

**PHY 711 Classical Mechanics and
Mathematical Methods
10-10:50 AM MWF Olin 103**

Plan for Lecture 29: Chap. 9 of F&W

Wave equation for sound in the linear approximation

1. Sound generation
 2. Sound scattering

11/8/2019

PHY 711 Fall 2019 – Lecture 29

1

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|----|-----------------|----------|-----------------------------------|-----|------------|
| 21 | Fri, 10/18/2019 | Chap. 7 | Contour integrals; Exam due | #15 | 10/23/2019 |
| 22 | Mon, 10/21/2019 | Chap. 7 | More about contour integrals | | |
| 23 | Wed, 10/23/2019 | Chap. 5 | Rigid body motion | #16 | 10/25/2019 |
| 24 | Fri, 10/25/2019 | Chap. 5 | Rigid body motion | #17 | 10/28/2019 |
| 25 | Mon, 10/28/2019 | Chap. 8 | Elastic two-dimensional membranes | #18 | 11/01/2019 |
| 26 | Wed, 10/30/2019 | Chap. 9 | Mechanics of 3 dimensional fluids | | |
| 27 | Fri, 11/01/2019 | Chap. 9 | Fluid mechanics | #19 | 11/04/2019 |
| 28 | Mon, 11/04/2019 | Chap. 9 | Sound waves | | |
| | Wed, 11/06/2019 | No class | NAWH out of town | | |
| 29 | Fri, 11/08/2019 | Chap. 9 | Sound waves; Project Topic due | #20 | 11/11/2019 |
| 30 | Mon, 11/11/2019 | | Non-linear waves and shocks | | |
| 31 | Wed, 11/13/2019 | | | | |
| 32 | Fri, 11/15/2019 | | | | |
| 33 | Mon, 11/18/2019 | | | | |
| 34 | Wed, 11/20/2019 | | | | |
| 35 | Fri, 11/22/2019 | | | | |
| 36 | Mon, 11/25/2019 | | | | |
| | Wed, 11/27/2019 | | Thanksgiving holiday | | |
| | Fri, 11/29/2019 | | Thanksgiving holiday | | |
| | Mon, 12/2/2019 | | Presentations I | | |
| | Wed, 12/4/2019 | | Presentations II | | |
| | Fri, 12/6/2019 | | Presentations III | | |

2

Solutions to wave equation:

$$\nabla^2 \Phi - \frac{1}{c^2} \frac{\partial^2 \Phi}{\partial t^2} = 0$$

Plane wave solution:

$$\Phi(\mathbf{r}, t) = A e^{i\mathbf{k} \cdot \mathbf{r} - i\omega t} \quad \text{where} \quad k^2 = \left(\frac{\omega}{c} \right)^2$$

Note that these sound waves are "longitudinal"

-- the velocity wave direction is along the propagation direction: $\delta \mathbf{v} = -\nabla \Phi = -iA \mathbf{k} e^{i\mathbf{k} \cdot \mathbf{r} - i\omega t}$

11/8/2019

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3

Some comments about Monday's lecture
Equations to lowest order in perturbation :

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = \mathbf{f}_{\text{applied}} - \frac{\nabla p}{\rho} \Rightarrow \frac{\partial \delta \mathbf{v}}{\partial t} = - \frac{\nabla \delta p}{\rho_0}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \Rightarrow \frac{\partial \delta \rho}{\partial t} + \rho_0 \nabla \cdot \delta \mathbf{v} = 0$$

In terms of the velocity potential:
 $\delta \mathbf{v} = -\nabla \Phi$

Note that:
 $\delta \mathbf{v} = -\nabla \Phi(\mathbf{r}, t) = -\nabla \tilde{\Phi}(\mathbf{r}, t)$

$$\frac{\partial \delta \mathbf{v}}{\partial t} = -\frac{\nabla \delta p}{\rho_0} \Rightarrow \nabla \left(-\frac{\partial \Phi}{\partial t} + \frac{\delta p}{\rho_0} \right) = 0 \quad \text{for } \tilde{\Phi}(\mathbf{r}, t) = \Phi(\mathbf{r}, t) + \int dt' K(t')$$

$$\frac{\partial \delta \rho}{\partial t} + \rho_0 \nabla \cdot \delta \mathbf{v} = 0 \Rightarrow \frac{\partial \delta \rho}{\partial t} - \rho_0 \nabla^2 \Phi = 0 \quad \Rightarrow \frac{\partial \delta \rho}{\partial t} - \frac{\partial \Phi}{\partial t} + \frac{\delta p}{\rho_0} = K(t)$$

11/8/2019

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4

4

Some comments about Monday's lecture -- continued

Expressing pressure in terms of the density :

$$p = p(s, \rho) = p_0 + \delta p \quad \text{where } s \text{ denotes the (constant) entropy}$$

$$p_0 = p(s, \rho_0)$$

$$\delta p = \left(\frac{\partial p}{\partial \rho} \right)_s \delta \rho \equiv c^2 \delta \rho$$

$$\nabla \left(-\frac{\partial \Phi}{\partial t} + \frac{\delta p}{\rho_0} \right) = 0 \Rightarrow \nabla \left(-\frac{\partial \Phi}{\partial t} + c^2 \frac{\delta \rho}{\rho_0} \right) = 0$$

$$\left(-\frac{\partial \Phi}{\partial t} + c^2 \frac{\delta \rho}{\rho_0} \right) = K(t) \Rightarrow -\frac{\partial^2 \Phi}{\partial t^2} + \frac{c^2}{\rho_0} \frac{\partial \delta \rho}{\partial t} = 0$$

$$\frac{\partial \delta \rho}{\partial t} - \rho_0 \nabla^2 \Phi = 0 \Rightarrow \frac{\partial^2 \Phi}{\partial t^2} - c^2 \nabla^2 \Phi = 0$$

11/8/2019

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5

5

Some comments about Monday's lecture -- continued

Wave equation for air :

Additional relations:

$$\frac{\partial^2 \Phi}{\partial t^2} - c^2 \nabla^2 \Phi = 0$$

$$\delta p = c^2 \delta \rho = \rho_0 \frac{\partial \Phi}{\partial t}$$

$$\text{Here, } c^2 = \left(\frac{\partial p}{\partial \rho} \right)_s$$

$$\Rightarrow \frac{\partial^2 \delta \rho}{\partial t^2} - c^2 \nabla^2 \delta \rho = 0$$

$$\mathbf{v} = -\nabla \Phi$$

$$\frac{\partial^2 \delta p}{\partial t^2} - c^2 \nabla^2 \delta p = 0$$

Boundary values:

Impenetrable surface with normal $\hat{\mathbf{n}}$ moving at velocity \mathbf{V} :

$$\hat{\mathbf{n}} \cdot \mathbf{V} = \hat{\mathbf{n}} \cdot \delta \mathbf{v} = -\hat{\mathbf{n}} \cdot \nabla \Phi$$

Free surface:

$$\delta p = 0 \Rightarrow \rho_0 \frac{\partial \Phi}{\partial t} = 0$$

11/8/2019

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6

6

Wave equation with source:

$$\nabla^2 \Phi - \frac{1}{c^2} \frac{\partial^2 \Phi}{\partial t^2} = -f(\mathbf{r}, t)$$

Solution in terms of Green's function :

$$\Phi(\mathbf{r}, t) = \int d^3 r' \int dt' G(\mathbf{r} - \mathbf{r}', t - t') f(\mathbf{r}', t')$$

where

$$\left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) G(\mathbf{r} - \mathbf{r}', t - t') = -\delta(\mathbf{r} - \mathbf{r}')\delta(t - t')$$

11/8/2019

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7

Wave equation with source -- continued:

We can show that :

$$G(\mathbf{r} - \mathbf{r}', t - t') = \frac{\delta\left(t' - \left(t \mp \frac{|\mathbf{r} - \mathbf{r}'|}{c}\right)\right)}{4\pi|\mathbf{r} - \mathbf{r}'|}$$

11/8/2019

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8

Derivation of Green's function for wave equation

$$\left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) G(\mathbf{r} - \mathbf{r}', t - t') = -\delta(\mathbf{r} - \mathbf{r}') \delta(t - t')$$

Recall that

$$\delta(t-t') = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\omega(t-t')} d\omega$$

11/8/2019

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6

9

Derivation of Green's function for wave equation -- continued

Define : $\tilde{G}(\mathbf{r}, \omega) = \int_{-\infty}^{\infty} G(\mathbf{r}, t) e^{i\omega t} dt$

$$G(\mathbf{r}, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{G}(\mathbf{r}, \omega) e^{-i\omega t} d\omega$$

$\tilde{G}(\mathbf{r}, \omega)$ must satisfy :

$$(\nabla^2 + k^2) \tilde{G}(\mathbf{r} - \mathbf{r}', \omega) = -\delta(\mathbf{r} - \mathbf{r}') \quad \text{where } k^2 = \frac{\omega^2}{c^2}$$

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10

10

Derivation of Green's function for wave equation -- continued

$$(\nabla^2 + k^2) \tilde{G}(\mathbf{r} - \mathbf{r}', \omega) = -\delta(\mathbf{r} - \mathbf{r}')$$

Solution assuming isotropy in $\mathbf{r} - \mathbf{r}'$:

$$\tilde{G}(\mathbf{r} - \mathbf{r}', \omega) = \frac{e^{\pm ik|\mathbf{r} - \mathbf{r}'|}}{4\pi|\mathbf{r} - \mathbf{r}'|}$$

Check -- Define $R \equiv |\mathbf{r} - \mathbf{r}'|$ and for $R > 0$:

$$(\nabla^2 + k^2) \tilde{G}(R, \omega) = \frac{1}{R} \frac{d^2}{dR^2} (R \tilde{G}(R, \omega)) + k^2 \tilde{G}(R, \omega) = 0$$

11/8/2019

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11

11

Derivation of Green's function for wave equation -- continued

For $R > 0$:

$$(\nabla^2 + k^2) \tilde{G}(R, \omega) = \frac{1}{R} \frac{d^2}{dR^2} (R \tilde{G}(R, \omega)) + k^2 \tilde{G}(R, \omega) = 0$$

$$\frac{d^2}{dR^2} (R \tilde{G}(R, \omega)) + k^2 (R \tilde{G}(R, \omega)) = 0$$

$$(R \tilde{G}(R, \omega)) = A e^{ikR} + B e^{-ikR}$$

$$\Rightarrow \tilde{G}(R, \omega) = A \frac{e^{ikR}}{R} + B \frac{e^{-ikR}}{R}$$

11/8/2019

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12

12

Derivation of Green's function for wave equation – continued
need to find A and B.

$$\text{Note that : } \nabla^2 \frac{1}{4\pi|\mathbf{r} - \mathbf{r}'|} = -\delta(\mathbf{r} - \mathbf{r}')$$

$$\Rightarrow A = B = \frac{1}{4\pi}$$

$$\tilde{G}(R, \omega) = \frac{e^{\pm ikR}}{4\pi R}$$

11/8/2019

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13

13

Derivation of Green's function for wave equation – continued

$$\begin{aligned} G(\mathbf{r} - \mathbf{r}', t - t') &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{G}(\mathbf{r} - \mathbf{r}', \omega) e^{-i\omega(t-t')} d\omega \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{e^{\pm ik|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|} e^{-i\omega(t-t')} d\omega \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{e^{\pm i\frac{\omega}{c}|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|} e^{-i\omega(t-t')} d\omega \end{aligned}$$

11/8/2019

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14

14

Derivation of Green's function for wave equation – continued

$$G(\mathbf{r} - \mathbf{r}', t - t') = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{e^{\pm i\frac{\omega}{c}|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|} e^{-i\omega(t-t')} d\omega$$

$$\text{Noting that } \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\omega u} d\omega = \delta(u)$$

$$\Rightarrow G(\mathbf{r} - \mathbf{r}', t - t') = \frac{\delta\left(t - \left(t' \mp \frac{|\mathbf{r} - \mathbf{r}'|}{c}\right)\right)}{4\pi|\mathbf{r} - \mathbf{r}'|}$$

11/8/2019

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15

15

→ In order to solve an inhomogeneous wave equation with a time harmonic forcing term, we can use the corresponding Green's function:

$$\tilde{G}(|\mathbf{r} - \mathbf{r}'|, \omega) = \frac{e^{\pm ik|\mathbf{r} - \mathbf{r}'|}}{4\pi|\mathbf{r} - \mathbf{r}'|}$$

In fact, this Green's function is appropriate for solving equations with boundary conditions at infinity. For solving problems with surface boundary conditions where we know the boundary values or their gradients, the Green's function must be modified.

11/8/2019

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16

16

Green's theorem

Consider two functions $h(\mathbf{r})$ and $g(\mathbf{r})$

$$\text{Note that: } \int_V (h\nabla^2 g - g\nabla^2 h) d^3 r = \oint_S (h\nabla g - g\nabla h) \cdot \hat{n} d^2 r$$

$$\nabla^2 \tilde{\Phi} + k^2 \tilde{\Phi} = -\tilde{f}(\mathbf{r}, \omega)$$

$$(\nabla^2 + k^2) \tilde{G}(|\mathbf{r} - \mathbf{r}'|, \omega) = -\delta(\mathbf{r} - \mathbf{r}')$$

$$h \leftrightarrow \tilde{\Phi}; \quad g \leftrightarrow \tilde{G}$$

$$\int_V (\tilde{\Phi}(\mathbf{r}, \omega) \delta(\mathbf{r} - \mathbf{r}') - \tilde{G}(|\mathbf{r} - \mathbf{r}'|, \omega) f(\mathbf{r}, \omega)) d^3 r =$$

$$\oint_S (\tilde{\Phi}(\mathbf{r}, \omega) \nabla \tilde{G}(|\mathbf{r} - \mathbf{r}'|, \omega) - \tilde{G}(|\mathbf{r} - \mathbf{r}'|, \omega) \nabla \tilde{\Phi}(\mathbf{r}, \omega)) \cdot \hat{n} d^2 r$$

11/8/2019

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17

17

$$\int_V (\tilde{\Phi}(\mathbf{r}, \omega) \delta(\mathbf{r} - \mathbf{r}') - \tilde{G}(|\mathbf{r} - \mathbf{r}'|, \omega) f(\mathbf{r}, \omega)) d^3 r = \\ \oint_S (\tilde{\Phi}(\mathbf{r}, \omega) \nabla \tilde{G}(|\mathbf{r} - \mathbf{r}'|, \omega) - \tilde{G}(|\mathbf{r} - \mathbf{r}'|, \omega) \nabla \tilde{\Phi}(\mathbf{r}, \omega)) \cdot \hat{n} d^2 r$$

Exchanging $\mathbf{r} \leftrightarrow \mathbf{r}'$:

$$\int_V (\tilde{\Phi}(\mathbf{r}', \omega) \delta(\mathbf{r} - \mathbf{r}') - \tilde{G}(|\mathbf{r} - \mathbf{r}'|, \omega) f(\mathbf{r}', \omega)) d^3 r' =$$

$$\oint_S (\tilde{\Phi}(\mathbf{r}', \omega) \nabla \tilde{G}(|\mathbf{r} - \mathbf{r}'|, \omega) - \tilde{G}(|\mathbf{r} - \mathbf{r}'|, \omega) \nabla \tilde{\Phi}(\mathbf{r}', \omega)) \cdot \hat{n} d^2 r'$$

If the integration volume V includes the point $\mathbf{r} = \mathbf{r}'$:

$$\tilde{\Phi}(\mathbf{r}, \omega) = \int_V \tilde{G}(|\mathbf{r} - \mathbf{r}'|, \omega) f(\mathbf{r}', \omega) d^3 r' +$$

$$\oint_S (\tilde{\Phi}(\mathbf{r}', \omega) \nabla \tilde{G}(|\mathbf{r} - \mathbf{r}'|, \omega) - \tilde{G}(|\mathbf{r} - \mathbf{r}'|, \omega) \nabla \tilde{\Phi}(\mathbf{r}', \omega)) \cdot \hat{n} d^2 r'$$

→ extra contributions from boundary

11/8/2019

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18

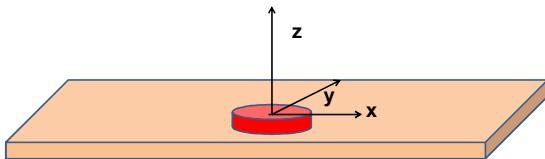
18

Wave equation with source:

$$\nabla^2 \Phi - \frac{1}{c^2} \frac{\partial^2 \Phi}{\partial t^2} = -f(\mathbf{r}, t)$$

Example:

$f(\mathbf{r}, t) \Rightarrow$ time harmonic piston of radius a , amplitude $\varepsilon \hat{\mathbf{z}}$
can be represented as boundary value of $\Phi(\mathbf{r}, t)$



11/8/2019

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19

Treatment of boundary values for time-harmonic force:

$$\tilde{\Phi}(\mathbf{r}, \omega) = \int_V \tilde{G}(|\mathbf{r} - \mathbf{r}'|, \omega) \tilde{f}(\mathbf{r}', \omega) d^3 r' + \oint_S (\tilde{\Phi}(\mathbf{r}', \omega) \nabla' \tilde{G}(|\mathbf{r} - \mathbf{r}'|, \omega) - \tilde{G}(|\mathbf{r} - \mathbf{r}'|, \omega) \nabla' \tilde{\Phi}(\mathbf{r}', \omega)) \cdot \hat{\mathbf{n}} d^2 r'$$

Boundary values for our example :

$$\left(\frac{\partial \tilde{\Phi}}{\partial z} \right)_{z=0} = \begin{cases} 0 & \text{for } x^2 + y^2 > a^2 \\ i\omega \varepsilon a & \text{for } x^2 + y^2 < a^2 \end{cases}$$

Note: Need Green's function with vanishing gradient at $z = 0$:

$$\tilde{G}(|\mathbf{r} - \mathbf{r}'|, \omega) = \frac{e^{ik|\mathbf{r} - \mathbf{r}'|}}{4\pi |\mathbf{r} - \mathbf{r}'|} + \frac{e^{ik|\bar{\mathbf{r}} - \bar{\mathbf{r}}'|}}{4\pi |\bar{\mathbf{r}} - \bar{\mathbf{r}}'|} \quad \text{where } \bar{z}' = -z'; \quad z > 0$$

11/8/2019

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20

$$\tilde{\Phi}(\mathbf{r}, \omega) = - \oint_{S: z=0} \tilde{G}(|\mathbf{r} - \mathbf{r}'|, \omega) \frac{\partial \tilde{\Phi}(\mathbf{r}', \omega)}{\partial z} dx' dy'$$

$$\tilde{G}(|\mathbf{r} - \mathbf{r}'|, \omega) = \frac{e^{ik|\mathbf{r} - \mathbf{r}'|}}{4\pi |\mathbf{r} - \mathbf{r}'|} + \frac{e^{ik|\bar{\mathbf{r}} - \bar{\mathbf{r}}'|}}{4\pi |\bar{\mathbf{r}} - \bar{\mathbf{r}}'|} \quad \text{where } \bar{z}' = -z'; \quad z > 0$$

$$\tilde{G}(|\mathbf{r} - \mathbf{r}'|, \omega)_{z=0} = \left. \frac{e^{ik|\mathbf{r} - \mathbf{r}'|}}{2\pi |\mathbf{r} - \mathbf{r}'|} \right|_{z=0}; \quad z > 0$$

11/8/2019

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21

$$\begin{aligned}\tilde{\Phi}(\mathbf{r}, \omega) &= - \oint_{S: z'=0} \tilde{G}(\mathbf{r} - \mathbf{r}', \omega) \frac{\partial \tilde{\Phi}(\mathbf{r}', \omega)}{\partial z'} dx' dy' \\ &= -i\omega \epsilon a \int_0^a r' dr' \int_0^{2\pi} d\phi' \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{2\pi |\mathbf{r}-\mathbf{r}'|} \Big|_{z'=0}\end{aligned}$$

Integration domain: $x' = r' \cos \phi'$
 $y' = r' \sin \phi'$

For $r \gg a$; $|\mathbf{r} - \mathbf{r}'| \approx r - \hat{\mathbf{r}} \cdot \mathbf{r}'$

Assume $\hat{\mathbf{r}}$ is in the yz plane; $\phi = \frac{\pi}{2}$

$$\hat{\mathbf{r}} = \sin \theta \hat{\mathbf{y}} + \cos \theta \hat{\mathbf{z}}$$

$$|\mathbf{r} - \mathbf{r}'| \approx r - \hat{\mathbf{r}} \cdot \mathbf{r}' = r - r' \sin \theta \sin \phi'$$

11/8/2019

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22

$$\tilde{\Phi}(\mathbf{r}, \omega) = -\frac{i\omega \epsilon a}{2\pi} \int_0^a r' dr' \int_0^{2\pi} d\phi' e^{-ikr' \sin \theta \sin \phi'}$$

$$\text{Note that: } \frac{1}{2\pi} \int_0^{2\pi} d\phi' e^{-iu \sin \phi'} = J_0(u)$$

$$\Rightarrow \tilde{\Phi}(\mathbf{r}, \omega) = -i\omega \epsilon a \frac{e^{ikr}}{r} \int_0^a r' dr' J_0(kr' \sin \theta)$$

$$\int_0^w u du J_0(u) = w J_1(w)$$

$$\Rightarrow \tilde{\Phi}(\mathbf{r}, \omega) = -i\omega \epsilon a^3 \frac{e^{ikr}}{r} \frac{J_1(ka \sin \theta)}{ka \sin \theta}$$

11/8/2019

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23

Energy flux: $\mathbf{j}_e = \delta \mathbf{v} p$

$$\begin{aligned}\text{Taking time average: } \langle \mathbf{j}_e \rangle &= \frac{1}{2} \Re(\delta \mathbf{v} p^*) \\ &= \frac{1}{2} \rho_0 \Re((-\nabla \Phi)(-\iota \omega \Phi)^*)\end{aligned}$$

Time averaged power per solid angle:

$$\left\langle \frac{dP}{d\Omega} \right\rangle = \langle \mathbf{j}_e \cdot \hat{\mathbf{r}} r^2 \rangle = \frac{1}{2} \rho_0 \epsilon^2 c^3 k^4 a^6 \left| \frac{J_1(ka \sin \theta)}{ka \sin \theta} \right|^2$$

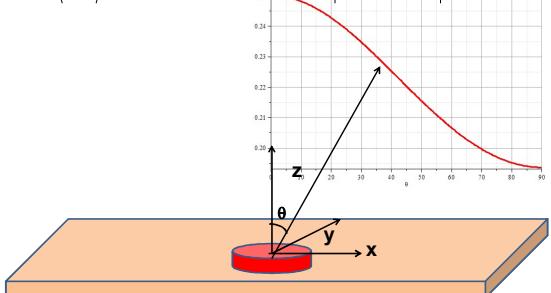
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24

Time averaged power per solid angle :

$$\left\langle \frac{dP}{d\Omega} \right\rangle = \langle \mathbf{j}_e \cdot \hat{\mathbf{r}} r^2 \rangle = \frac{1}{2} \rho_0 c^2 c^3 k^4 a^6 \left| \frac{J_1(k a \sin \theta)}{k a \sin \theta} \right|^2$$



11/8/2019

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25

25

Scattering of sound waves –
for example, from a rigid cylinder

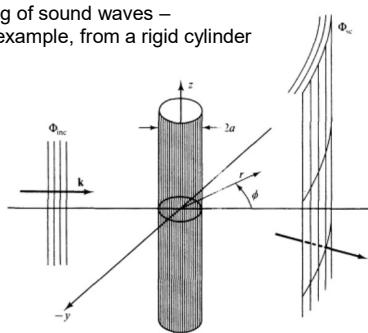


Figure 51.8 Scattering from a rigid cylinder.

Figure from Fetter and Walecka pg. 337

11/8/2019

PHY 711 Fall 2019 -- Lecture 29

26

26

Scattering of sound waves –
for example, from a rigid cylinder

Velocity potential --

$$\Phi(\mathbf{r}) = \Phi_{inc}(\mathbf{r}) + \Phi_{sc}(\mathbf{r}) \quad \Phi_{inc}(\mathbf{r}) = e^{i\mathbf{k} \cdot \mathbf{r}}$$

Helmholz equation in cylindrical coordinates:

$$(\nabla^2 + k^2)\Phi(\mathbf{r}) = 0 = \left(\frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial}{\partial \phi^2} + \frac{\partial}{\partial z^2} + k^2 \right) \Phi(\mathbf{r})$$

$$\text{Assume: } \Phi(\mathbf{r}) = \sum_{m=-\infty}^{\infty} e^{im\phi} R_m(r)$$

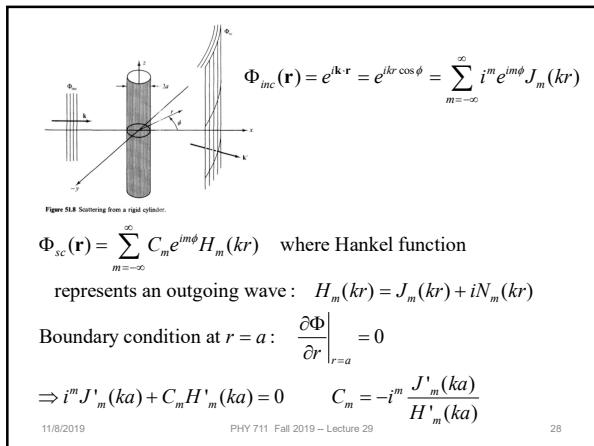
$$\text{where } \left(\frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - \frac{m^2}{r^2} + k^2 \right) R_m(r) = 0$$

11/8/2019

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27

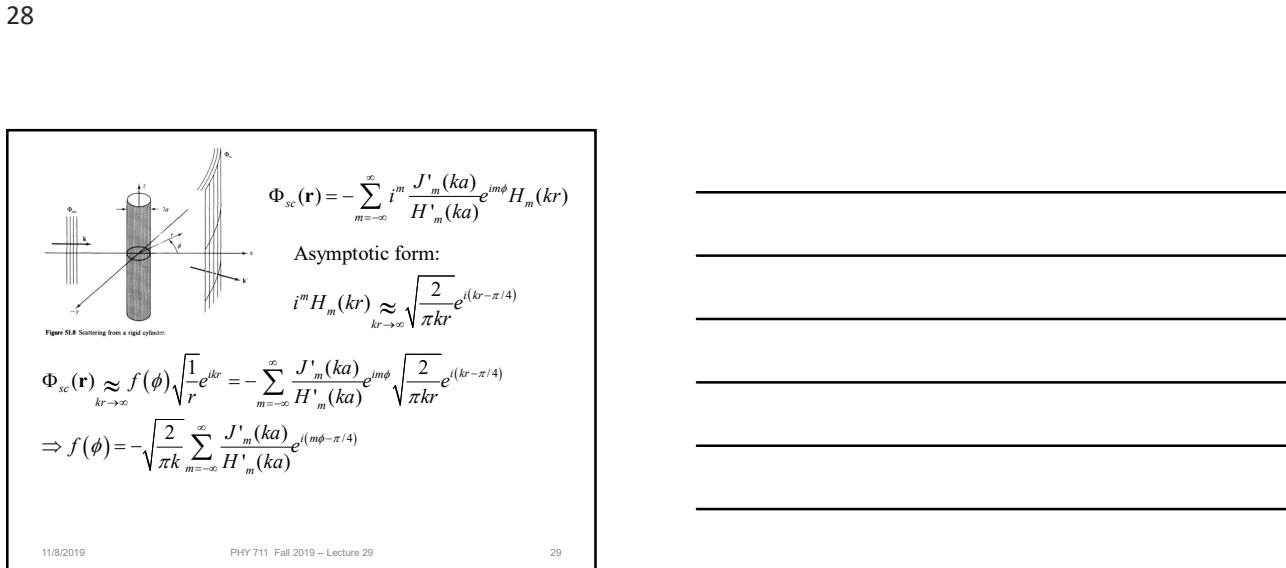
27



11/8/2019

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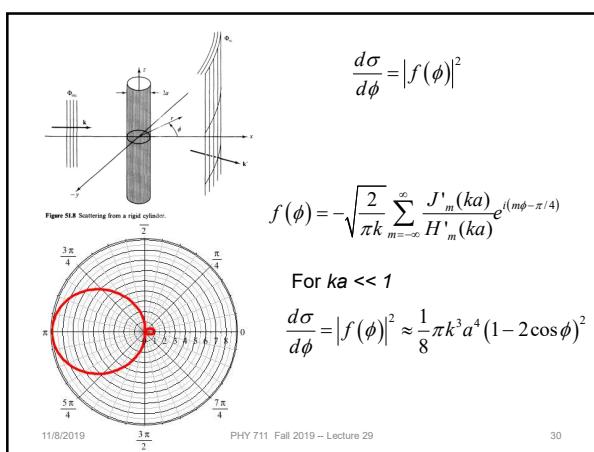
28



11/8/2019

PHY 711 Fall 2019 -- Lecture 29

29



11/8/2019

PHY 711 Fall 2019 -- Lecture 29

30