

PHY 711 Classical Mechanics and Mathematical Methods
10-10:50 AM MWF Olin 103

Plan for Lecture 34

Viscous fluids – Chap. 12 in F & W

1. Viscous stress tensor
2. Navier-Stokes equation
3. Example for incompressible fluid

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21	Fri, 10/18/2019	Chap. 7	Contour integrals; Exam due	#15	10/23/2019
22	Mon, 10/21/2019	Chap. 7	More about contour integrals		
23	Wed, 10/23/2019	Chap. 5	Rigid body motion	#16	10/25/2019
24	Fri, 10/25/2019	Chap. 5	Rigid body motion	#17	10/28/2019
25	Mon, 10/28/2019	Chap. 8	Elastic two-dimensional membranes	#18	11/01/2019
26	Wed, 10/30/2019	Chap. 9	Mechanics of 3 dimensional fluids		
27	Fri, 11/01/2019	Chap. 9	Fluid mechanics	#19	11/04/2019
28	Mon, 11/04/2019	Chap. 9	Sound waves		
	Wed, 11/06/2019	No class	NAWH out of town		
29	Fri, 11/08/2019	Chap. 9	Sound waves; Project Topic due	#20	11/11/2019
30	Mon, 11/11/2019	Chap. 9	Non-linear waves and shocks	#21	11/15/2019
31	Wed, 11/13/2019	Chap. 10	Surface waves in water	#22	11/18/2019
32	Fri, 11/15/2019	Chap. 10	Surface waves -- non linear effects		
33	Mon, 11/18/2019	Chap. 11	Heat conduction	#23	11/22/2019
34	Wed, 11/20/2019	Chap. 12	Effects of viscosity on fluid dynamics		
35	Fri, 11/22/2019				
36	Mon, 11/25/2019				
	Wed, 11/27/2019		Thanksgiving holiday		
	Fri, 11/29/2019		Thanksgiving holiday		
	Mon, 12/2/2019		Presentations I		
	Wed, 12/4/2019		Presentations II		
	Fri, 12/6/2019		Presentations III		

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Events

Colloquium: "From Enzyme-Catalyzed Living Polymerization Reactions to Microfluidic Shear Wave Resonator Sensors" - Wednesday Nov. 13, 2019 at 3 PM
 Professor Stefan Zauchner Department of Mechanical Engineering and Materials Science Duke University George P. Williams, Jr. Lecture Hall (Olin 101) Wednesday, Nov. 13, 2019, at 3:00 PM There will be a reception following the talk.

Colloquium: "Understanding and Tailoring Molecular Co-adsorption in Nanoporous Materials" - Wednesday Nov. 20, 2019 at 3 PM
 Kui Yan, PhD Research Scientist The University of Texas at Austin Department of Materials Science and Engineering George P. Williams, Jr. Lecture Hall (Olin 101), Wednesday, Nov 20, 2019 at 3:00 PM There will be a reception following the talk.

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Equations for motion of non-viscous fluid

Newton-Euler equation of motion:

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \rho(\mathbf{v} \cdot \nabla) \mathbf{v} = \rho \mathbf{f}_{\text{applied}} - \nabla p$$

Continuity equation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad \Rightarrow \mathbf{v} \left(\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) \right) = 0$$

Add two equations:

$$\underbrace{\rho \frac{\partial \mathbf{v}}{\partial t} + \frac{\partial \rho}{\partial t} \mathbf{v}}_{\partial(\rho \mathbf{v})} + \underbrace{\rho(\mathbf{v} \cdot \nabla) \mathbf{v} + \rho \nabla \cdot (\rho \mathbf{v})}_{\sum_{j=1}^3 \frac{\partial}{\partial x_j} (\rho v_j \mathbf{v})} = \rho \mathbf{f}_{\text{applied}} - \nabla p$$

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Equations for motion of non-viscous fluid -- continued

Newton-Euler equation in terms of momentum:

$$\frac{\partial(\rho \mathbf{v})}{\partial t} + \sum_{j=1}^3 \frac{\partial(\rho v_j \mathbf{v})}{\partial x_j} = \rho \mathbf{f}_{\text{applied}} - \nabla p$$

$$\frac{\partial(\rho \mathbf{v})}{\partial t} + \sum_{j=1}^3 \frac{\partial(\rho v_j \mathbf{v})}{\partial x_j} + \nabla p = \rho \mathbf{f}_{\text{applied}}$$

Fluid momentum: $\rho \mathbf{v}$

Stress tensor: $T_{ij} \equiv \rho v_i v_j + p \delta_{ij}$

i^{th} component of Newton-Euler equation:

$$\frac{\partial(\rho v_i)}{\partial t} + \sum_{j=1}^3 \frac{\partial T_{ij}}{\partial x_j} = \rho f_i$$

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Now consider the effects of viscosity

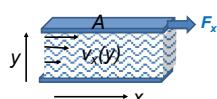
In terms of stress tensor:

$$T_{ij} = T_{ij}^{\text{ideal}} + T_{ij}^{\text{viscous}}$$

$$T_{ij}^{\text{ideal}} = \rho v_i v_j + p \delta_{ij} = T_{ji}^{\text{ideal}}$$

Newton's "law" of viscosity

$$\frac{F_x}{A} = \eta \frac{\partial v_x}{\partial y}$$



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Effects of viscosity

Argue that viscosity is due to shear forces in a fluid of the form:

$$\frac{F_{drag}}{A} = \eta \frac{\partial v_x}{\partial y}$$

Formulate viscosity stress tensor with traceless and diagonal terms:

$$T_{kl}^{\text{viscous}} = -\eta \left(\frac{\partial v_k}{\partial x_l} + \frac{\partial v_l}{\partial x_k} - \frac{2}{3} \delta_{kl} (\nabla \cdot \mathbf{v}) \right) - \zeta \delta_{kl} (\nabla \cdot \mathbf{v})$$

Total stress tensor: $T_{kl} = T_{kl}^{\text{ideal}} + T_{kl}^{\text{viscous}}$

$$T_{kl}^{\text{ideal}} = \rho v_k v_l + p \delta_{kl}$$

$$T_{kl}^{\text{viscous}} = -\eta \left(\frac{\partial v_k}{\partial x_l} + \frac{\partial v_l}{\partial x_k} - \frac{2}{3} \delta_{kl} (\nabla \cdot \mathbf{v}) \right) - \zeta \delta_{kl} (\nabla \cdot \mathbf{v})$$

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Effects of viscosity -- continued

Incorporating generalized stress tensor into Newton-Euler equations

$$\frac{\partial(\rho v_i)}{\partial t} + \sum_{j=1}^3 \frac{\partial T_{ij}}{\partial x_j} = \rho f_i$$

$$\frac{\partial(\rho v_i)}{\partial t} + \sum_{j=1}^3 \frac{\partial(\rho v_i v_j)}{\partial x_j} = \rho f_i - \frac{\partial p}{\partial x_i} + \eta \sum_{j=1}^3 \frac{\partial^2 v_i}{\partial x_j^2} + \left(\zeta + \frac{1}{3} \eta \right) \sum_{j=1}^3 \frac{\partial^2 v_j}{\partial x_i \partial x_j}$$

Continuity condition

$$\frac{\partial \rho}{\partial t} + \sum_{j=1}^3 \frac{\partial(\rho v_j)}{\partial x_j} = 0$$

Vector form (Navier-Stokes equation)

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = \mathbf{f} - \frac{1}{\rho} \nabla p + \frac{\eta}{\rho} \nabla^2 \mathbf{v} + \frac{1}{\rho} \left(\zeta + \frac{1}{3} \eta \right) \nabla (\nabla \cdot \mathbf{v})$$

Continuity condition

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

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Newton-Euler equations for viscous fluids

Navier-Stokes equation

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = \mathbf{f} - \frac{1}{\rho} \nabla p + \frac{\eta}{\rho} \nabla^2 \mathbf{v} + \frac{1}{\rho} \left(\zeta + \frac{1}{3} \eta \right) \nabla (\nabla \cdot \mathbf{v})$$

Continuity condition

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

Typical viscosities at 20° C and 1 atm:

Fluid	η/ρ (m²/s)	η (Pa s)
Water	1.00×10^{-6}	1×10^{-3}
Air	14.9×10^{-6}	0.018×10^{-3}
Ethyl alcohol	1.52×10^{-6}	1.2×10^{-3}
Glycerine	1183×10^{-6}	1490×10^{-3}

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Example – steady flow of an incompressible fluid in a long pipe with a circular cross section of radius R

Navier-Stokes equation

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = \mathbf{f} - \frac{1}{\rho} \nabla p + \frac{\eta}{\rho} \nabla^2 \mathbf{v} + \frac{1}{\rho} \left(\zeta + \frac{1}{3} \eta \right) \nabla (\nabla \cdot \mathbf{v})$$

Continuity condition

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad \text{Note that } \nabla \times (\nabla \times \mathbf{v}) = \nabla (\nabla \cdot \mathbf{v}) - \nabla^2 \mathbf{v}$$

Incompressible fluid $\Rightarrow \nabla \cdot \mathbf{v} = 0$

$$\text{Steady flow} \Rightarrow \frac{\partial \mathbf{v}}{\partial t} = 0$$

$$\text{Irrational flow} \Rightarrow \nabla \times \mathbf{v} = 0$$

$$\text{No applied force} \Rightarrow \mathbf{f} = 0$$

$$\text{Neglect non-linear terms} \Rightarrow \nabla (v^2) = 0$$

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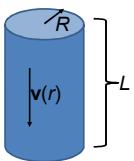
Example – steady flow of an incompressible fluid in a long pipe with a circular cross section of radius R -- continued

Navier-Stokes equation becomes:

$$0 = -\frac{1}{\rho} \nabla p + \frac{\eta}{\rho} \nabla^2 \mathbf{v}$$

$$\text{Assume that } \mathbf{v}(\mathbf{r}, t) = v_z(r) \hat{\mathbf{z}}$$

$$\frac{\partial p}{\partial z} = \eta \nabla^2 v_z(r) \quad (\text{independent of } z)$$



$$\text{Suppose that } \frac{\partial p}{\partial z} = -\frac{\Delta p}{L} \quad (\text{uniform pressure gradient})$$

$$\Rightarrow \nabla^2 v_z(r) = -\frac{\Delta p}{\eta L}$$

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Example – steady flow of an incompressible fluid in a long pipe with a circular cross section of radius R -- continued

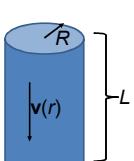
$$\nabla^2 v_z(r) = -\frac{\Delta p}{\eta L}$$

$$\frac{1}{r} \frac{d}{dr} r \frac{dv_z(r)}{dr} = -\frac{\Delta p}{\eta L}$$

$$v_z(r) = -\frac{\Delta p r^2}{4 \eta L} + C_1 \ln(r) + C_2$$

$$\Rightarrow C_1 = 0 \quad v_z(R) = 0 = -\frac{\Delta p R^2}{4 \eta L} + C_2$$

$$v_z(r) = \frac{\Delta p}{4 \eta L} (R^2 - r^2)$$



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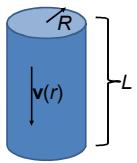
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Example – steady flow of an incompressible fluid in a long pipe with a circular cross section of radius R -- continued

$$v_z(r) = \frac{\Delta p}{4\eta L} (R^2 - r^2)$$

Mass flow rate through the pipe:

$$\frac{dM}{dt} = 2\pi\rho \int_0^R r dr v_z(r) = \frac{\Delta p \rho \pi R^4}{8\eta L}$$



Poiseuille formula;

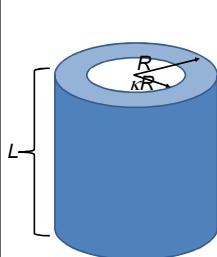
→ Method for measuring η

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Example – steady flow of an incompressible fluid in a long tube with a circular cross section of outer radius R and inner radius κR



$$\nabla^2 v_z(r) = -\frac{\Delta p}{\eta L}$$

$$\frac{1}{r} \frac{d}{dr} r \frac{dv_z(r)}{dr} = -\frac{\Delta p}{\eta L}$$

$$v_z(r) = -\frac{\Delta p r^2}{4\eta L} + C_1 \ln(r) + C_2$$

$$v_z(R) = 0 = -\frac{\Delta p R^2}{4\eta L} + C_1 \ln(R) + C_2$$

$$v_z(\kappa R) = 0 = -\frac{\Delta p \kappa^2 R^2}{4\eta L} + C_1 \ln(\kappa R) + C_2$$

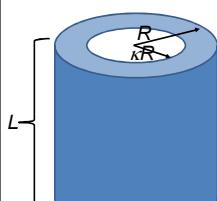
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Example – steady flow of an incompressible fluid in a long tube with a circular cross section of outer radius R and inner radius κR -- continued

Solving for C_1 and C_2 :



$$v_z(r) = \frac{\Delta p R^2}{4\eta L} \left(1 - \left(\frac{r}{R} \right)^2 - \frac{1 - \kappa^2}{\ln \kappa} \ln \left(\frac{r}{R} \right) \right)$$

Mass flow rate through the pipe:

$$\frac{dM}{dt} = 2\pi\rho \int_{\kappa R}^R r dr v_z(r) = \frac{\Delta p \rho \pi R^4}{8\eta L} \left(1 - \kappa^4 + \frac{(1 - \kappa^2)^2}{\ln \kappa} \right)$$

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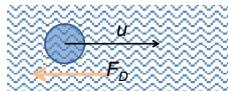
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More discussion of viscous effects in incompressible fluids

Stokes' analysis of viscous drag on a sphere of radius R moving at speed u in medium with viscosity η :

$$F_D = -\eta(6\pi Ru)$$



Plan:

1. Consider the general effects of viscosity on fluid equations
2. Consider the solution to the linearized equations for the case of steady-state flow of a sphere of radius R
3. Infer the drag force needed to maintain the steady-state flow

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Newton-Euler equation for incompressible fluid, modified by viscous contribution (Navier-Stokes equation):

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = \mathbf{f}_{\text{applied}} - \frac{\nabla p}{\rho} + \frac{\eta}{\rho} \nabla^2 \mathbf{v}$$

ν Kinematic viscosity

Typical kinematic viscosities at 20° C and 1 atm:

Fluid	ν (m ² /s)
Water	1.00×10^{-6}
Air	14.9×10^{-6}
Ethyl alcohol	1.52×10^{-6}
Glycerine	1183×10^{-6}

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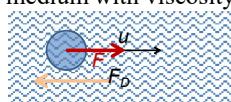
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Stokes' analysis of viscous drag on a sphere of radius R moving at speed u in medium with viscosity η :

$$F_D = -\eta(6\pi Ru)$$



Effects of drag force on motion of

particle of mass m with constant force F :

$$F - 6\pi R \eta u = m \frac{du}{dt} \quad \text{with } u(0) = 0$$

$$\Rightarrow u(t) = \frac{F}{6\pi R \eta} \left(1 - e^{-\frac{6\pi R \eta}{m} t} \right)$$

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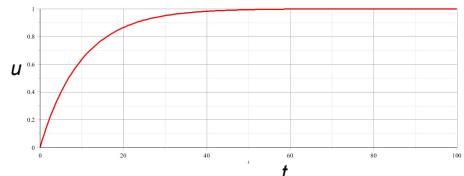
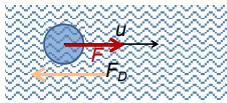
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Effects of drag force on motion of particle of mass m with constant force F :

$$F - 6\pi R \eta u = m \frac{du}{dt} \quad \text{with } u(0) = 0$$

$$\Rightarrow u(t) = \frac{F}{6\pi R \eta} \left(1 - e^{-\frac{6\pi R \eta t}{m}} \right)$$



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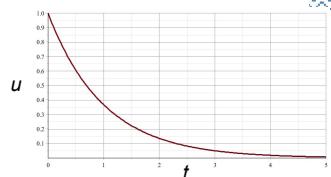
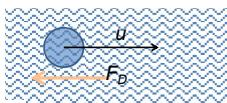
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Effects of drag force on motion of particle of mass m with an initial velocity with $u(0) = U_0$ and no external force

$$-6\pi R \eta u = m \frac{du}{dt}$$

$$\Rightarrow u(t) = U_0 e^{-\frac{6\pi R \eta t}{m}}$$



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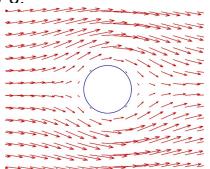
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Recall: PHY 711 -- Assignment #19 Nov. 1, 2019

Determine the form of the velocity potential for an incompressible fluid representing uniform velocity in the \mathbf{z} direction at large distances from a spherical obstruction of radius a . Find the form of the velocity potential and the velocity field for all $r > a$. Assume that for $r = a$, the velocity in the radial direction is 0 but the velocity in the azimuthal direction is not necessarily 0.

$$\nabla^2 \Phi = 0$$

$$\Phi(r, \theta) = -v_0 \left(r + \frac{a^3}{2r^2} \right) \cos \theta$$



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Newton-Euler equation for incompressible fluid,
modified by viscous contribution (Navier-Stokes equation):

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = \mathbf{f}_{\text{applied}} - \frac{\nabla p}{\rho} + \frac{\eta}{\rho} \nabla^2 \mathbf{v}$$

Continuity equation: $\nabla \cdot \mathbf{v} = 0$

Assume steady state: $\Rightarrow \frac{\partial \mathbf{v}}{\partial t} = 0$

Assume non-linear effects small

Initially set $\mathbf{f}_{\text{applied}} = 0$;

$$\Rightarrow \nabla p = \eta \nabla^2 \mathbf{v}$$

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$$\nabla p = \eta \nabla^2 \mathbf{v}$$

Take curl of both sides of equation:

$$\nabla \times (\nabla p) = 0 = \eta \nabla^2 (\nabla \times \mathbf{v})$$

Assume (with a little insight from Landau):

$$\mathbf{v} = \nabla \times (\nabla \times f(r) \mathbf{u}) + \mathbf{u}$$

where $f(r) \xrightarrow[r \rightarrow \infty]{} 0$

Note that:

$$\nabla \times (\nabla \times \mathbf{A}) = \nabla (\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$$

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Digression

Some comment on assumption: $\mathbf{v} = \nabla \times (\nabla \times f(r) \mathbf{u}) + \mathbf{u}$

$$\nabla \times (\nabla \times \mathbf{A}) = \nabla (\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$$

Here $\mathbf{A} = f(r) \mathbf{u}$

$$\nabla \times \mathbf{v} = \nabla \times (\nabla \times (\nabla \times \mathbf{A})) = -\nabla \times (\nabla^2 \mathbf{A})$$

Also note: $\nabla p = \eta \nabla^2 \mathbf{v}$

$$\Rightarrow \nabla \times \nabla p = \nabla \times \eta \nabla^2 \mathbf{v} \quad \text{or} \quad \nabla^2 (\nabla \times \mathbf{v}) = 0$$

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$$\mathbf{v} = \nabla \times (\nabla \times f(r) \mathbf{u}) + \mathbf{u}$$

$$\mathbf{u} = u \hat{\mathbf{z}}$$

$$\nabla \times (\nabla \times f(r) \hat{\mathbf{z}}) = \nabla(\nabla \cdot f(r) \hat{\mathbf{z}}) - \nabla^2 f(r) \hat{\mathbf{z}}$$

$$\nabla \times \mathbf{v} = 0 \quad \Rightarrow \nabla^2 (\nabla \times \mathbf{v}) = 0$$

$$\nabla^4 (\nabla \times f(r) \hat{\mathbf{z}}) = 0 \quad \Rightarrow \nabla^4 (\nabla f(r) \times \hat{\mathbf{z}}) = 0 \quad \Rightarrow \nabla^4 f(r) = 0$$

$$f(r) = C_1 r^2 + C_2 r + C_3 + \frac{C_4}{r}$$

$$v_r = u \cos \theta \left(1 - \frac{2}{r} \frac{df}{dr} \right) = u \cos \theta \left(1 - 4C_1 - \frac{2C_2}{r} + \frac{2C_4}{r^3} \right)$$

$$v_\theta = -u \sin \theta \left(1 - \frac{d^2 f}{dr^2} - \frac{1}{r} \frac{df}{dr} \right) = -u \sin \theta \left(1 - 4C_1 - \frac{C_2}{r} - \frac{C_4}{r^3} \right)$$

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Some details:

$$\nabla^4 f(r) = 0 \quad \Rightarrow \left(\frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr} \right)^2 f(r) = 0$$

$$f(r) = C_1 r^2 + C_2 r + C_3 + \frac{C_4}{r}$$

$$\mathbf{v} = u (\nabla \times (\nabla \times f(r) \hat{\mathbf{z}}) + \hat{\mathbf{z}})$$

$$= u (\nabla (\nabla \cdot (f(r) \hat{\mathbf{z}})) - \nabla^2 f(r) \hat{\mathbf{z}} + \hat{\mathbf{z}})$$

Note that: $\hat{\mathbf{z}} = \cos \theta \hat{\mathbf{r}} - \sin \theta \hat{\theta}$

$$\mathbf{v} = u \left(\nabla \left(\frac{df}{dr} \cos \theta \right) - (\nabla^2 (f(r)) - 1) (\cos \theta \hat{\mathbf{r}} - \sin \theta \hat{\theta}) \right)$$

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$$v_r = u \cos \theta \left(1 - \frac{2}{r} \frac{df}{dr} \right) = u \cos \theta \left(1 - 4C_1 - \frac{2C_2}{r} + \frac{2C_4}{r^3} \right)$$

$$v_\theta = -u \sin \theta \left(1 - \frac{d^2 f}{dr^2} - \frac{1}{r} \frac{df}{dr} \right) = -u \sin \theta \left(1 - 4C_1 - \frac{C_2}{r} - \frac{C_4}{r^3} \right)$$

To satisfy $\mathbf{v}(r \rightarrow \infty) = \mathbf{u}$: $\Rightarrow C_1 = 0$ To satisfy $\mathbf{v}(R) = 0$ solve for C_2, C_4

$$v_r = u \cos \theta \left(1 - \frac{3R}{2r} + \frac{R^3}{2r^3} \right)$$

$$v_\theta = -u \sin \theta \left(1 - \frac{3R}{4r} - \frac{R^3}{4r^3} \right)$$

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$$v_r = u \cos \theta \left(1 - \frac{3R}{2r} + \frac{R^3}{2r^3} \right)$$

$$v_\theta = -u \sin \theta \left(1 - \frac{3R}{4r} - \frac{R^3}{4r^3} \right)$$

Determining pressure :

$$\nabla p = \eta \nabla^2 \mathbf{v} = -\eta \nabla \left(u \cos \theta \left(\frac{3R}{2r^2} \right) \right)$$

$$\Rightarrow p = p_0 - \eta u \cos \theta \left(\frac{3R}{2r^2} \right)$$

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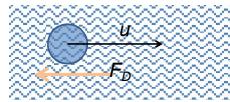
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$$p = p_0 - \eta u \cos \theta \left(\frac{3R}{2r^2} \right)$$

Corresponds to:

$$F_D \cos \theta = (p(R) - p_0) 4\pi R^2$$

$$\Rightarrow F_D = -\eta u (6\pi R)$$



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