PHY 337/637 Analytical Mechanics
12:30-1:45 PM TR in Olin103
(Aug. 28-Oct. 17, 2023)

Lecture notes for Lecture 1 -- Chaps. 1 & 2 of Cline

Introduction, Motivation, and Review

1. Course structure and plan
2. Brief review of Newtonian mechanics
3. What is “analytical mechanics” and why should we study it?

Course website -- https://users.wfu.edu/natalie/f23phy337/

PHY 337/637 Analytical Mechanics

TR 12:30 -1:45 PM	**OPL 103**	**http://www.wfu.edu/~natalie/f23phy337/**

Instructor: Natalie Holzwarth	**Office:**300 OPL	**e-mail:**natalie@wfu.edu

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- General information
- Syllabus and homework assignments
- Lecture Notes

Fall 2023 Schedule
for N. A. W. Holzwarth

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<tr><th></th><th>Monday</th><th>Tuesday</th><th>Wednesday</th><th>Thursday</th><th>Friday</th></tr>
<tr><td>9:00-10:00</td><td>Lecture Preparation</td><td rowspan="2">Physics Research</td><td>Lecture Preparation</td><td rowspan="2">Physics Research</td><td>Lecture Preparation</td></tr>
<tr><td>10:00-11:00</td><td>Classical Mechanics: PHY 711</td><td>Classical Mechanics: PHY 711</td><td>Classical Mechanics: PHY 711</td></tr>
<tr><td>11:00-12:30</td><td>Office Hours</td><td>Lecture Preparation</td><td>Office Hours</td><td>Lecture Preparation</td><td rowspan="2">Office Hours</td></tr>
<tr><td>12:30-1:00</td><td rowspan="6">Physics Research</td><td rowspan="2">Analytical Mechanics: PHY 337/637</td><td rowspan="6">Physics Research</td><td rowspan="2">Analytical Mechanics: PHY 337/637</td></tr>
<tr><td>1:00-1:45</td><td rowspan="2">Condensed Matter Seminar</td></tr>
<tr><td>1:45-2:00</td><td rowspan="2">Office Hours</td><td rowspan="2">Office Hours</td></tr>
<tr><td>2:00-2:30</td><td rowspan="3">Physics Research</td></tr>
<tr><td>2:30-4:00</td><td rowspan="2">Physics Research</td><td>Physics Research</td></tr>
<tr><td>4:00-5:00</td><td>Physics Department Colloquium</td></tr>
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Note – Colloquium starts this week.

Note that the schedule for PHY 337/637 occurs during Aug 28 - Oct 17.

Physics Colloquium Series

Welcome to Fall 2023

Introductions, Presentations, and Announcements

4 PM in Olin 101

(Pre-Colloquium Reception in Olin Lobby at 3:30 PM)

WAKE FOREST
UNIVERSITY

August 31, 2023

General Information

This course is a one-half semester introduction to mathematical optimization tools (the calculus of variation) and their application to the analysis of the physics of motion (Lagrangian and Hamiltonian formalisms). The textbook is available online: Variational Principles in Classical Mechanics by Professor Douglas Cline from the University of Rochester. Students who prefer a print version of the textbook can order it from the website. Since this is a one-half semester course, we will focus on Chapters 5 - 8 and 13

The course will consist of the following components:

- In person meetings TR 12:30-1:45 PM in Olin 103.* Starting with the second meeting, the sessions will focus on discussion of the material, particularly answering your prepared and spontaneous questions.
- Asynchronous review of annotated lecture notes and corresponding textbook sections. Starting with Lecture 2, the annotated lecture notes will be available one day before the corresponding synchronous online discussion. For each class meeting, students will be expected to submit (by email) at least one question for class discussion at least 3 hours before the in person class meetings. The practice of formulating questions is one of the important tools of science. Note that there are no "silly" questions.
- Participation in weekly tutorial sessions, discussing example problems from each chapter from the textbook covered in class, concepts and theories, and possibly material associated with homework problems.
- Homework sets. Typically there will be one homework problem associated with each class meeting that is due the following Monday and which will be graded by the instructor.
- There will be 1 or 2 exams associated with this course. Details to be determined.

It is likely that your grade for the course will depend upon the following factors:

Class participation	35%
Problem sets*	35%
Exams	30%

*In general, there will a new assignment after each lecture, so that for optimal learning, it would be best to complete each assignment before the next scheduled lecture. According to the honor system, all work submitted for grading purposes should represent the student's own best efforts.

Dates of note:

- Classes begin: Mon. Aug. 28, 2023
- Fall break: Oct. 12-15, 2023
- Last class meeting for course: Tue Oct. 17, 2023

On-line textbook: Variational Principles in Classical Mechanics by Professor Douglas Cline from the University of Rochester

- Participation in weekly tutorial sessions, discussing example problems from each chapter from the textbook covered in class, concepts and theories, and possibly material associated with homework problems.

Graduate student in charge of tutorials –
Arezoo Nemeny namea22@wfu.edu

Chapter 2 of textbook

2: Review of Newtonian Mechanics

Douglas Cline
University of Rochester

2.1: Introduction to Newtonian Mechanics

Newtonian mechanics is based on application of Newton's Laws of motion which assume that the concepts of distance, time, and mass, are absolute, that is, motion is in an inertial frame. The Newtonian idea of the complete separation of space and time, and the concept of the absoluteness of time, are violated by the Theory of Relativity. However, for most practical applications, relativistic effects are negligible and Newtonian mechanics is an adequate description at low velocities.

2.2: Newton's Laws of motion

Newton's laws, expressed in terms of linear momentum, are: (1) Law of inertia: A body remains at rest or in uniform motion unless acted upon by a force. (2) Equation of motion: A body acted upon by a force moves in such a manner that the time rate of change of momentum equals the force. and (3) Action and reaction: If two bodies exert forces on each other these forces are equal in magnitude and opposite in direction.

2.E: Review of Newtonian Mechanics (Exercises)

10. A particle of mass m moving in one dimension has potential energy $U(x) = U_0[2(\frac{x}{a})^2 - (\frac{x}{a})^4]$, where U_0 and a are positive constants.

a. Find the force $F(x)$ that acts on the particle.

b. Sketch $U(x)$. Find the positions of stable and unstable equilibrium.

c. What is the angular frequency ω of oscillations about the point of stable equilibrium?

d. What is the minimum speed the particle must have at the origin to escape to infinity?

e. At $t = 0$ the particle is at the origin and its velocity is positive and equal to the escape velocity. Find $x(t)$ and sketch the result.

Course schedule

In the table below, **Reading** refers to the chapters in the Cline textbook , **PP** refers to textbook section listing practice problems to be discussed at the course tutorials, and **Assign** is a link to the graded homework for the lecture. The graded homeworks are due each Monday following the associated lecture.

(Preliminary schedule -- subject to frequent adjustment.)

	Date	Reading	Topic	PP	Assign
1	Tu, 8/29/2023	Ch. 1 & 2	Introduction, history, and motivation	2E	#1
2	Th, 8/31/2023	Ch. 5	Introduction to Calculus of variation		
3	Tu, 9/05/2023	Ch. 5	Calculus of variation		
4	Th, 9/07/2023	Ch. 5	Calculus of variation		
5	Tu, 9/12/2023				
6	Th, 9/14/2023				
7	Tu, 9/19/2023				

Topics to be covered –

- Calculus of variation (necessary mathematical tool)
- Lagrangian mechanics
- Hamiltonian mechanics
- Rigid body motion

PHY 337/637 – Assignment #1

Assigned: 08/29/2023 Due: 09/05/2023

This problem concerns the analysis of a function V of single variable x in terms of its maximum and minimum values. In this case V represents potential energy in units of Joules and x represents position in units of meters. In this case, for $0 \leq x \leq 3$,

$$V(x) = x^3 - 4x^2 + 5x - 2.$$

1. For this $V(x)$, find the values of x_m for which $\frac{dV}{dx}(x_m) = 0$.
2. For each of these values of x_m, identify whether $V(x_m)$ represents a maximum or minimum of the function $V(x)$.

Some of the contributors to basic physics principles

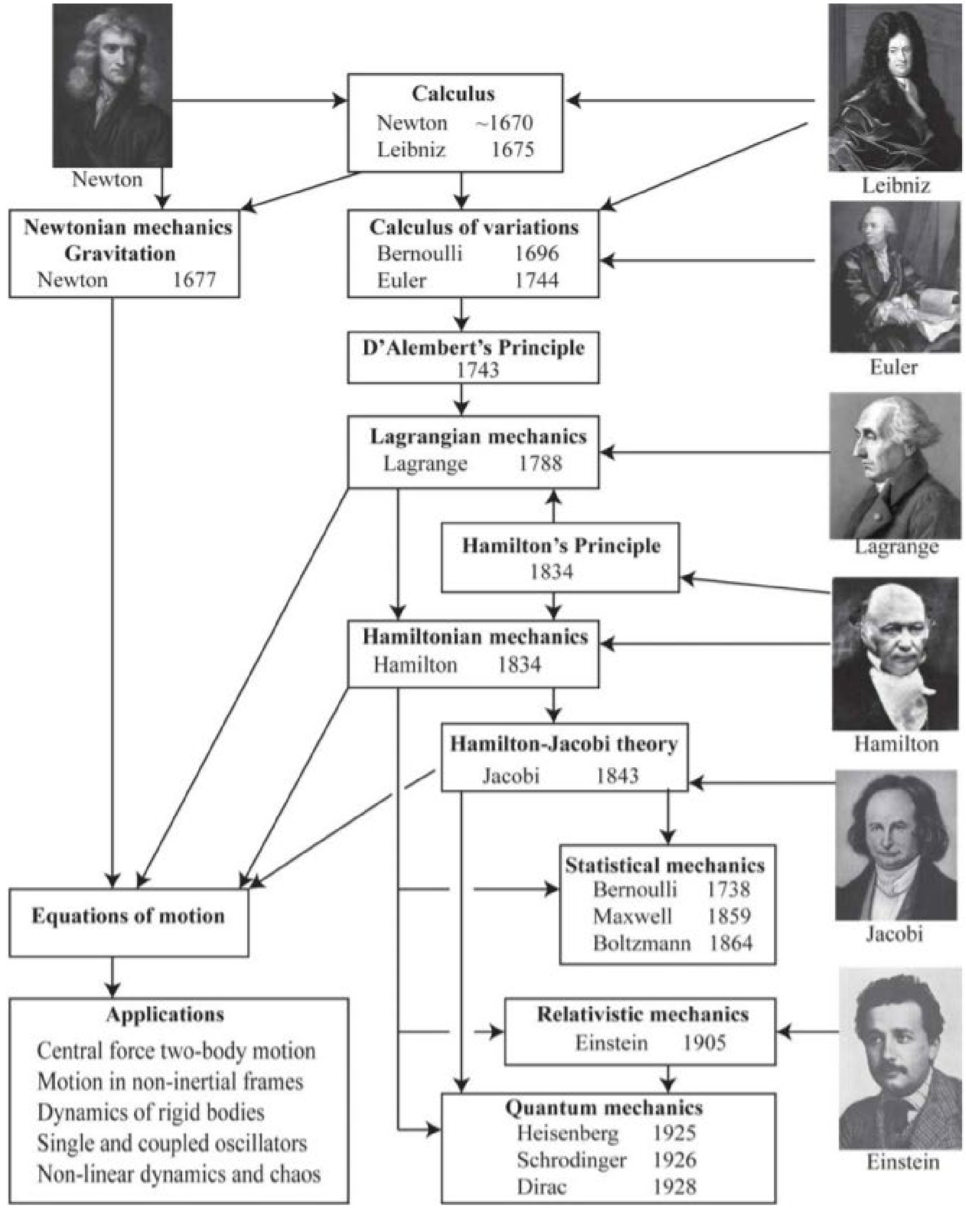
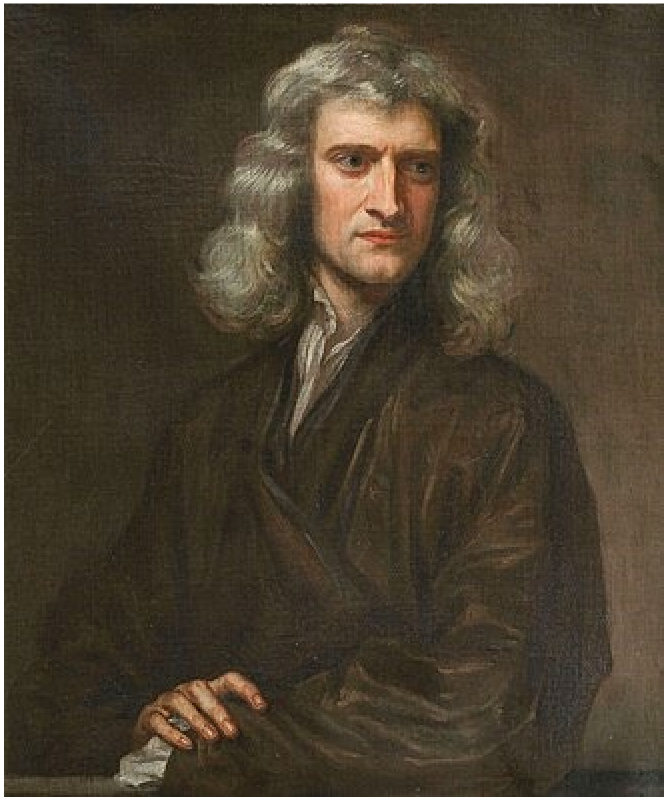


Figure 1.4.1: Chronological roadmap of the parallel development of the Newtonian and Variational-principles approaches to classical mechanics.



According to Wikipedia –
Sir Isaac Newton (1642-1726**)**
English mathematician, physicist, astronomer, alchemist, theologian, and author who was described in his time as a natural philosopher. He was a key figure in the Scientific Revolution and the Enlightenment that followed.

- **Whatever you learn in this course – Newton's laws are still true.**
- **Except when stated otherwise, we will focus on the physics of a single point particle of mass *m*.**

Newton's second law: $\mathbf{F} = m\mathbf{a}$

Vector quantities: $\mathbf{F} = F_x\hat{\mathbf{x}} + F_y\hat{\mathbf{y}} + F_z\hat{\mathbf{z}}$

$$\mathbf{a} = a_x\hat{\mathbf{x}} + a_y\hat{\mathbf{y}} + a_z\hat{\mathbf{z}}$$

$$\mathbf{a} = \frac{d^2x}{dt^2}\hat{\mathbf{x}} + \frac{d^2y}{dt^2}\hat{\mathbf{y}} + \frac{d^2z}{dt^2}\hat{\mathbf{z}}$$

Newton's second law: $\mathbf{F} = m\mathbf{a}$

$$F_x\hat{\mathbf{x}} + F_y\hat{\mathbf{y}} + F_z\hat{\mathbf{z}} = m\frac{d^2x}{dt^2}\hat{\mathbf{x}} + m\frac{d^2y}{dt^2}\hat{\mathbf{y}} + m\frac{d^2z}{dt^2}\hat{\mathbf{z}}$$

$$\Rightarrow m\frac{d^2x}{dt^2} = F_x \quad \text{and} \quad m\frac{d^2y}{dt^2} = F_y \quad \text{and} \quad m\frac{d^2z}{dt^2} = F_z$$

If $F_x, F_y,$ and F_z are known, we can find the particle trajectory $\mathbf{r}(t) = x(t)\hat{\mathbf{x}} + y(t)\hat{\mathbf{y}} + z(t)\hat{\mathbf{z}}$ provided that ??

Example -- $F_x = -Kx \quad F_y = F_z = 0$ and $y(t) = 0 = z(t)$

where K is a given constant. Given initial conditions:

$$x(t=0) = X_0 \text{ and } \frac{dx}{dt}(t=0) = 0$$

$$m\frac{d^2x}{dt^2} = -Kx$$

Solution: $$x(t) = X_0 \cos\left(\sqrt{\frac{K}{m}}t\right)$$

Newton's second law: $\mathbf{F} = m\mathbf{a}$

➔Related notions of energy

$$\mathbf{F} = m\mathbf{a} \quad \Rightarrow \quad \mathbf{F} \cdot d\mathbf{r} = m\mathbf{a} \cdot d\mathbf{r}$$

where $\quad d\mathbf{r} \equiv dx\hat{\mathbf{x}} + dy\hat{\mathbf{y}} + dz\hat{\mathbf{z}}$

$$\int_{\mathbf{r}_{\text{ref}}}^{\mathbf{r}} \mathbf{F} \cdot d\mathbf{r} = \int_{\mathbf{r}_{\text{ref}}}^{\mathbf{r}} m\mathbf{a} \cdot d\mathbf{r}$$

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$$-V(\mathbf{r}) \qquad = \frac{1}{2} m \frac{d\mathbf{r}(t)}{dt} \cdot \frac{d\mathbf{r}(t)}{dt} - \frac{1}{2} m \frac{d\mathbf{r}(t_{ref})}{dt} \cdot \frac{d\mathbf{r}(t_{ref})}{dt}$$

Is this always true?

Some details --

For a conservative force $\mathbf{F} = -\nabla V(\mathbf{r})$

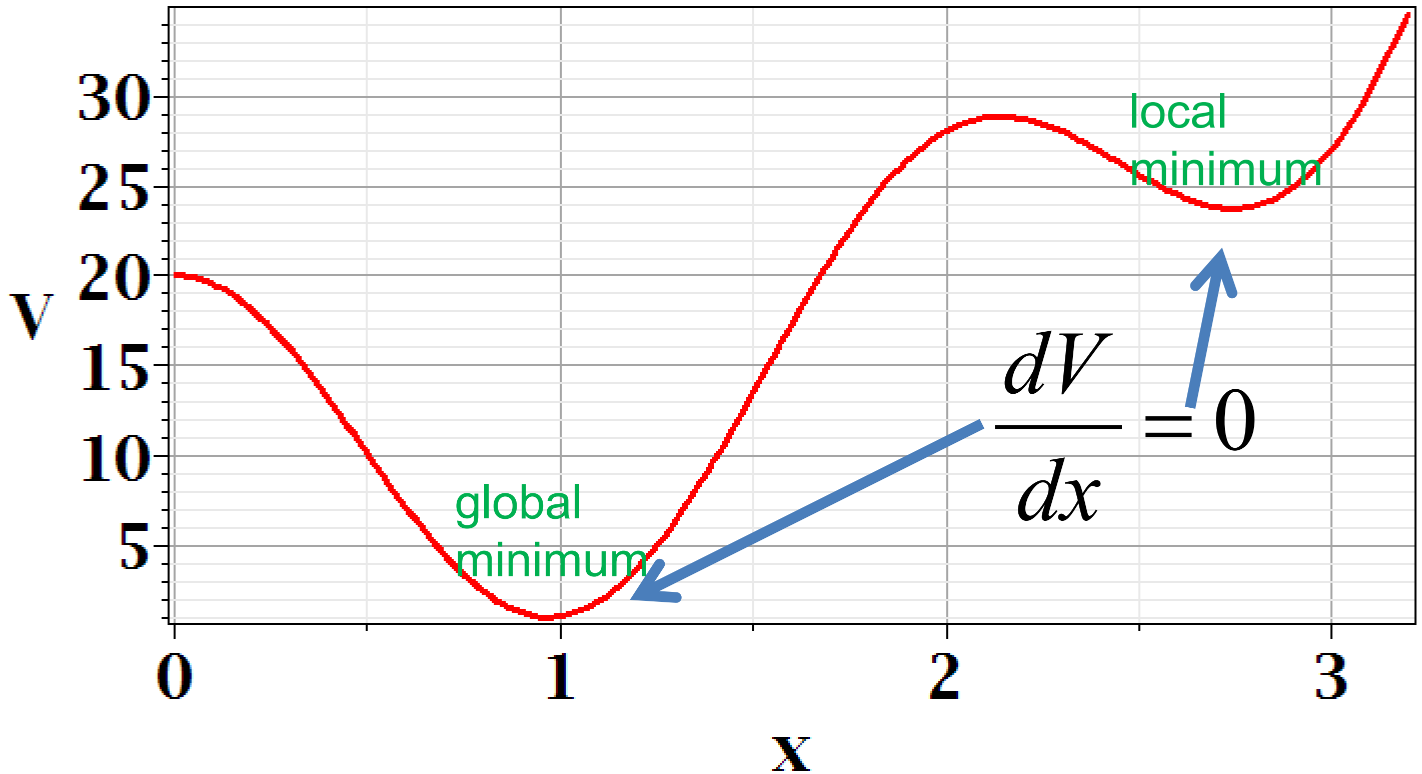
where $\int_{\mathbf{r}_{\rm ref}}^{\mathbf{r}} \mathbf{F} \cdot d\mathbf{r} \equiv -V(\mathbf{r})$

Note that not all forces are "conservative".

$$\int_{\mathbf{r}_{\rm ref}}^{\mathbf{r}} m\mathbf{a} \cdot d\mathbf{r} = \int_{t_{\rm ref}}^{t} m \frac{d^2\mathbf{r}}{dt^2} \cdot \frac{d\mathbf{r}}{dt} dt = \frac{1}{2} m \int_{t_{\rm ref}}^{t} \frac{d}{dt}\left(\frac{d\mathbf{r}}{dt} \cdot \frac{d\mathbf{r}}{dt} \right) dt$$

$$= \frac{1}{2} m \frac{d\mathbf{r}(t)}{dt} \cdot \frac{d\mathbf{r}(t)}{dt} - \frac{1}{2} m \frac{d\mathbf{r}(t_{ref})}{dt} \cdot \frac{d\mathbf{r}(t_{ref})}{dt}$$

Consider a potential function *V(x)* with the following shape:



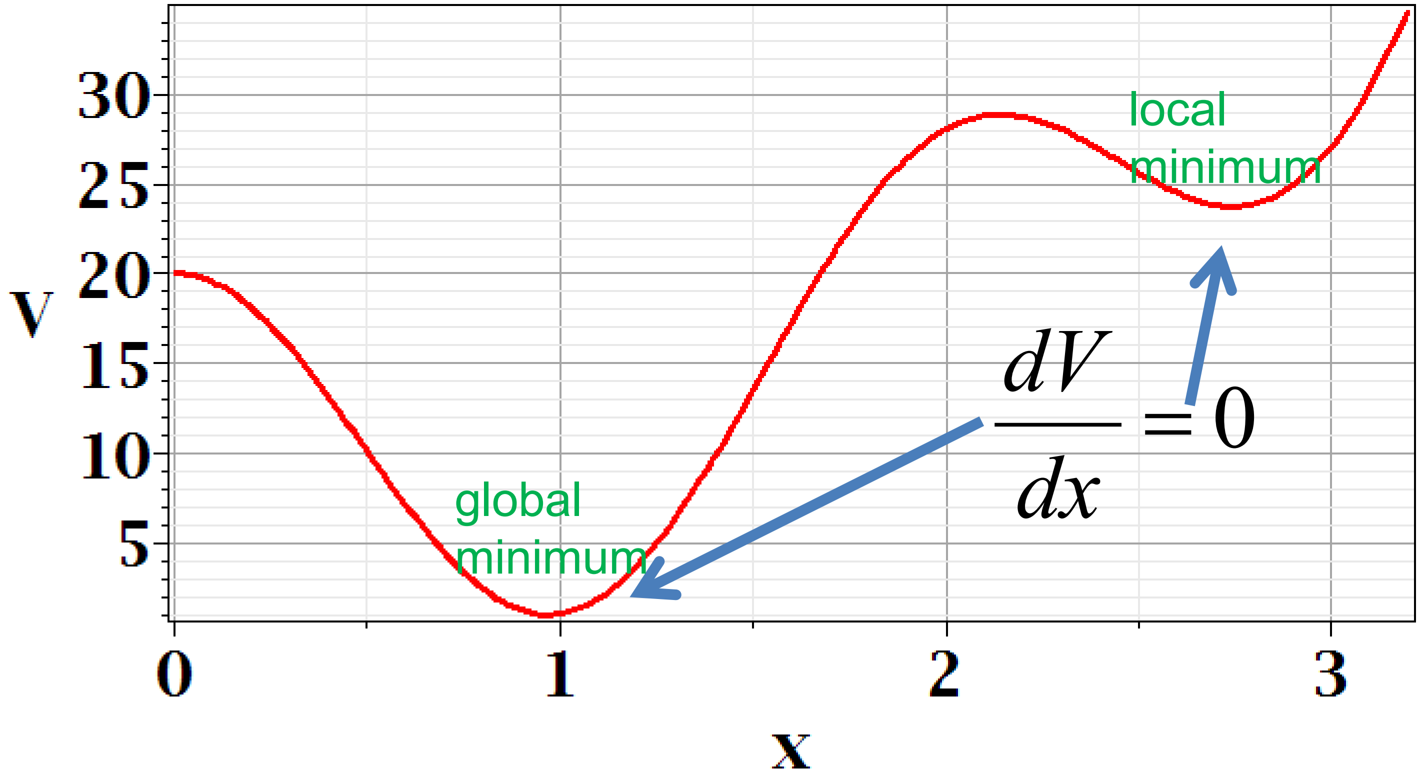
In this case, we see that there are some values of x for which $\frac{dV}{dx}=0$

Why might this be interesting/important?

Minimization of a simple function

Given a function $V(x)$, find the value(s) of x for which $V(x)$ is minimized (or maximized).

Necessary condition : $\frac{dV}{dx} = 0$



We have already seen that the search for minimum (or maximum) values of function is

1. Mathematically well-defined
2. Analytic and numerical methods are well-developed
3. Often physically significant

The calculus of variation is based on finding minimum values – in a different way.   This will be discussed in Lecture 2 and in Chapter 5 of your textbook.