Announcements

1. This week: read Chapters 16-18 – wave motion

   work HW #31 & #32

   online quiz for Wed. & Fri.

   start reviewing Chapters 1-22 for final exam

2. 4^{th} hour exam

   extra credit for reworking exam

   presentations?

3. Final exam

   dates: Thurs. 12/11/02 at 9 AM, Sat. 12/14/02 at 2 PM

The phenomenon of wave motion

Physical mechanisms for mechanical waves

The wave equation $\boxed{\dfrac{\partial^2 y}{\partial t^2} = v^2 \dfrac{\partial^2 y}{\partial x^2}}$

Solutions of the wave equation – mathematical representations of waves

Peer instruction question

Which of the following properties of a wave are characteristic of the medium in which the wave is traveling?

(A) Its frequency

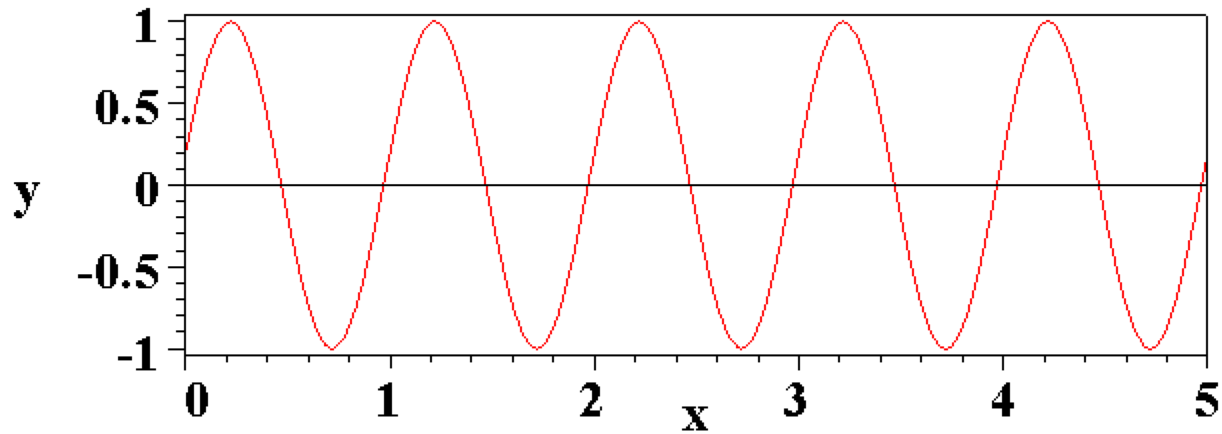
(B) Its wavelength

(C) Its velocity

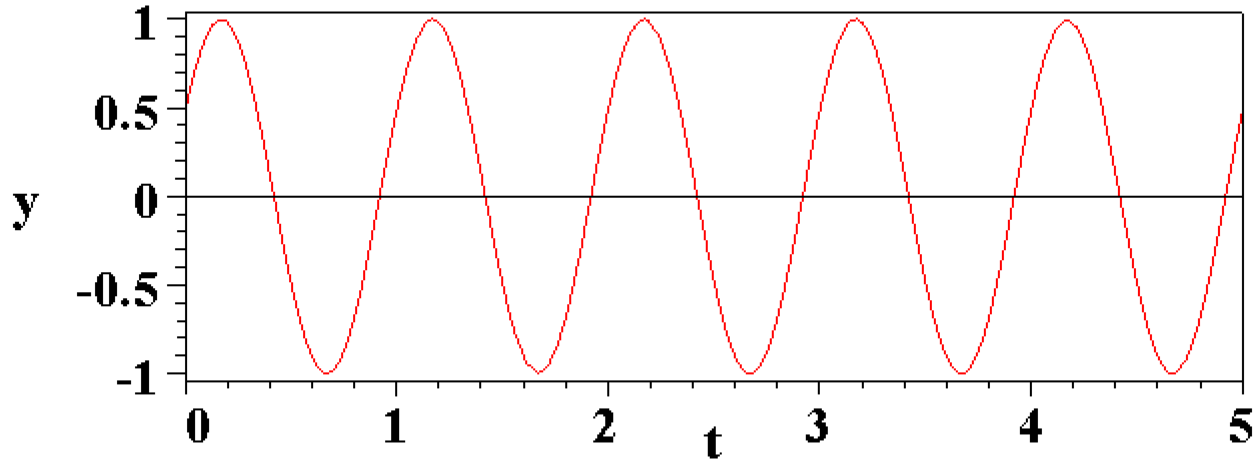
(D) All of the above

Mechanical waves occur in continuous media.   They are characterized by a value (y) which changes in both time (t) and position (x).

Example -- periodic wave

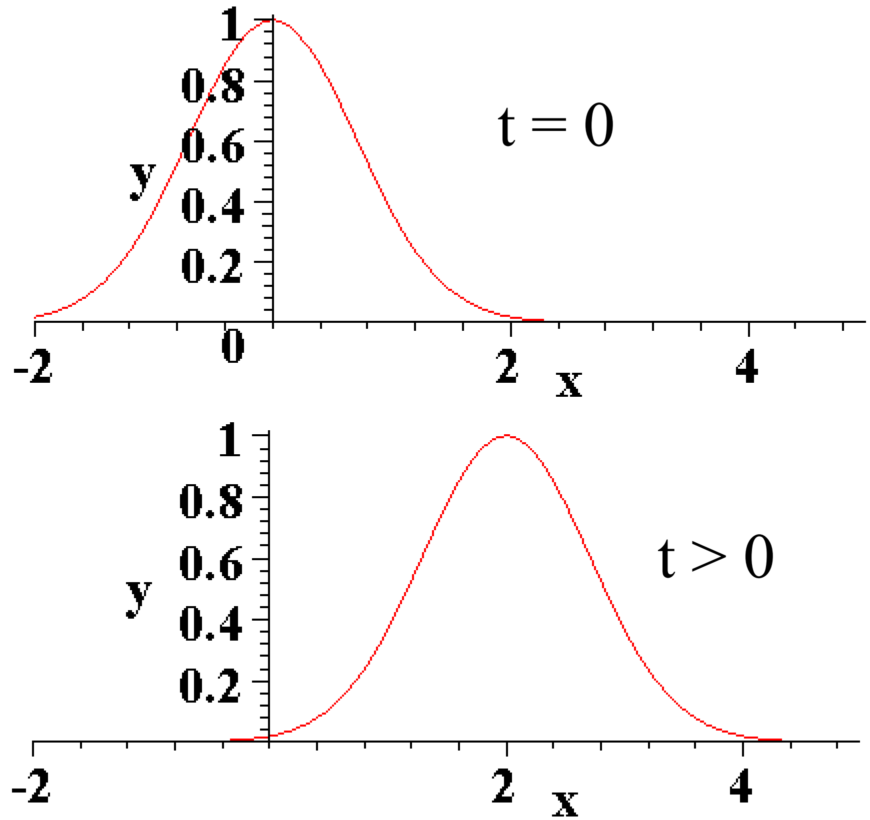


$y(t_0,x)$



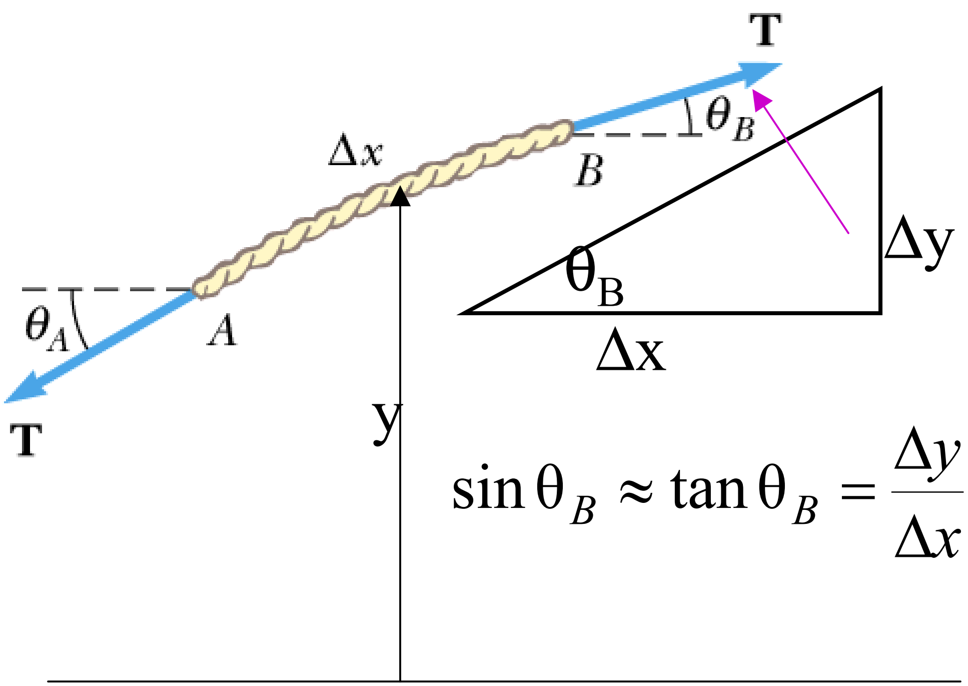
$y(t,x_0)$

General traveling wave –



Basic physics behind wave motion --

example: transverse wave on a string with tension T and mass per unit length μ



$$\sin\theta_B \approx \tan\theta_B = \left.\frac{\Delta y}{\Delta x}\right|_B$$

$$m\frac{d^2y}{dt^2} = T\sin\theta_B - T\sin\theta_A$$

$$m \approx \mu\Delta x$$

$$\Rightarrow \mu\Delta x\frac{d^2y}{dt^2} \approx T\left(\left.\frac{\Delta y}{\Delta x}\right|_B - \left.\frac{\Delta y}{\Delta x}\right|_A\right)$$

$$\lim_{\Delta x\to 0}\frac{1}{\Delta x}\left(\left.\frac{\Delta y}{\Delta x}\right|_B - \left.\frac{\Delta y}{\Delta x}\right|_A\right) = \frac{\partial^2 y}{\partial x^2}$$

$$\boxed{\frac{\partial^2 y}{\partial t^2} = \frac{T}{\mu}\frac{\partial^2 y}{\partial x^2}}$$

The wave equation:

$$\boxed{\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}}$$

where $v \equiv \sqrt{\frac{T}{\mu}}$ (for a string)

Solutions: $y(x,t) = f\,(x \pm vt)$

function of any shape

Note: $\frac{\partial f(u)}{\partial x} = \frac{\partial f(u)}{\partial u}\frac{\partial u}{\partial x}$

$$\frac{\partial^2 f(u)}{\partial x^2} = \frac{\partial^2 f(u)}{\partial u^2}\left(\frac{\partial u}{\partial x}\right)^2 = \frac{\partial^2 f(u)}{\partial u^2}$$

$$\frac{\partial^2 f(u)}{\partial t^2} = \frac{\partial^2 f(u)}{\partial u^2}\left(\frac{\partial u}{\partial t}\right)^2 = \frac{\partial^2 f(u)}{\partial u^2}v^2$$

Examples of solutions to the wave equation:

Moving "pulse": $y(x,t) = y_0 e^{-(x-vt)^2}$

Periodic wave: $y(x,t) = y_0 \sin\left(k\left(x - vt\right) + \varphi\right)$

"wave vector"
<u>not</u> spring constant!!!

$$k = \frac{2\pi}{\lambda}$$

$$kv = \frac{2\pi}{T} = 2\pi f = \omega$$

$$y(x,t) = y_0 \sin\left(2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right) + \varphi\right) \qquad \frac{\lambda}{T} = v$$

Periodic waves:

phase (radians)

$$y(x,t) = y_0 \sin\left(2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right) + \varphi\right) \qquad \frac{\lambda}{T} = v$$

velocity (m/s)

period (s); $T = 1/f$

wave length (m)

Amplitude

Combinations of waves ("superposition")

Note that :

$$\sin A \pm \sin B = 2\sin\left[\tfrac{1}{2}(A \pm B)\right]\cos\left[\tfrac{1}{2}(A \mp B)\right]$$

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$$y_{right}(x,t) = y_0 \sin\left(2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right) + \varphi\right) \qquad y_{left}(x,t) = y_0 \sin\left(2\pi\left(\frac{x}{\lambda} + \frac{t}{T}\right) + \varphi\right)$$

"Standing" wave:

$$y_{right}(x,t) + \mathrm{y}_{left}(x,t) = 2y_0 \sin\left(\frac{2\pi x}{\lambda} + \varphi\right)\cos\left(\frac{2\pi t}{T}\right)$$

Constraints of standing waves: ($\phi = 0$)

$$y_{right}(x,t) + \mathrm{y}_{left}(x,t) = 2y_0 \sin\left(\frac{2\pi x}{\lambda}\right)\cos\left(\frac{2\pi t}{T}\right)$$

for string :

$$L = \frac{n\lambda}{2}$$

$$\frac{\lambda}{T} = v \quad \Rightarrow T = \frac{2L}{nv} \quad \text{and} \quad f = \frac{nv}{2L}$$