

(A) Increase (B) Decrease (C) Remain the same

$$P_i V_i^{\gamma} = P_f V_f^{\gamma}$$

$$P_i V_i = nRT_i \Rightarrow P_i = nR\frac{T_i}{V_i}$$

$$T_i V_i^{\gamma\text{-}1} = T_f V_f^{\gamma\text{-}1}$$

$$T_f = T_i\left(\frac{V_i}{V_f}\right)^{\gamma\text{-}1}$$