PHY 711 Classical Mechanics and Mathematical Methods
10-10:50 AM MWF Olin 103

Plan for Lecture 2: -- Chap. 1 of FW

1. **Brief comment on quiz**

2. **Introduction to scattering theory**

3. **Example of scattering of hard spheres**

4. **General particle interactions**

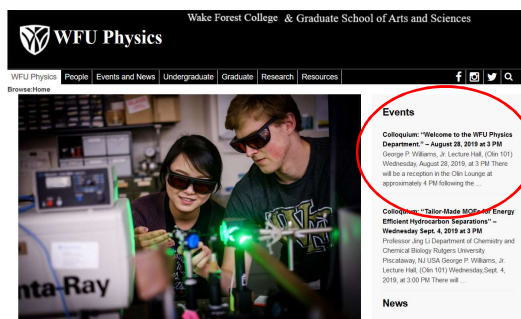5. **Laboratory and center of mass reference fame**

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Comment about Physics Colloquia

http://www.physics.wfu.edu

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Colloquium: "Welcome to the WFU Physics Department." – August 28, 2019 at 3 PM

Posted on August 8, 2019
George P. Williams, Jr. Lecture Hall, (Olin 101)
Wednesday, August 28, 2019, at 3 PM

There will be a reception in the Olin Lounge at approximately 4 PM following the colloquium. All interested persons are cordially invited to attend.

PROGRAM

The purpose of this first seminar is to help new, returning, and prospective students (including both undergraduate and graduate students), faculty, and staff to become acquainted with each other and with the Physics Department. We will meet in the George P. Williams, Jr. Lecture Hall (Olin 101) at 3:00 PM for presentations by some undergraduate students highlighting their summer research experiences, followed by general welcoming statements and departmental announcements.

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PHY 711 Classical Mechanics and Mathematical Methods

MWF 10 AM-10:50 AM │ OPL 103 │ http://www.wfu.edu/~natalie/f19phy711/

Instructor: Natalie Holzwarth │ Phone:758-5510 │ Office:300 OPL │ e-mail:natalie@wfu.edu

Course schedule

(Preliminary schedule -- subject to frequent adjustment.)

	Date	F&W Reading	Topic	Assignment	Due
1	Mon, 8/26/2019	Chap. 1	Introduction	#1	8/30/2019
2	Wed, 8/28/2019	Chap. 1	Scattering theory	#2	9/02/2019
3	Fri, 8/30/2019	Chap. 1	Scattering theory		
4	Mon, 9/02/2019	Chap. 1	Scattering theory		
5	Wed, 9/04/2019	Chap. 1	Scattering theory		
6	Fri, 9/06/2019	Chap. 2	Non-inertial coordinate systems		
7	Mon, 9/9/2019	Chap. 3	Calculus of Variation		
8	Wed, 9/11/2019	Chap. 3	Calculus of Variation		

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PHY 711 -- Assignment #2

Aug. 28, 2019

Read Chapter 1 in **Fetter & Walecka**.

1. In class, we started the derivation of the differential cross section for the elastic scattering of a beam of particles of mass m having an initial velocity $\mathbf{u_i}$ hitting a stationary spherical hard sphere target having mass M (uniformly distributed) with mutual radius R, scattered at an angle θ in the laboratory frame of reference. Complete the derivation to find the expression for the differential cross section as a function of R.

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Comment on quiz questions

1.
$$g(t) = \int_0^t \left(x^2 + t\right) dx \qquad \frac{dg}{dt} = \int_0^t \frac{d\left(x^2 + t\right)}{dt} dt + \left(x^2 + t\right)\Big|_{x=t}$$

$$= \int_0^t dt + \left(t^2 + t\right) = t^2 + 2t$$

2. Evaluate the integral $\oint \frac{dz}{z}$ for a closed contour about the origin.

Suppose that $z = e^{i\theta} \qquad dz = e^{i\theta} i d\theta \qquad \oint \frac{dz}{z} = \int_0^{2\pi} \frac{e^{i\theta} i d\theta}{e^{i\theta}} = 2\pi i$

3. $\dfrac{df}{dx} = f \qquad \Rightarrow f(x) = A e^x \qquad f(x) = 1 \quad \Rightarrow A = 1$

4. $\sum_{n=1}^{N} a^n = \dfrac{a - a^{N+1}}{1 - a} \qquad$ Let $S \equiv \sum_{n=1}^{N} a^n \quad$ Note that $\quad aS - S = a^{N+1} - a$

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Scattering theory:
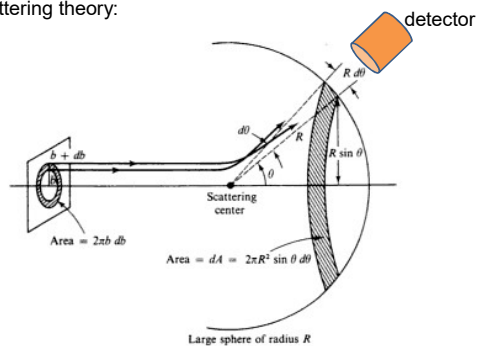
detector

Figure 5.5 The scattering problem and relation of cross section to impact parameter.
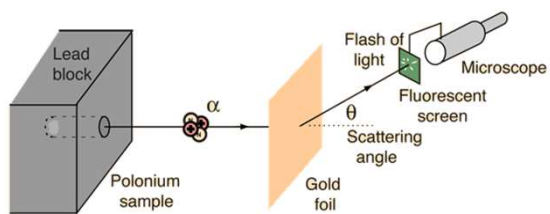
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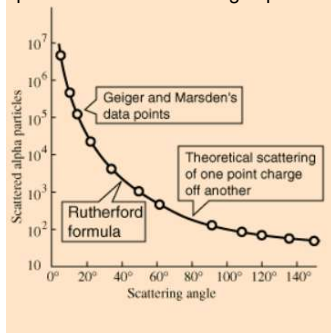Example: Diagram of Rutherford scattering experiment
http://hyperphysics.phy-astr.gsu.edu/hbase/rutsca.html

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Graph of data from scattering experiment

From website: http://hyperphysics.phy-astr.gsu.edu/hbase/Nuclear/rutsca2.html

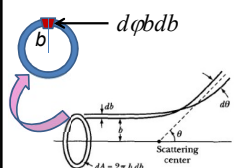8/28/2019   PHY 711 Fall 2019 -- Lecture 2   9

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Standardization of scattering experiments --

Differential cross section

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{\text{Number of detected particles at } \theta \text{ per target particle}}{\text{Number of incident particles per unit area}}$$

= Area of incident beam that is scattered into detector

at angle θ Impact parameter: b

$d\varphi b db$

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{d\varphi\, b\, db}{d\varphi \sin\theta\, d\theta} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$$

Figure from Marion & Thorton, Classical Dynamics

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Note: The notion of cross section is common to many areas of physics including classical mechanics, quantum mechanics, optics, etc. Only in the **classical mechanics** can we calculate it from a knowledge of the particle trajectory as it relates to the scattering geometry.
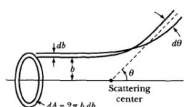
$d\varphi b db$

Figure from Marion & Thorton, Classical Dynamics

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{d\varphi\, b\, db}{d\varphi \sin\theta\, d\theta} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$$

Note: We are assuming that the process is isotropic in φ
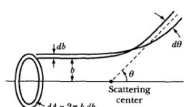
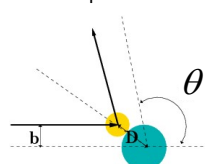8/28/2019   PHY 711 Fall 2019 -- Lecture 2   11

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Simple example – collision of hard spheres
having mutual radius D; very large target mass

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$$

Microscopic view: $b(\theta) = ?$

$$b(\theta) = D\sin\left(\frac{\pi}{2} - \frac{\theta}{2}\right)$$

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{D^2}{4}$$

θ

b

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Simple example – collision of hard spheres -- continued

Total scattering cross section:

$$\sigma = \int \left(\frac{d\sigma}{d\Omega}\right) d\Omega$$

Area $= 2\pi b\, db$

Area $= dA = 2\pi R^2 \sin\theta\, d\theta$

Large sphere of radius R

Hard sphere:

θ

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{D^2}{4}$$

$$\sigma = \pi D^2$$

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More details of hard sphere scattering –

Hidden in the analysis are assumptions about the scattering process such as:
- No external forces ➔ linear momentum is conserved
- No dissipative phenomena ➔ energy is conserved
- No torque on the system ➔ angular momentum is conserved
- Target particle is much more massive than scattering particle
- Other assumptions??

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More general treatment of hard sphere scattering

mass m

$\mathbf{v}$
β θ
$\mathbf{u}$ α
α
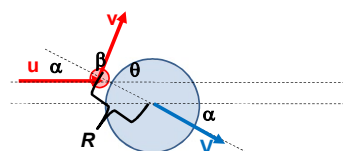$\mathbf{V}$
R

mass M

Impact parameter: $b = R \sin\alpha$

Differential cross section $\dfrac{d\sigma}{d\Omega} = \dfrac{b}{\sin\theta}\dfrac{db}{d\theta}$

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Continued treatment of hard sphere scattering

Conservation of momentum

$$mu\cos\alpha = MV - mv\cos\beta$$

$$mu\sin\alpha = mv\sin\beta$$

Conservation of energy
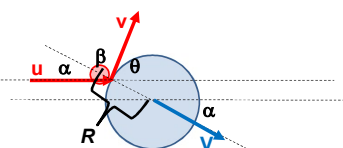
$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + \frac{1}{2}MV^2$$

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Continued treatment of hard sphere scattering

Relationships between the angles:

$$\alpha + \beta + \theta = \pi$$

Solution strategy:
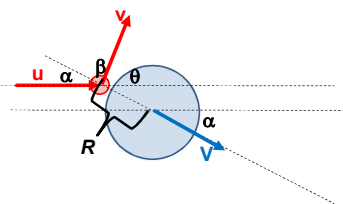
Define $x \equiv \dfrac{m}{M}$

Solve for $b(\theta)$

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Continued treatment of hard sphere scattering

Possibly useful identity

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

When the dust clears :

for $x < 1$

$$\frac{d\sigma}{d\Omega} = \frac{R^2}{4}\left(2x\cos\theta + \frac{1 + x^2\cos 2\theta}{\sqrt{1 - x^2\sin^2\theta}}\right)$$
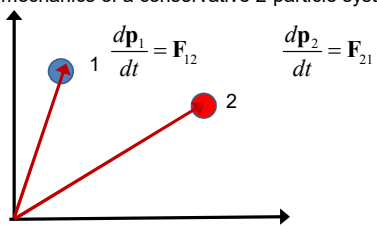
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Relationship of scattering cross-section to particle interactions --
Classical mechanics of a conservative 2-particle system.

$$\frac{d\mathbf{p}_1}{dt} = \mathbf{F}_{12} \qquad \frac{d\mathbf{p}_2}{dt} = \mathbf{F}_{21}$$

1

2

$$\mathbf{F}_{12} = -\nabla_1 V\left(\mathbf{r}_1 - \mathbf{r}_2\right) \quad \Rightarrow E = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + V\left(\mathbf{r}_1 - \mathbf{r}_2\right)$$

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Typical two-particle interactions –

Central potential:　　$V\left(\mathbf{r}_1 - \mathbf{r}_2\right) = V\left(\left|\mathbf{r}_1 - \mathbf{r}_2\right|\right) \equiv V(r)$

Hard sphere:　　$V(r) = \begin{cases} \infty & r \le a \\ 0 & r > a \end{cases}$

Coulomb or gravitational:　　$V(r) = \dfrac{K}{r}$

Lennard-Jones:　　$V(r) = \dfrac{A}{r^{12}} - \dfrac{B}{r^{6}}$

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