Announcements

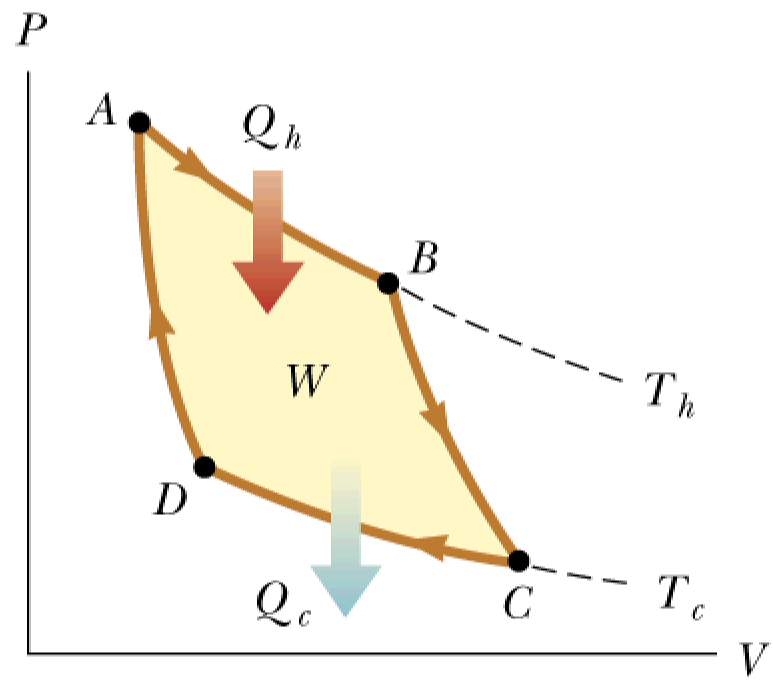
1. Physics colloquium this week (4 PM Thursday) on modeling of protein structures
   www.wfu.edu/physics/seminars/jchen.html
2. Reminder – 4th hour exam – Monday, Nov. 25th
   - Most votes in favor of more time –
     - please email if you would like to start the exam early or when if later
     - **Honor code means that you will *not* discuss the exam with anyone**
   - Extra practice sessions & presentations
3. Today's topic – Carnot cycle, concept of entropy

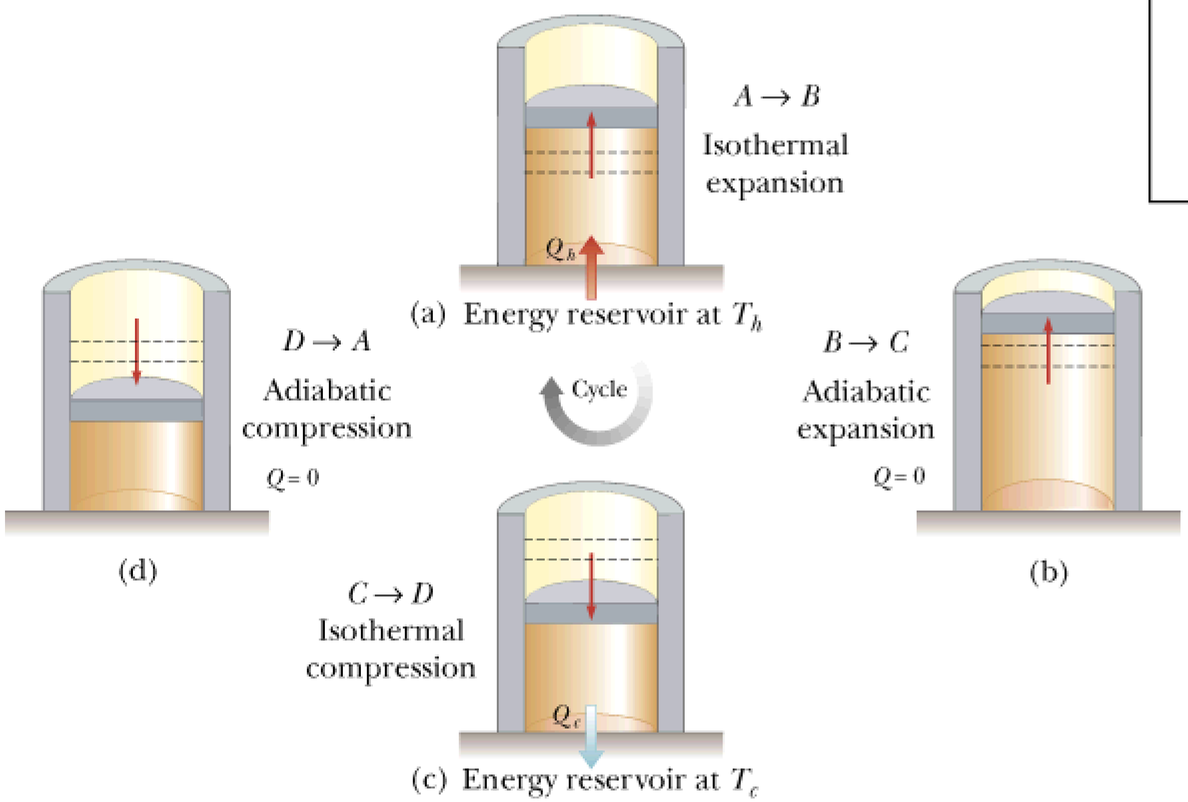
Online Quiz for Lecture 30
Heat Engines and efficiency of cycles -- Nov. 20, 2002

Consider one mole of an ideal gas with $\gamma = 1.3$, initially have temperature, pressure, and volume equal to T_i, P_i, and V_i, respectively.

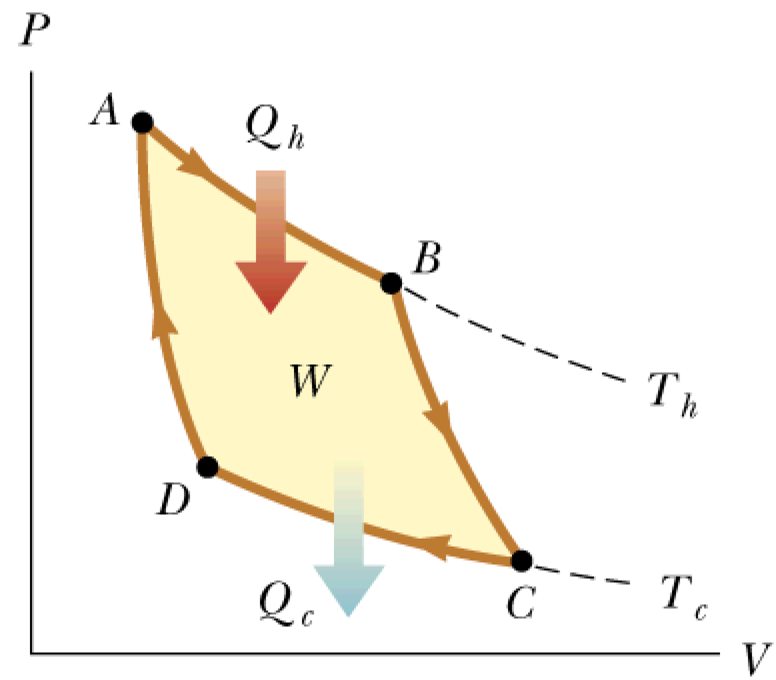
1. If the gas increases its pressure to $4P_i$ at constant volume, what is the final temperature? (a) $T_i/4$ (b) $T_i/2$ (c) T_i (d) $2T_i$ (e) $4T_i$
2. If the gas increases its volume to $4V_i$ at constant pressure, what is the final temperature? (a) $T_i/4$ (b) $T_i/2$ (c) T_i (d) $2T_i$ (e) $4T_i$
3. If the gas increases its volume $4V_i$ adiabatically, what is the final temperature? (a) 0.16 T_i (b) 0.66 T_i (c) T_i (d) 1.52 T_i (e) 6.06 T_i

Carnot process





Carnot cycle



	A→B	B→C	C→D	D→A
Q	$nRT_h \ln\left(\frac{V_B}{V_A}\right)$	0	$-nRT_c \ln\left(\frac{V_C}{V_D}\right)$	0
W	$nRT_h \ln\left(\frac{V_B}{V_A}\right)$	$\frac{nR(T_h - T_c)}{\gamma - 1}$	$-nRT_c \ln\left(\frac{V_C}{V_D}\right)$	$-\frac{nR(T_h - T_c)}{\gamma - 1}$
ΔE_{int}	0	$-\frac{nR(T_h - T_c)}{\gamma - 1}$	0	$\frac{nR(T_h - T_c)}{\gamma - 1}$

$$e = \frac{Q_{AB} + Q_{CD}}{Q_{AB}}$$

$$= 1 - \frac{T_c}{T_h} \frac{\ln(V_D / V_C)}{\ln(V_B / V_A)}$$

$$= 1 - \frac{T_c}{T_h}$$

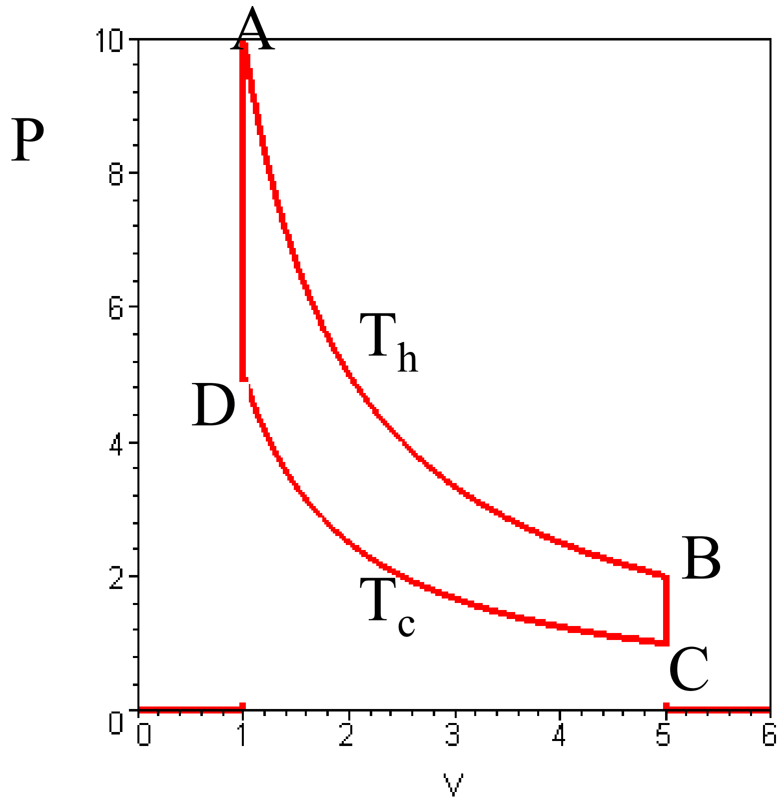
Examples

Efficiency of a Carnot engine operating between the temperatures of T_c=0°C and T_h=100°C:

$$e = 1 - \frac{273.15}{373.15} = 26.8\%$$

➔ For a Carnot engine, it is clear that we cannot achieve e=100%; not possible to completely transform heat into work. It is possible to show that the Carnot engine is the most efficient that one can construct between the two operating temperatures T_c and T_h.

Stirling engine



$$e = \frac{Q_{AB} + Q_{CD}}{Q_{AB} + Q_{DA}}$$

	A→B	B→C	C→D	D→A
Q	$nRT_h \ln\left(\frac{V_B}{V_A}\right)$	$-\frac{nR(T_h - T_c)}{\gamma - 1}$	$-nRT_c \ln\left(\frac{V_C}{V_D}\right)$	$\frac{nR(T_h - T_c)}{\gamma - 1}$
W	$nRT_h \ln\left(\frac{V_B}{V_A}\right)$	0	$-nRT_c \ln\left(\frac{V_C}{V_D}\right)$	0
ΔE_{int}	0	$-\frac{nR(T_h - T_c)}{\gamma - 1}$	0	$\frac{nR(T_h - T_c)}{\gamma - 1}$

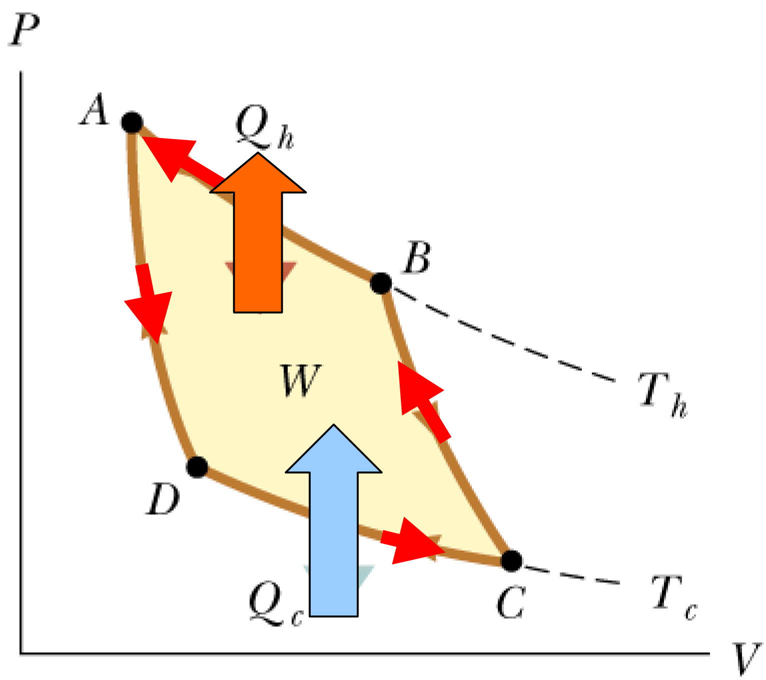
Example:

$T_h = 3T_c \quad V_B = V_C = 5V_A = 5V_D \quad \gamma = 1.3$

$$e = \frac{nR(T_h - T_c)\ln(V_B / V_A)}{nRT_h \ln(V_B / V_A) + \frac{nR(T_h - T_c)}{\gamma - 1}} = 50.6\%$$

$$e_{\text{Carnot}} = 66.7\%$$

Carnot cycle for cooling and heating



"coefficient of performance"

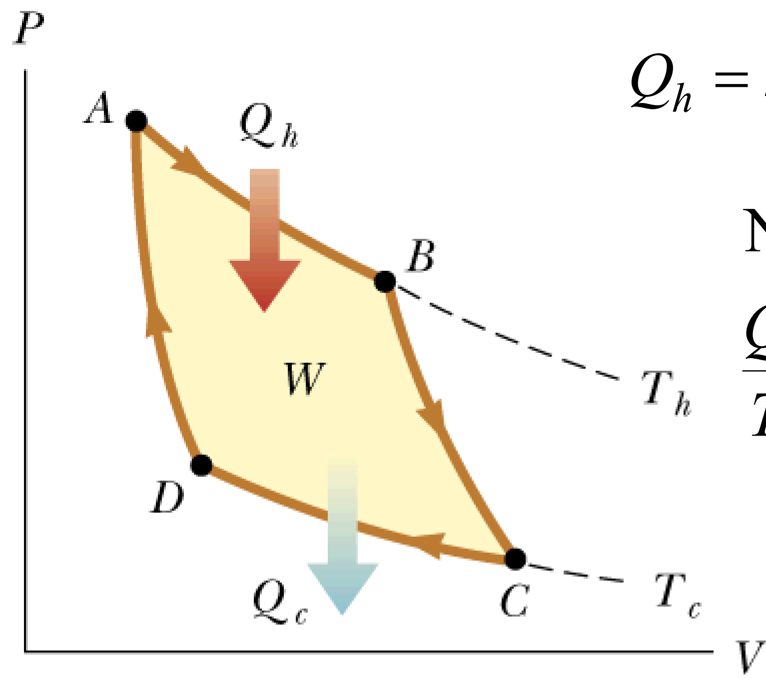
$COP_{heating}=|Q_h/W|=T_h/(T_h-T_c)$

$COP_{cooling}=|Q_c/W|=T_c/(T_h-T_c)$

Example: Suppose that on a cold winter day, a heat pump has a compressor which brings outdoor air at $T_c=-3^oC$ into a room at $T_h=22^oC$. What is the COP?

COP=295.15/25=11.8

More about Carnot cycle



$$Q_h = nRT_h \ln\left(\frac{V_B}{V_A}\right) \qquad Q_c = -nRT_c \ln\left(\frac{V_B}{V_A}\right)$$

Notice that :

$$\frac{Q_h}{T_h} + \frac{Q_c}{T_c} = \frac{|Q_h|}{T_h} - \frac{|Q_c|}{T_c} 0$$

$$S_{AB} = nR \ln\left(\frac{V_B}{V_A}\right)$$

$$S_{BC} = 0$$

$$S_{CD} = -nR \ln\left(\frac{V_B}{V_A}\right)$$

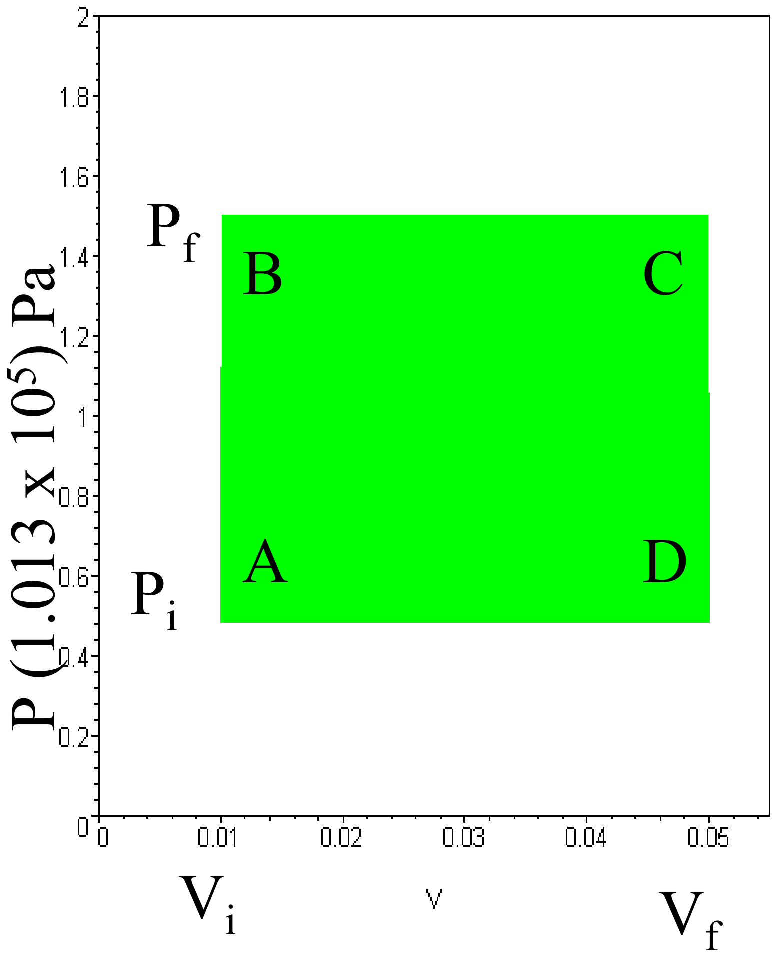
$$S_{DA} = 0$$

Define entropy :

$$S_{AB} = \int_A^B \frac{dQ}{T}$$

Peer instruction question:

Consider the "square cycle" shown below. What can you say about the entropy change in each cycle:



(A) $S_{ABCDA} = 0$

(B) $S_{ABCDA} > 0$

(C) $S_{ABCDA} < 0$

Other examples of entropy calculations:

Ideal gas:

Isovolumetric process:

$$dQ = nC_V dT = \frac{nR}{\gamma - 1} dT$$

$$S = \frac{nR}{\gamma - 1} \int_A^B \frac{dT}{T} = \frac{nR}{\gamma - 1} \ln\left(\frac{T_B}{T_A}\right)$$

Isobaric process:

$$dQ = nC_P dT = \frac{\gamma nR}{\gamma - 1} dT$$

$$S = \frac{\gamma nR}{\gamma - 1} \int_A^B \frac{dT}{T} = \frac{\gamma nR}{\gamma - 1} \ln\left(\frac{T_B}{T_A}\right)$$

Melting of solid having mass *m* and latent heat *L* at melting temperature T_M:

$$S = \int_0^m \frac{Ldm}{T} = \frac{Lm}{T_M}$$

4. [SB5 22.P.16.] A 20%-efficient real engine is used to speed up a train from rest to 5.00 m/s. It is known that an ideal (Carnot) engine having the same cold and hot reservoirs would accelerate the same train from rest to a speed of 6.50 m/s using the same amount of fuel. Assuming that the engines use air at 300 K as a cold reservoir, find the temperature of the steam serving as the hot reservoir.

[0.2] ______ K

2. [SB5 22.P.21.] A 1.60 liter gasoline engine with a compression ratio of 8.20 has a power output of 92 hp. Assume that the engine operates in an idealized Otto cycle and that the fuel-air mixture behaves like an ideal gas with $\gamma = 1.40$.

Find the energy absorbed each second.

[0.16666666666666667] ______ kW

Find the energy exhausted each second.

[0.16666666666666667] ______ kW