PHY 113 – Additional notes for Chapter 1 (Problem Set # 1)

Here are some notes about error analysis. This will be discussed also in your laboratory work. Some helpful comments follow:

Some degree of error is associated with any measurement. For example, Suppose your ruler has centimeter and millimeter markings. If you measured one side of your text you could say that its length is $l_1 \pm \delta l_1$ (for example 22.2 ± 0.2 cm). Suppose the second length is measured as $l_2 \pm \delta l_2$ (for example 26.2 ± 0.2 cm), while the thickness is $t \pm \delta t$ (for example 6.3 ± 0.2 cm). If you now wanted to compute the expected volume of your text, that would be

$$V = l_1 \cdot l_2 \cdot t. \tag{1}$$

To get an idea of the error in your calculation you need to think about the error in each length measurement. Symbollically you can write this as the difference between the possible values and the expected value given in Eq. 1,

$$\delta V \equiv (l_1 \pm \delta l_1) \cdot (l_2 \pm \delta l_2) \cdot (t \pm \delta t) - l_1 \cdot l_2 \cdot t. \tag{2}$$

Expanding this, we find

$$\delta V \approx \pm \delta l_1 \cdot l_2 \cdot t \pm l_1 \cdot \delta l_2 \cdot t \pm l_1 \cdot l_2 \cdot \delta t \pm \ldots, \tag{3}$$

where the terms we have omitted, such as $\delta l_1 \cdot \delta l_2 \cdot t$ are expected to be much smaller than the terms we kept. If we want an estimate of the maximum possible error, then we can we should replace $\pm$ with $+$ and it is convenient to divide the estimate of the error in V by the the estimated value of V so that the expression becomes the very compact result:

$$\frac{\delta V}{V} = \frac{\delta l_1}{l_1} + \frac{\delta l_2}{l_2} + \frac{\delta t}{t}. \tag{4}$$

For the particular numbers quoted above, the fractional error is

$$\frac{\delta V}{V} = \frac{0.2}{22.2} + \frac{0.2}{26.2} + \frac{0.2}{6.3} = 0.048 \equiv \%4.8, \tag{5}$$

or $V = 3664 \pm 177$ cm^3.

In this case, the fractional error is equal to the sum of the fractional errors in each of the length measurements. This is not a general result, but frequently the error analysis simplifies to a compact result when expressed in terms of the fractional error of each measurement.

Homework problem # 4 makes use of some of these ideas. In that case you have a sphere of radius r with an estimated error of δr. Since the volume of the sphere is $V = \frac{4\pi}{3} r^3$, and $(r \pm \delta r)^3 = r^3 \pm 3r^2\delta r + r(\delta r)^2 \pm (\delta r)^3 \approx r^3 \pm 3r^2 \delta r$, we see that

$$\frac{\delta V}{V} = 3\frac{\delta r}{r} \tag{6}$$

for this case. The full problem asks you to estimate the error in the density which is the mass/volmume ($\rho = m/V$). From the previous argument, can you convince yourself that

$$\frac{\delta \rho}{\rho} = \frac{\delta m}{m} + 3\frac{\delta r}{r}? \tag{7}$$