Announcements

1. **Second exam scheduled for Oct. 28th -- practice exams now available -- http://www.wfu.edu/~natalie/f03phy113/extrapractice/**
2. **Thursday – review of Chapters 9-14**
3. **Today's lecture –**

   Universal law of gravitation

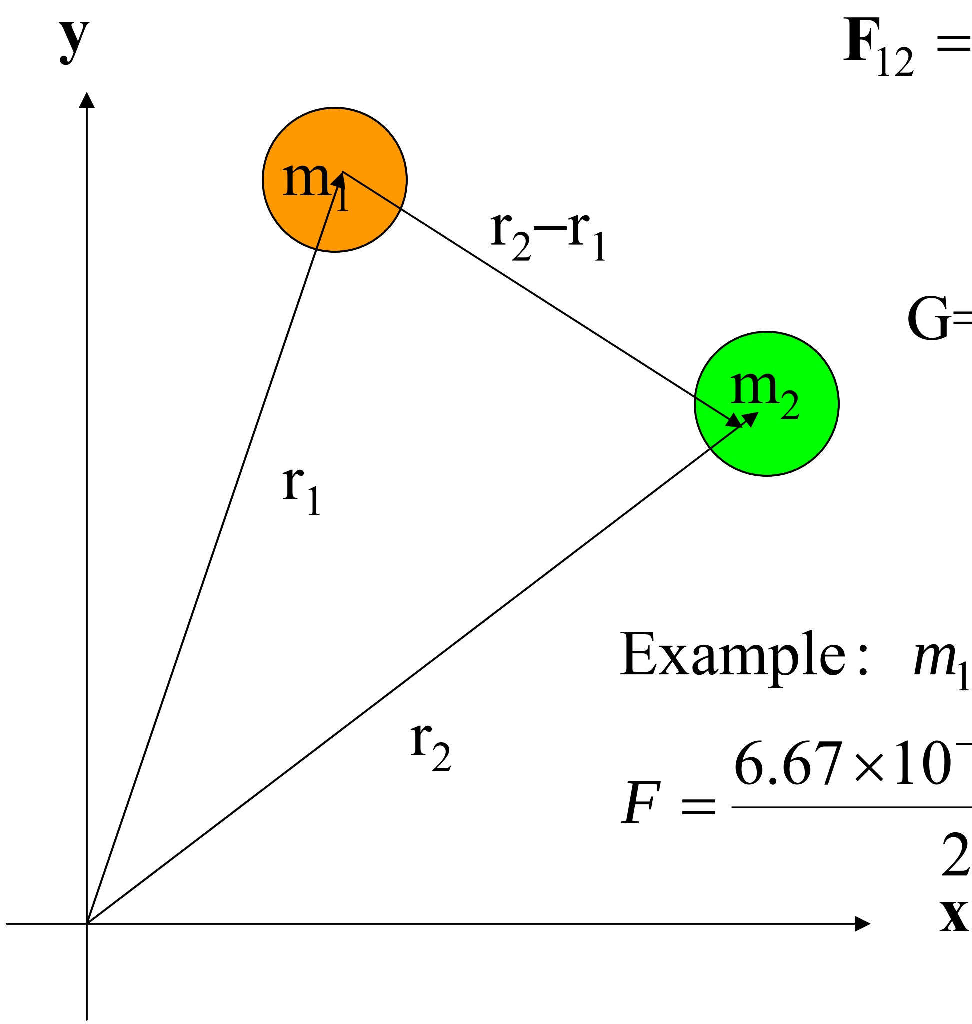
   Gravity near the planet's surface

   Gravitational potential energy

   Planetary and satelite motion

Newton's law of gravitation: m_2 *attracts* m_1 according to:

$$\mathbf{F}_{12} = \frac{G m_1 m_2 \hat{\mathbf{r}}_{12}}{r_{12}^2}$$

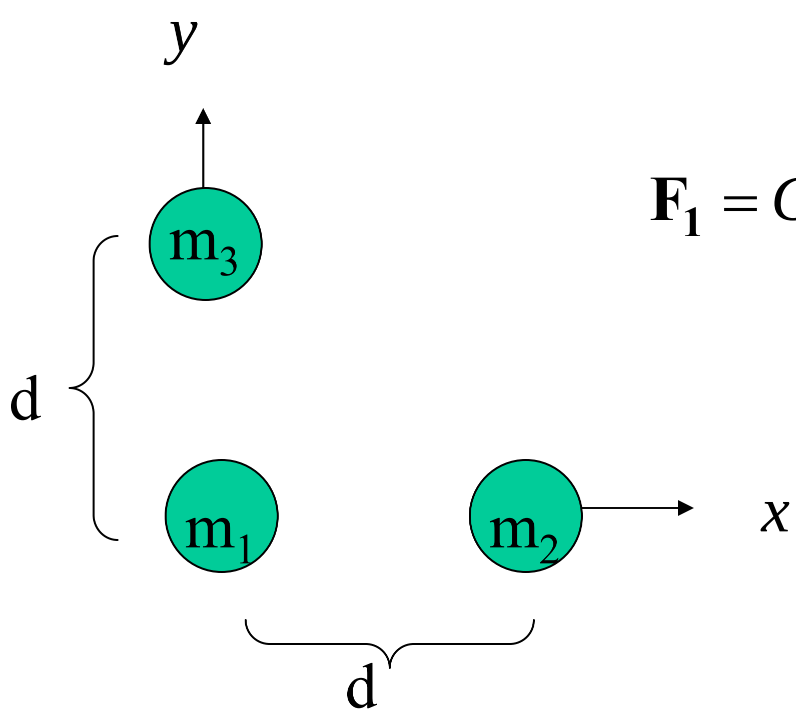


$G = 6.67 \times 10^{-11}\ \mathrm{N\ m^2/kg^2}$

$$\text{Example:}\quad m_1 = m_2 = 70\,\mathrm{kg};\quad r_{12} = 2\,\mathrm{m}:$$

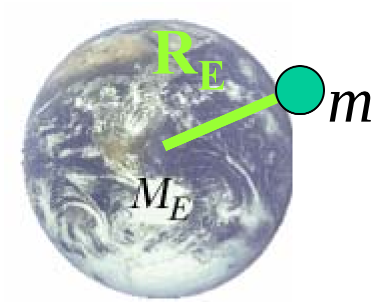
$$F = \frac{6.67\times 10^{-11}\cdot 70\cdot 70}{2^2}\,N = 8.17\times 10^{-8}\,N$$

Vector nature of Gravitational law:



$$\mathbf{F_1} = G\frac{m_1}{d^2}\left(m_2\mathbf{i} + m_3\mathbf{j}\right)$$

Gravitational force of the Earth



$$F = \frac{GM_E m}{R_E^2}$$

$$\Rightarrow g = \frac{GM_E}{R_E^2} = \frac{6.67\times10^{-11}\cdot5.98\times10^{24}}{(6.37\times10^{6})^2}\,\mathrm{m/s^2} = 9.8\mathrm{m/s^2}$$

Question:

Suppose you are flying in an airplane at an altitude of 35000ft~11km above the Earth's surface. What is the acceleration due to Earth's gravity?

$$F = \frac{GM_E m}{(R_E + h)^2} = ma$$

$$a = \frac{GM_E}{(R_E + h)^2} = \frac{6.67\times 10^{-11}\cdot 5.98\times 10^{24}}{((6.37+0.011)\times 10^6)^2}\,\mathrm{m/s^2} = 9.796\mathrm{m/s^2}$$

a/g = 0.997

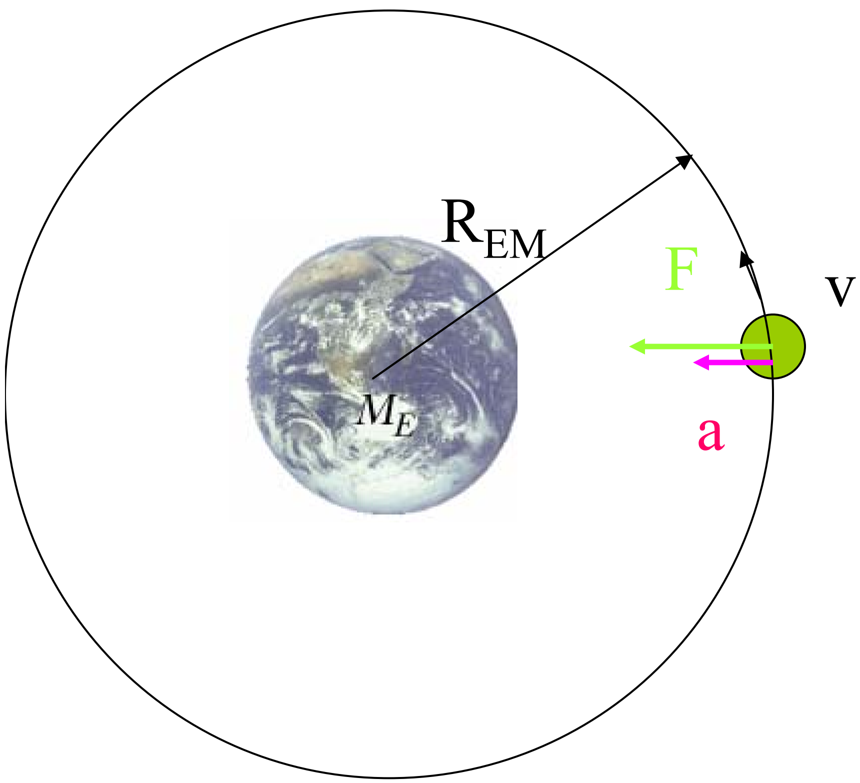
Attraction of moon to the Earth:

$$F = \frac{GM_E M_M}{R_{EM}^2} = \frac{6.67\times10^{-11}\cdot 5.98\times10^{24}\cdot 7.36\times10^{22}}{(3.84\times10^8)^2}\,\mathrm{N} = 1.99\times10^{20}\,\mathrm{N}$$

Acceleration of moon toward the Earth:

$F = M_M\, a$ ➔ $a = 1.99\mathrm{x}20^{20}$ N/$7.36\mathrm{x}10^{22}$kg $=0.0027$ m/s^2

Stable circular orbit of two gravitationally attracted objects (such as the moon and the Earth)



$$a = \frac{v^2}{R_{EM}} = \frac{GM_E}{R_{EM}^2}$$

$$v = \omega R_{EM} = \frac{2\pi}{T} R_{EM}$$

$$T = 2\pi\sqrt{\frac{R_{EM}^3}{GM_E}}$$

$$= 2\pi\sqrt{\frac{(3.84\times 10^8)^3}{6.67\times 10^{-11}\cdot 5.98\times 10^{24}}}$$

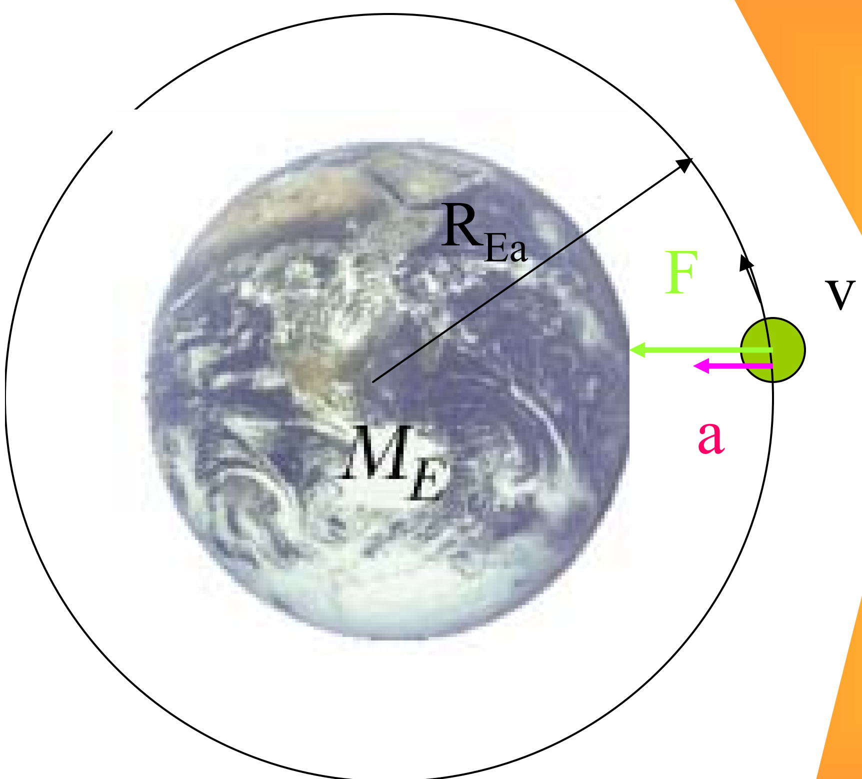
$$= 2367353.953\,\mathrm{s} = 27.4\ \mathrm{days}$$

Peer instruction question

In the previous discussion, we saw how the moon orbits the Earth in a stable circular orbit because of the radial gravitational attraction of the moon and Newton's second law: F=ma, where a is the centripetal acceleration of the moon in its circular orbit. Is this the same mechanism which stabilizes airplane travel? Assume that a typical cruising altitude of an airplane is 11 km above the Earth's surface and that the Earth's radius is 6370 km.

(a) Yes (b) No

Stable (??) circular orbit of two gravitationally attracted objects (such as the airplane and the Earth)



$v^2 \quad = \frac{GM_E}{R_E^2}$

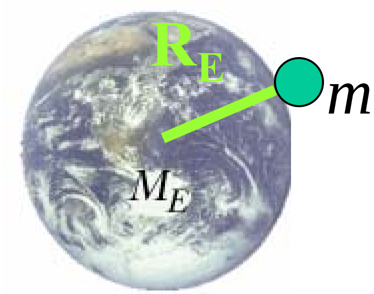
$=1.4$

$\text{km/s} \approx$

Newton's law of gravitation: $$\mathbf{F}_{12} = \frac{G m_1 m_2 \hat{\mathbf{r}}_{12}}{r_{12}^2}$$

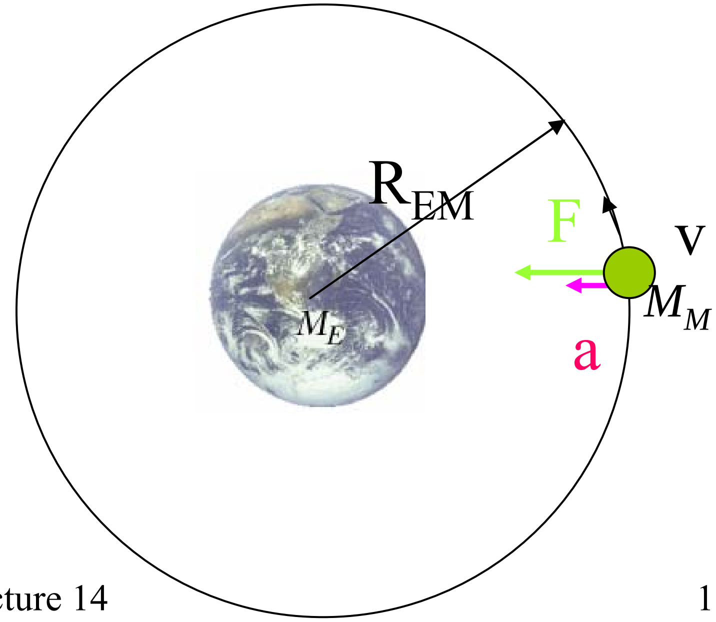
Earth's gravity:

$$F = \frac{G M_E m}{R_E^2}$$



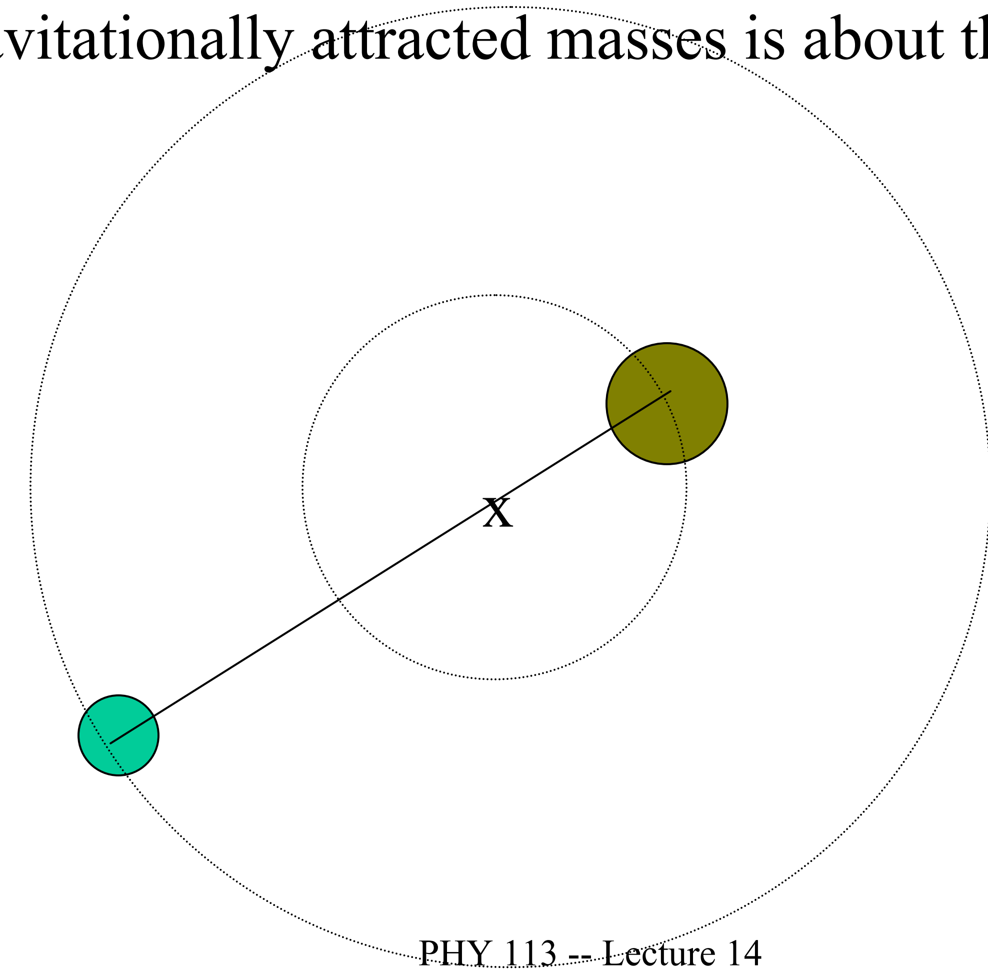
$$\Rightarrow g = \frac{G M_E}{R_E^2} = \frac{6.67\times10^{-11}\cdot 5.98\times10^{24}}{(6.37\times10^6)^2}\,\mathrm{m/s^2} = 9.8\,\mathrm{m/s^2}$$

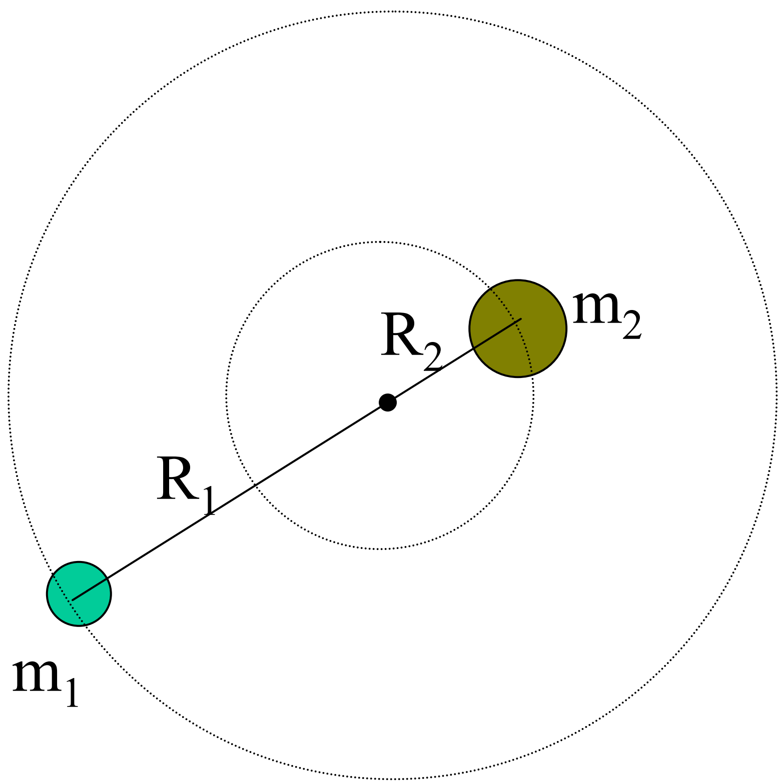
Stable circular orbits of gravitational attracted objects:



More details

If we examine the circular orbit more carefully, we find that the correct analysis is that the stable circular orbit of two gravitationally attracted masses is about their center of mass.





Radial forces on m$_1$:

$$F_{r1} = \frac{Gm_1 m_2}{(R_1 + R_2)^2} = m_1 a_{r1} = m_1 \frac{v_1^2}{R_1}$$

$$v_1 = \frac{2\pi R_1}{T_1}$$

$$T_1 = 2\pi \sqrt{\frac{R_1 (R_1 + R_2)^2}{Gm_2}}$$

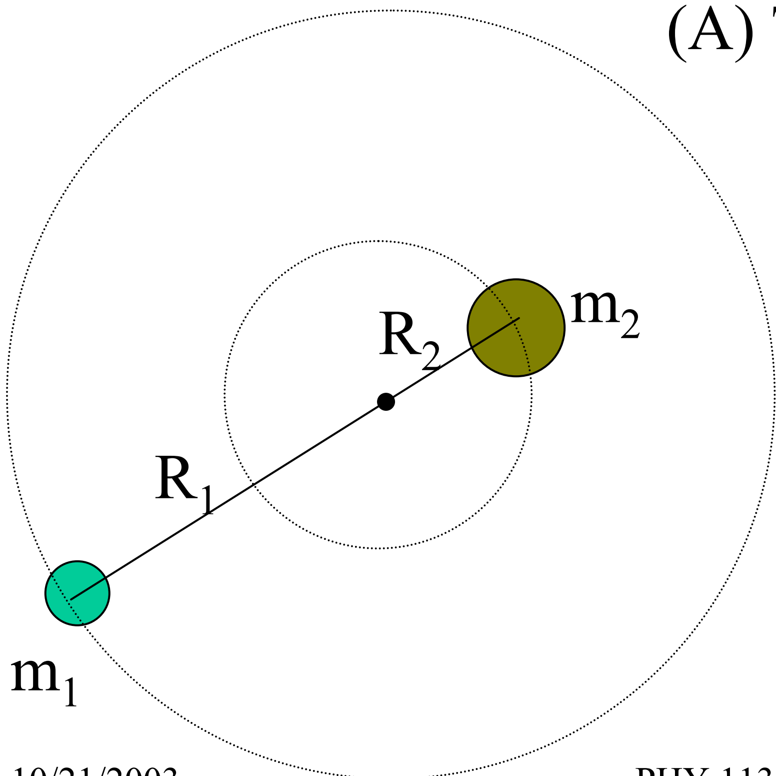
T_2 ?

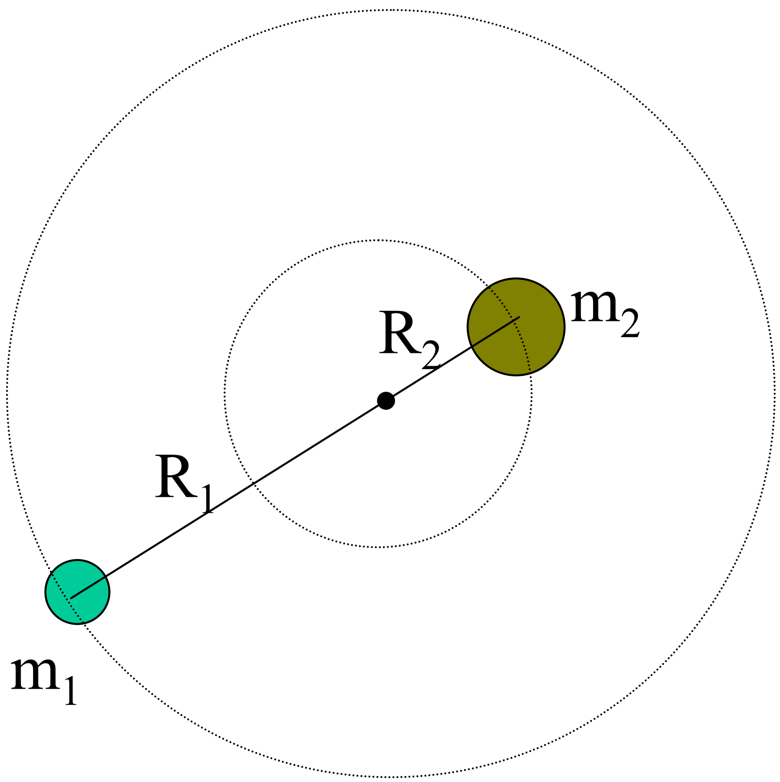
Tangential forces ?

Peer instruction question

What is the relationship between the periods T_1 and T_2 of the two gravitationally attracted objects rotating about their center of mass? (Assume that $m_1 < m_2$.)

(A) $T_1=T_2$ (B) $T_1<T_2$ (C) $T_1>T_2$





$$m_1 R_1 = m_2 R_2$$

$$m_1 \frac{v_1^2}{R_1} = m_1 R_1 \omega_1^2 = \frac{G m_1 m_2}{(R_1 + R_2)^2} = m_2 R_2 \omega_2^2$$

$$\Rightarrow \omega_1 = \omega_2$$

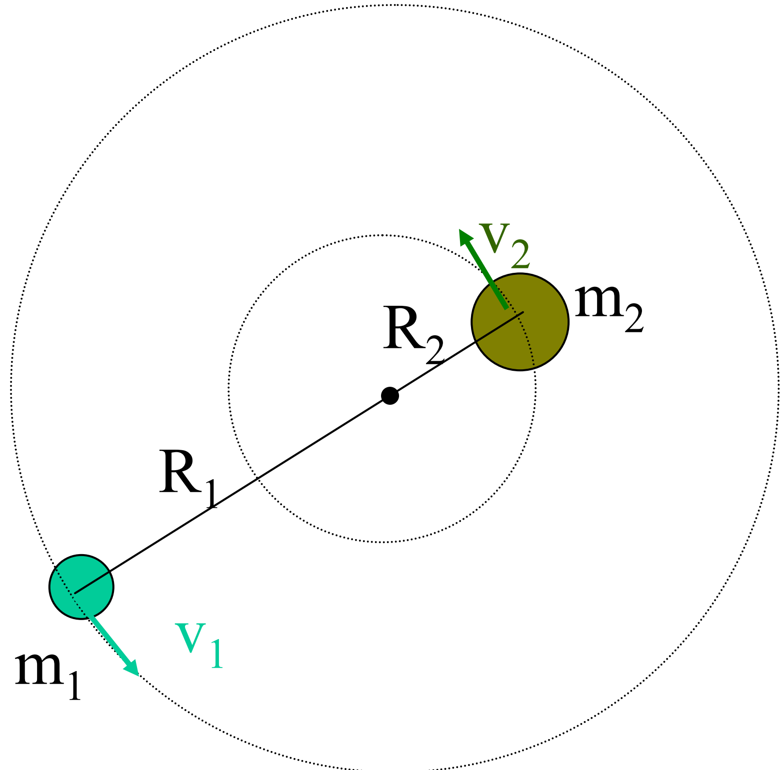
$$\boxed{T_1 = T_2 = 2\pi \sqrt{\frac{(R_1 + R_2)^3}{G(m_1 + m_2)}}}$$

What is the physical basis for stable circular orbits?

1. Newton's second law? $\mathbf{F} = m\mathbf{a} = \frac{d\mathbf{p}}{dt}$
2. Conservation of mechanical energy? $E = K + U =$ (const)
3. Conservation of linear momentum? $\mathbf{p} =$ (const)
4. Torqued motion? $\tau = I\,\alpha$?
5. Conservation of angular momentum? $\mathbf{L} =$ (const)

$$\boldsymbol{\tau} = \frac{d\mathbf{L}}{dt} = 0$$

$$\Rightarrow \mathbf{L} = (\text{const})$$



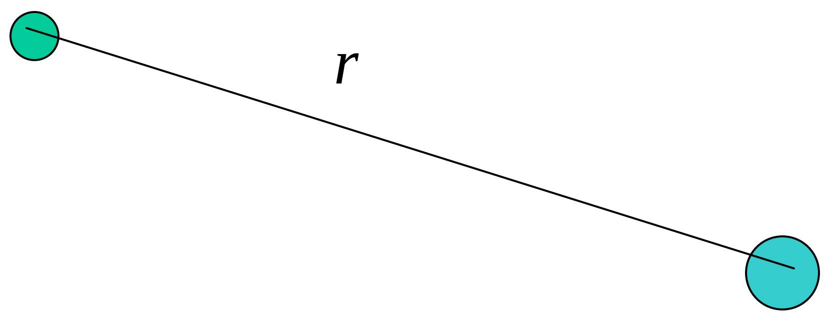
$L_1 = m_1 v_1 R_1$

$L_2 = m_2 v_2 R_2$

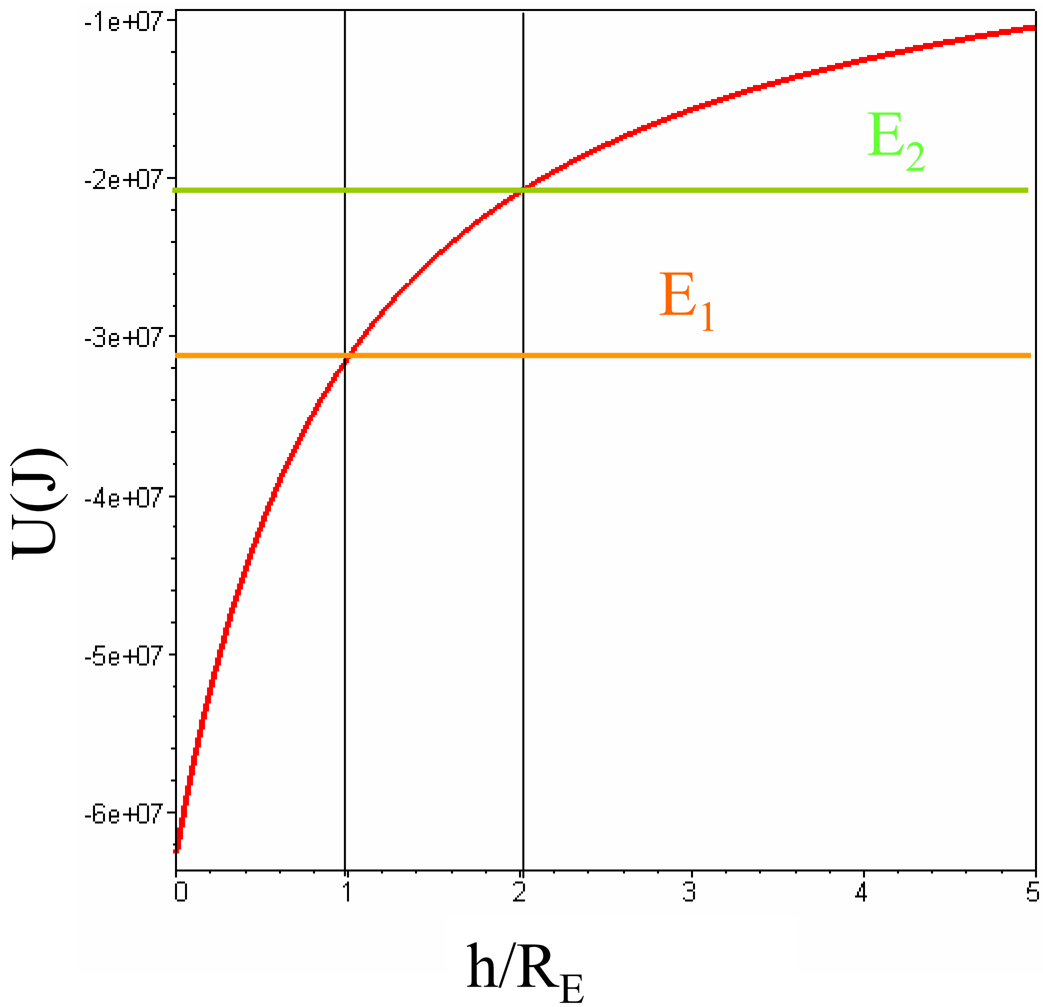
Question:

How are the magnitudes of L_1 and L_2 related?

The potential energy associated with the gravitational force.



$$U(r) = -\int_{r_{ref}}^{r} \frac{Gm_1m_2}{r^2}dr = -\int_{\infty}^{r} \frac{Gm_1m_2}{r^2}dr = -\frac{Gm_1m_2}{r}$$



Total mechanical energy for circular orbits:

(assume M >> m)

$$E = \tfrac{1}{2} m v^2 + U(r)$$

$$\frac{mv^2}{r} = \frac{GMm}{r^2} \quad \Rightarrow v^2 = \frac{GM}{r}$$

$$U(r) = -\frac{GMm}{r}$$

$$\boxed{E = -\frac{GMm}{2r}}$$

Peer instruction question

What is wrong with the previous analysis?

A. Nothing is wrong. (The description of circular motion due to gravitational attraction is complete.)

B. E depends on r and therefore must not be constant.

C. E can only be constant if r is constant, but it is not obvious why r is constant.

D. Conservation of angular moment will come to the rescue.

Angular momentum: **L** = **r** x m**v**

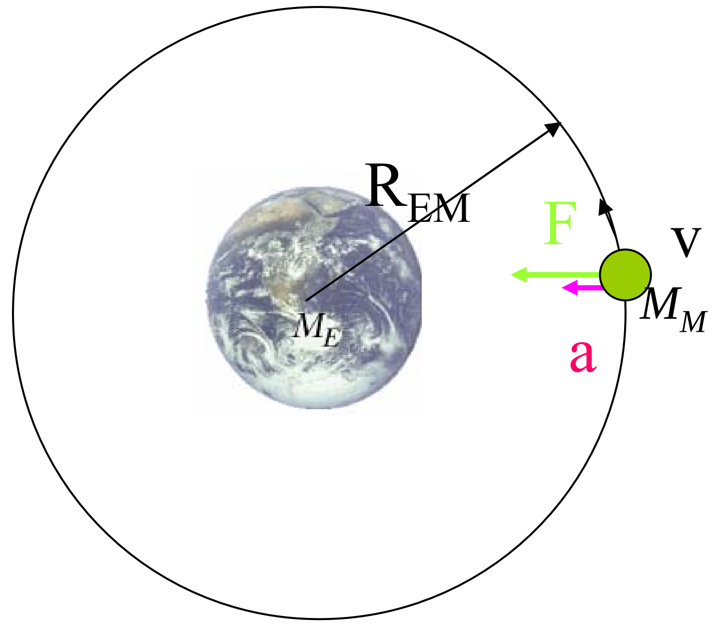
For circular orbit:

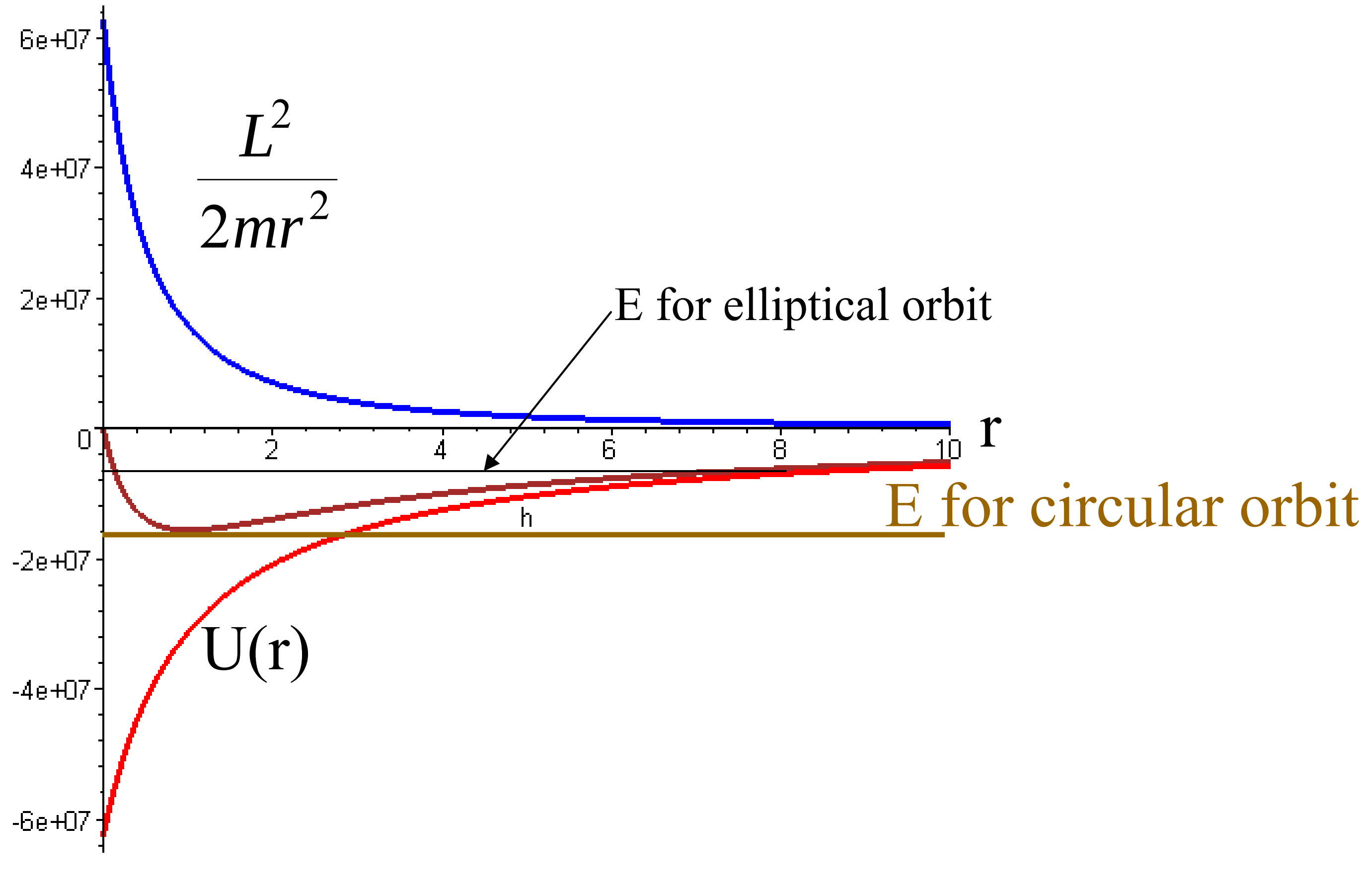
$$L = R_{ME} M_M v$$

$$v = \frac{L}{M_M R_{EM}}$$

$$K = \tfrac{1}{2} M_M v^2$$

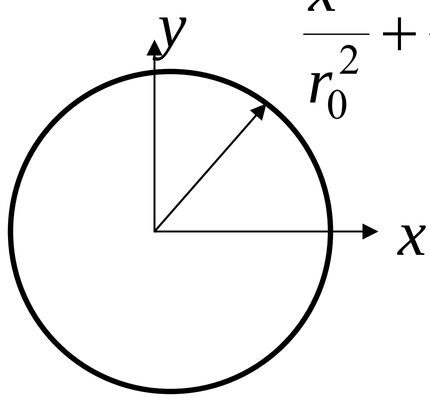
$$= \frac{L^2}{2 M_M R_{EM}^2}$$




6e+07
4e+07
2e+07
0
-2e+07
-4e+07
-6e+07
2
4
6
8
10
r
$\frac{L^2}{2mr^2}$
E for elliptical orbit
h
E for circular orbit
U(r)

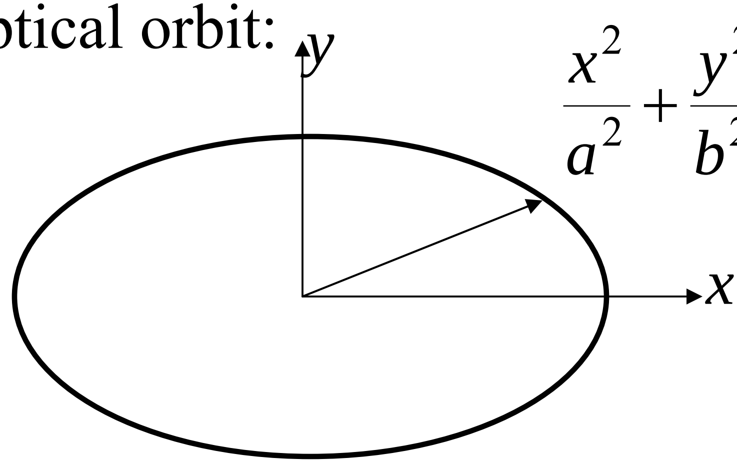
Circular orbit:

$$\frac{x^2}{r_0^2}+\frac{y^2}{r_0^2}=1$$



Elliptical orbit:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$



Satellites orbiting earth (approximately circular orbits):

$$T = 2\pi\sqrt{\frac{R_E^3}{GM_E}}\left(1 + h/R_E\right)^{3/2} = 5058\left(1 + h/R_E\right)^{3/2} s$$

$R_E \sim 6370$ km

Examples:

Satellite	h (km)	T (hours)	v (mi/h)
Geosynchronous	35790	~24	6900
NOAA polar orbitor	800	~1.7	16700
Hubble	600	~1.6	16900
Inter. space station*	390	~1.5	17200

*Link: http://liftoff.msfc.nasa.gov/temp/StationLoc.html

Sample question:

Suppose that the space shuttle ($m=10^5$kg) was initially in the same orbit as the International space station (h_i=390km) and the engines are fired to give it exactly the amount of energy ΔW to raise it to the same orbit as the Hubble space telescope (h_f= 600km). What is the energy ΔW?

You can show that the energy of a satellite of mass m in a circular orbit of height h above the Earth's surface is given by:

$$E_{mech} = K + U = -\frac{GM_E m}{2(R_E + h)}$$

$$\Delta W = -\frac{GM_E m}{2R_E}\left(\frac{1}{\left(1+{h_f}/{R_E}\right)} - \frac{1}{\left(1+{h_i}/{R_E}\right)}\right) = 89{,}000J$$