PHY 712 Electrodynamics
10-10:50 AM MWF Olin 103

Notes for Lecture 1:

Reading: Appendix 1 and Chapters I&1

1. Course structure and expectations
2. Units – SI vs Gaussian
3. Electrostatics – Poisson equation

Comment on upcoming physics colloquia – https://physics.wfu.edu/wfu-phy-news/colloquium/seminar-spring-2025/

Physics Colloquium Schedule – Spring 2025

Previous and Future Colloquia

All colloquia will be held at 4 PM in Olin 101 (unless noted otherwise). Refreshments will be available at 3:30 PM in Olin Lobby prior to each seminar. For additional information contact wfuphys@wfu.edu.

Thurs. Jan. 16, 2025 — Dr. Tong Wang, Brigham and Women's Hospital, Harvard Medical School — "Quantitative methods for microbiome research and precision nutrition" (Host: D. Kim-Shapiro)

Thurs. Jan. 23, 2025 — Dr. Caitlin Witt, Center for Interdisciplinary Exploration and Research in Astrophysics (CIERA), Adler Planetarium, Northwestern University (Host: D. Kim-Shapiro)

Thurs. Jan. 30, 2025 — Dr. Allen Scheie, MPA-Q division Los Alamos National Laboratory (Host: D. Kim-Shapiro)

Thurs. Feb. 6, 2025 — Dr. Jingang Li, University of California, Berkeley — "Advancing Optical Manipulation and Measurement at the Nanoscale" (Host: D. Kim-Shapiro)

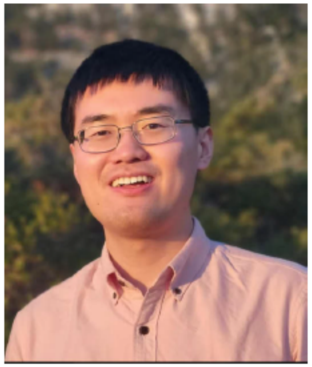
Physics Colloquium

- Thursday -

January 16, 2025

Quantitative methods for microbiome research and precision nutrition

Due to highly personalized biological and lifestyle characteristics, individuals may have different metabolic responses to specific foods and nutrients. In particular, the gut microbiota, a collection of trillions of microorganisms living in our gastrointestinal tract, is highly personalized and plays a key role in our metabolic responses to foods and nutrients. Characterizing the metabolic profile of a microbial community and accurately predicting metabolic responses to dietary interventions based on individuals' gut microbial compositions are crucial for understanding its impact on the host and hold great promise for precision nutrition. Here, I will present my past research that investigates the complex microbiome and precision nutrition using different computational methods, such as mathematical models, multi-omics data analysis, and deep learning methods. After that, I will include my vision for future research that solves ecological and biomedical problems through computational approaches.



Dr. Tong Wang
Brigham and Women's Hospital
Harvard Medical School

4 PM Olin 101

Reception 3:30
Olin Lobby

Colloquium 4:00

Spring 2025 Schedule
for N. A. W. Holzwarth

	Monday	Tuesday	Wednesday	Thursday	Friday
9:00-10:00	Lecture Preparation / Office Hours	Physics Research	Lecture Preparation / Office Hours	Physics Research	Lecture Preparation / Office Hours
10:00-11:00	Electrodynamics: PHY 712		Electrodynamics: PHY 712		Electrodynamics: PHY 712
11:00-12:00	Office Hours		Office Hours		Condensed Matter Seminar
12:00-4:00	Physics Research		Physics Research		Physics Research
4:00-5:00				Physics Department Colloquium	

If the posted "Office hours" are not convenient to your Schedule, NAWH is happy to meet with you at other times. Please send email to natalie@wfu.edu to set up a mutually convenient schedule.

http://users.wfu.edu/natalie/s25phy712/

PHY 712 Electrodynamics

MWF 10-10:50 AM	**Olin 103**	Webpage: **http://www.wfu.edu/~natalie/s25phy712/**

Instructor: Natalie Holzwarth	**Office:**300 OPL	**e-mail:**natalie@wfu.edu

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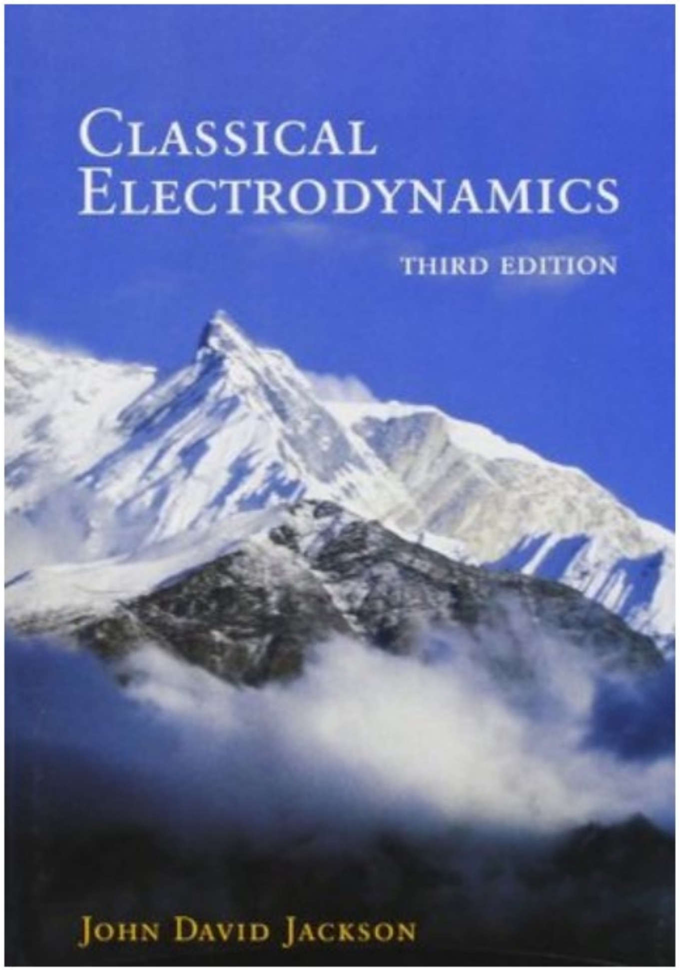
- General information
- Syllabus and homework assignments
- Lecture notes
- Some presentation ideas

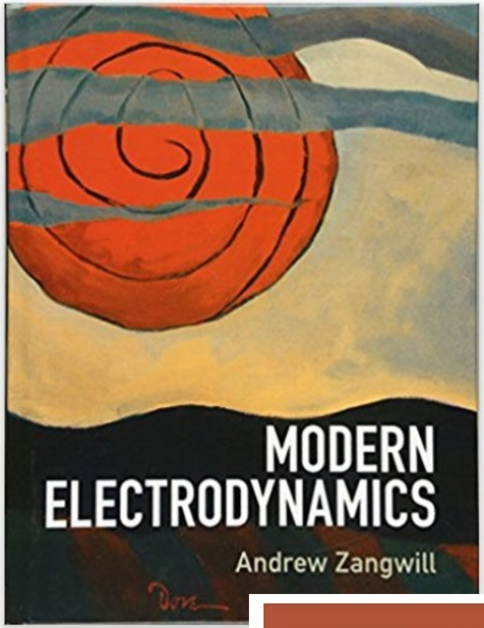
Last modfied: Thursday, 09-Jan-2025 14:56:01 EST

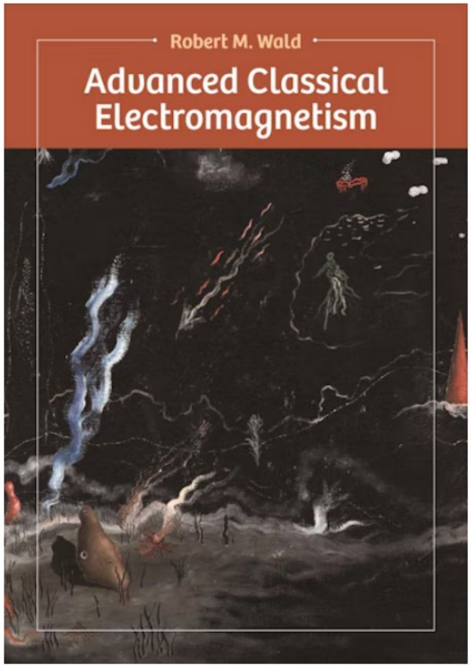
Optional supplements

Textbook



Third edition





http://users.wfu.edu/natalie/s25phy712/info

General Information

This course is a one semester survey of Electrodynamics at the graduate level, using the textbook: **Classical Electrodynamics**, 3rd edition, by John David Jackson (John Wiley & Sons, Inc., 1999) -- "**JDJ**". (link to errata for early printings) Note that it is necessary to get the **third** edition in order to synchronize with the class lectures and homework. The more recent textbook: **Modern Electrodynamics**, by Andrew Zangwill (Cambridge University Press, 2013) will be used as a supplement. LINK An even more recent textbook: **Advanced Classical Electromagnetism**, by Robert M. Wald (Princeton University Press, 2022) may be of interest to some of you. LINK

The course will consist of the following components:

- In person meetings in Olin 103 MWF 10-10:50 AM. Zoom connections can be made available if requested, but not on a regular basis. The class sessions will focus on discussion of the material, particularly answering your prepared and spontaneous questions.
- Asynchronous review of annotated lecture notes and corresponding textbook sections. The reading assignment and annotated lecture notes will be available one day before the corresponding synchronous online discussion. For each class meeting, students will be expected to submit (by email) at least one question for class discussion at least 3 hours before the class meeting.
- Homework sets. Typically there will be one homework problem associated with each class meeting.
- There will be two take-home exams, one at mid-term and the other during finals week.
- There will be one project on a chosen topic related to electrodynamics.
- It is highly recommended that each student arrange for weekly one-on-one meetings with the instructor to discuss the course material, homework, and/or projects. These may be face-to-face or online as appropriate.

It is likely that your grade for the course will depend upon the following factors:

Class participation	15%
Problem sets*	35%
Project	15%
Exams	35%

*In general, there will a new assignment after each lecture, so that for optimal learning, it would be best to complete each assignment before the next scheduled lecture. According to the honor system, all work submitted for grading purposes should represent the student's own best efforts.

➔ Email your questions >= 1 hour before each class, schedule weekly one-on-one meetings, and/or attend office hours

Today's questions   from Paoblo:

I am not sure how to link the definition for Gauss' Law for continuum distributions and for discrete charge distributions. Is it possible to write the continuum definition such that it simplifies to the discrete one? Maybe using delta functions? Same question would apply for the superposition of electric fields. If we had a discrete system which can initially be modeled as continuous, how would the distribution function be rewritten as we randomly took out particles from the system? Also, looking at Gauss' differential form, for a point charge, it would suggest that it "dilutes" inside the Gaussian surface, it works for Gauss' Law but is not correct. How should I interpret this?

Some short (incomplete) answers – The discrete and continuous formulations have to be equivalent.   In today's lecture, some details – such as boundary values have been omitted.

Some Ideas for Computational Project

The purpose of the "Computational Project" is to provide an opportunity for you to study a topic of your choice in greater depth. The general guideline for your choice of project is that it should have something to do with electrodynamics, and there should be some degree of computation or analysis with the project. The completed project will include a short write-up and a ~20min presentation to the class. You may design your own project or use one of the following list (which will be updated throughout the term).

- Evaluate the Ewald sum of various ionic crystals using Maple or a programing language. (Template available in Fortran code.)
- Work out the details of the finite difference or finite element methods.
- Work out the details of the hyperfine Hamiltonian as discussed in Chapter 5 of Jackson.
- Work out the details of Jackson problem 7.2 and related problems.
- Work out the details of reflection and refraction from birefringent materials.
- Analyze the Kramers-Kronig transform of some optical data or calculations.
- Determine the classical electrodynamics associated with an infrared or optical laser.
- Analyze the radiation intensity and spectrum from an interesting source such as an atomic or molecular transition, a free electron laser, etc.
- Work out the details of Jackson problem 14.15.

http://users.wfu.edu/natalie/s25phy712/homework/

Course schedule for Spring 2025

(Preliminary schedule -- subject to frequent adjustment.)

	Lecture date	JDJ Reading	Topic	HW	Due date
1	Mon: 01/13/2025	Chap. 1 & Appen.	Introduction, units and Poisson equation	#1	01/15/2025
2	Wed: 01/15/2025	Chap. 1	Electrostatic energy calculations	#2	01/17/2025
3	Fri: 01/17/2025	Chap. 1	Electrostatic energy calculations	#3	01/22/2025
	Mon: 01/20/2025	No Class	Martin Luther King Jr. Holiday		

There will be a grade penalty for late homework

PHY 712 – Problem Set #1

Assigned: 01/13/2025 Due: 01/15/2025

Read Chapters I and 1 and Appendix 1 in **Jackson**.

1. In SI units, the electrostatic interaction energy between two point particles is given by:

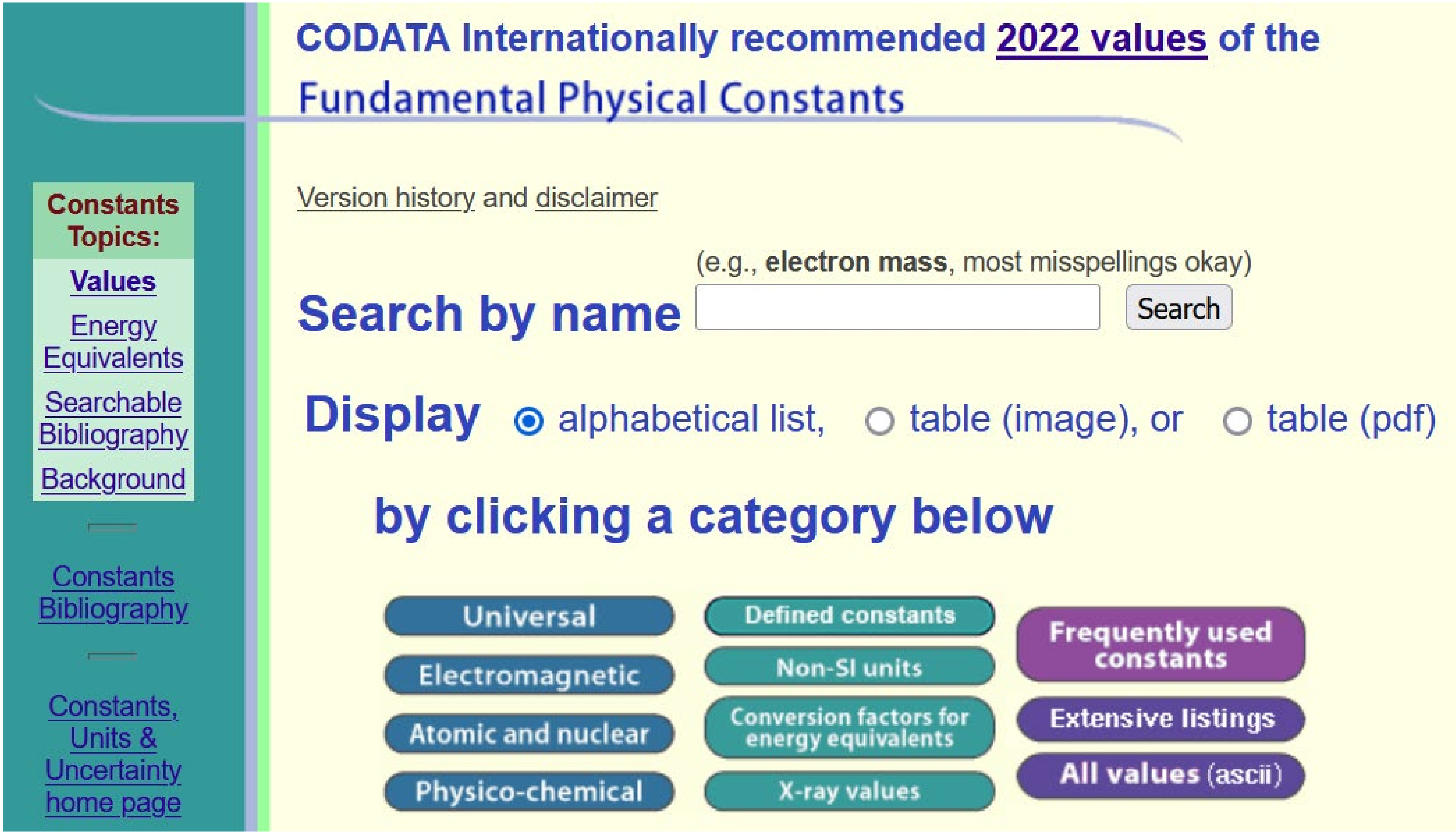
$$E_{ES} = \frac{e^2}{4\pi\varepsilon_0}\frac{z_1 z_2}{r},$$

where e denotes the elementary charge unit (in Coulombs), ε_0 denotes the permittivity of vacuum, z_i denotes the charge of particle i in units of e, and r denotes the separation of the two particles in units of m. Suppose that $z_1 = z_2 = 1$ and that $r = 2$ Å.

   (a) Using the standard values of the constants from the NIST website (`https://physics.nist.gov/cuu/Constants/`), determine E_{ES} in SI units (Joules).

   (b) Using the standard values of the constants from the NIST website (`https://physics.nist.gov/cuu/Constants/`), determine E_{ES} in eV units.

https://physics.nist.gov/cuu/Constants/



The NIST Reference on
Constants, Units, and Uncertainty

Constants Topics:

Values

Energy Equivalents

Searchable Bibliography

Background

Constants Bibliography

Constants, Units & Uncertainty home page

Fundamental Physical Constants

Conversion from J to eV

Conversion equation:	1 J = x eV $x = 1/\{e\}$
Value of conversion factor:	x = 6.241 509 074... x 10^{18}
Your input value:	1.000 000 000 000 00... J
Your converted value:	6.241 509 074 460 76... x 10^{18} eV

Note: {*Q*} is the numerical value of ***Q*** when ***Q*** is expressed in SI units

Source: 2022 CODATA recommended values

Definition of uncertainty

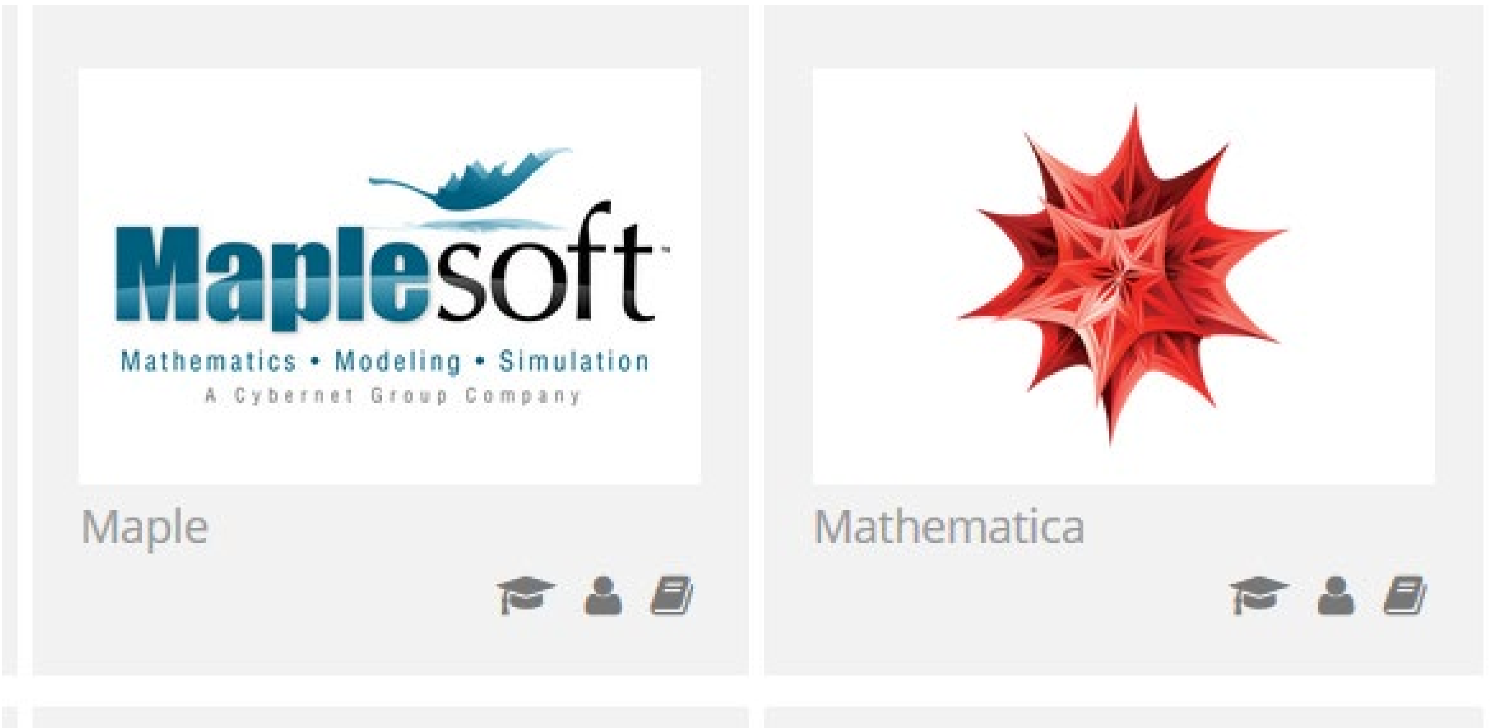
Basis of conversion factors for energy equivalents

Tentative additional information –

Spring break March 10-14
Mid term grades due March 10 (based on HW only)
Take-home mid-term exam March 17-21 (no class)
Mon Apr 28 – Last day of class
May 1-7 – Final exams

Remember to check your algebraic manipulation software --

https://software.wfu.edu/audience/students/



Material discussed in Appendix of textbook --

Units - SI vs Gaussian

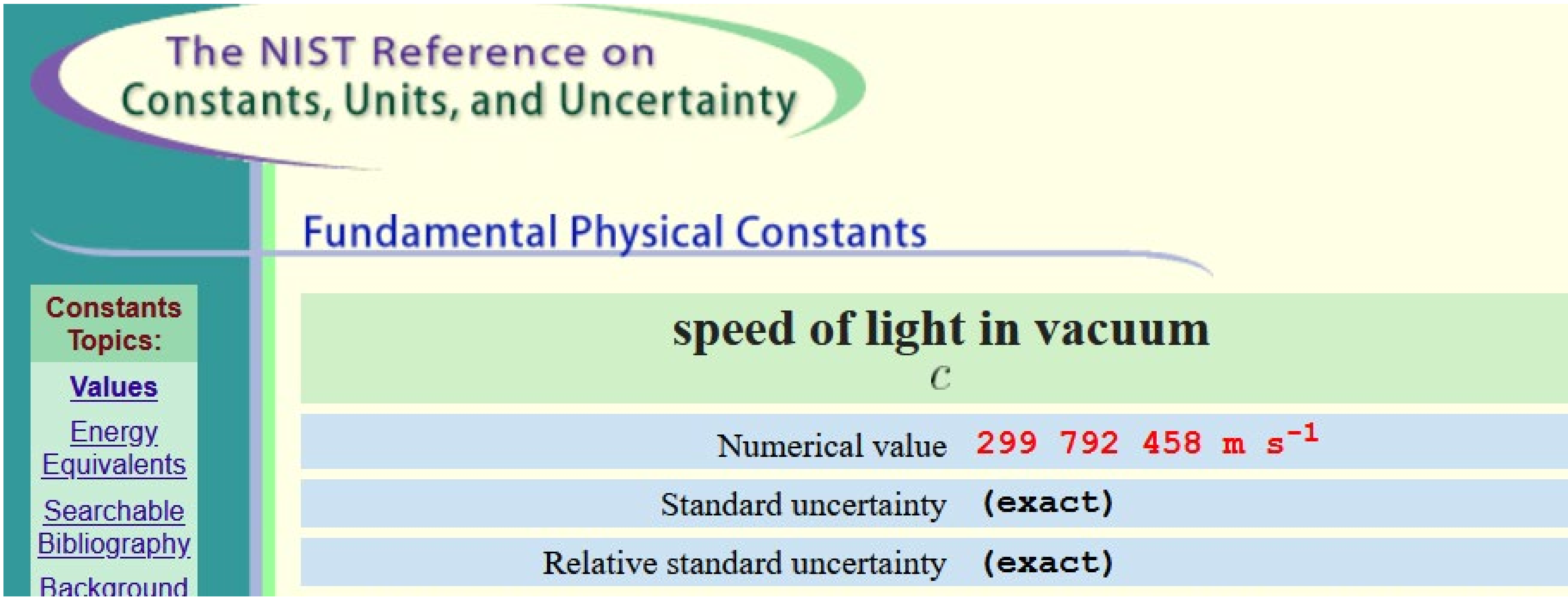
Coulomb's Law

$$F = K_C \frac{q_1 q_2}{r_{12}^2}. \tag{1}$$

Ampere's Law

$$F = K_A \frac{i_1 i_2}{r_{12}^2}\, d\mathbf{s_1} \times d\mathbf{s_2} \times \hat{\mathbf{r}}_{\mathbf{12}}, \tag{2}$$

In the equations above, the current and charge are related by $i_1 = dq_1/dt$ for all unit systems. The two constants K_C and K_A are related so that their ratio K_C/K_A has the units of $(m/s)^2$ and it is *experimentally* known that the ratio has the value $K_C/K_A = c^2$, where c is the speed of light.


The NIST Reference on
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Fundamental Physical Constants
Constants Topics:
Values
Energy Equivalents
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speed of light in vacuum
c
Numerical value 299 792 458 m s-1
Standard uncertainty (exact)
Relative standard uncertainty (exact)

Units - SI vs Gaussian – continued

The choices for these constants in the SI and Gaussian units are given below:

	CGS (Gaussian)	SI
K_C	1	$\frac{1}{4\pi\epsilon_0}$
K_A	$\frac{1}{c^2}$	$\frac{\mu_0}{4\pi}$

Rectangular Snip

Here, $\frac{\mu_0}{4\pi} \equiv 10^{-7} N/A^2$ and $\frac{1}{4\pi\epsilon_0} = c^2 \cdot 10^{-7} N/A^2 = 8.98755 \times 10^9 N \cdot m^2/C^2$.

Units - SI vs Gaussian – continued

Below is a table comparing SI and Gaussian unit systems. The fundamental units for each system are so labeled and are used to define the derived units.

Variable	SI		Gaussian		SI/Gaussian
	Unit	Relation	Unit	Relation	
length	m	fundamental	cm	fundamental	100
mass	kg	fundamental	gm	fundamental	1000
time	s	fundamental	s	fundamental	1
force	N	$kg \cdot m/s^2$	$dyne$	$gm \cdot cm/s^2$	10^5
current	A	fundamental	$statampere$	$statcoulomb/s$	$\frac{1}{10c}$
charge	C	$A \cdot s$	$statcoulomb$	$\sqrt{dyne \cdot cm^2}$	$\frac{1}{10c}$

Units - SI vs Gaussian – continued

One advantage of the Gaussian system is that the field vectors: $\mathbf{E}, \mathbf{D}, \mathbf{B}, \mathbf{H}, \mathbf{P}, \mathbf{M}$ all have the same physical dimensions., In vacuum, the following equalities hold: $\mathbf{B} = \mathbf{H}$ and $\mathbf{E} = \mathbf{D}$. Also, in the Gaussian system, the dielectric and permittivity constants ϵ and μ are dimensionless.

Many materials respond to fields according to the linear response approximation:

$\mathbf{D}=\epsilon\mathbf{E}$ in vacuum $\epsilon=1$ (for cgs Gaussian); $\epsilon=\epsilon_0$ (for SI)

$\mathbf{B}=\mu\mathbf{H}$ in vacuum n $\mu=1$ (for cgs Gaussian); $\mu=\mu_0$ (for SI)

Units - SI vs Gaussian – continued

One advantage of the Gaussian system is that the field vectors: $\mathbf{E}, \mathbf{D}, \mathbf{B}, \mathbf{H}, \mathbf{P}, \mathbf{M}$ all have the same physical dimensions., In vacuum, the following equalities hold: $\mathbf{B} = \mathbf{H}$ and $\mathbf{E} = \mathbf{D}$. Also, in the Gaussian system, the dielectric and permittivity constants ϵ and μ are dimensionless.

As we will see throughout the course, the E and B fields represent the basic electric and magnetic fields while the other fields include electric and magnetic effects of matter.

Basic equations of electrodynamics

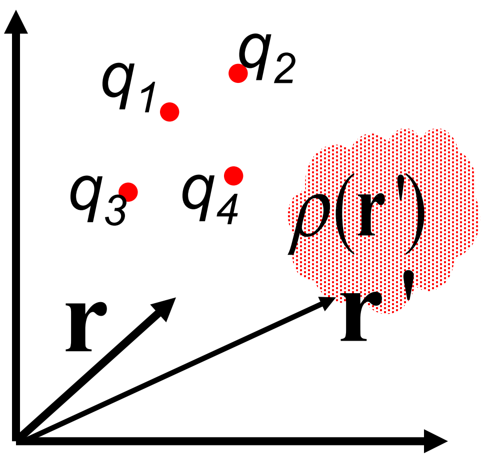
CGS (Gaussian)	SI
$\nabla \cdot \mathbf{D} = 4\pi\rho$	$\nabla \cdot \mathbf{D} = \rho$
$\nabla \cdot \mathbf{B} = 0$	$\nabla \cdot \mathbf{B} = 0$
$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
$\nabla \times \mathbf{H} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t}$	$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$
$\mathbf{F} = q(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B})$	$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$
$u = \frac{1}{8\pi}(\mathbf{E}\cdot\mathbf{D} + \mathbf{B}\cdot\mathbf{H})$	$u = \frac{1}{2}(\mathbf{E}\cdot\mathbf{D} + \mathbf{B}\cdot\mathbf{H})$
$\mathbf{S} = \frac{c}{4\pi}(\mathbf{E} \times \mathbf{H})$	$\mathbf{S} = (\mathbf{E} \times \mathbf{H})$

Units choice for this course:

SI units for Jackson in Chapters 1-10

Gaussian units for Jackson in Chapters 11-16

Electrostatics



Force on particle i due to particles j :

$$\mathbf{F}_i(\mathbf{r}_i) = q_i\mathbf{E}(\mathbf{r}_i) = \frac{1}{4\pi\epsilon_0}\sum_j q_i q_j \frac{\mathbf{r}_i - \mathbf{r}_j}{|\mathbf{r}_i - \mathbf{r}_j|^3}$$

electric field: $$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\sum_j q_j \frac{\mathbf{r} - \mathbf{r}_j}{|\mathbf{r} - \mathbf{r}_j|^3}$$

continuum field: $$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int d^3r'\rho(\mathbf{r}')\frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3}$$

Electrostatics

Discrete versus continuous charge distributions

In terms of Dirac delta function:

$$\rho(\mathbf{r}) = \sum_i q_i \; \delta(\mathbf{r} - \mathbf{r}_i)$$

Digression: Note that in cartesian coordinates --

$$\delta(\mathbf{r} - \mathbf{r}_i) = \delta(x - x_i)\delta(y - y_i)\delta(z - z_i)$$

in spherical polar coordinates --

$$\delta(\mathbf{r} - \mathbf{r}_i) = \frac{1}{r^2}\delta(r - r_i)\delta(\cos\theta - \cos\theta_i)\delta(\phi - \phi_i)$$

Differential equations --

Electrostatics

$$\nabla \cdot \mathbf{E} = \rho/\epsilon_0$$

$$\nabla \times \mathbf{E} = 0$$

Electrostatic potential

$$\mathbf{E} = -\nabla \Phi(r).$$

$$\nabla^2 \Phi(r) = -\rho(r)/\epsilon_0.$$

Relationship between integral and differential forms of electrostatics --

Differential form

$$\nabla^2\Phi(\mathbf{r}) = -\rho(\mathbf{r})/\epsilon_0$$

Integral form

$$\Phi(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int d^3r' \frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}$$

Relationship between integral and differential forms of electrostatics --

Need to show: $$\nabla^2 \left(\frac{1}{|\mathbf{r}-\mathbf{r}'|} \right) = -4\pi\delta^3(\mathbf{r}-\mathbf{r}').$$

Noting that

$$\int_{\substack{\text{small sphere}\\ \text{about } \mathbf{r}'}} d^3r\ \delta^3(\mathbf{r}-\mathbf{r}') f(\mathbf{r}) = f(\mathbf{r}'),$$

we see that we must show that

$$\int_{\substack{\text{small sphere}\\ \text{about } \mathbf{r}'}} d^3r\ \nabla^2 \left(\frac{1}{|\mathbf{r}-\mathbf{r}'|} \right) f(\mathbf{r}) = -4\pi f(\mathbf{r}').$$

We introduce a small radius a such that:

$$\frac{1}{|\mathbf{r}-\mathbf{r}'|} = \lim_{a\to 0} \frac{1}{\sqrt{|\mathbf{r}-\mathbf{r}'|^2 + a^2}}.$$

For a fixed value of a,

$$\nabla^2 \frac{1}{\sqrt{|\mathbf{r}-\mathbf{r}'|^2 + a^2}} = \frac{-3a^2}{(|\mathbf{r}-\mathbf{r}'|^2 + a^2)^{5/2}}.$$

Some details --

Let $|\mathbf{r}-\mathbf{r}'| \equiv u$ $\qquad \nabla^2 = \frac{\partial^2}{\partial u^2} + \frac{2}{u}\frac{\partial}{\partial u}$

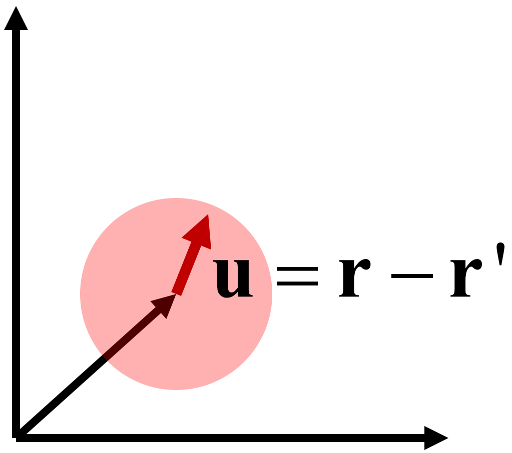
$$\nabla^2 \frac{1}{\sqrt{u^2+a^2}} = \left(\frac{\partial^2}{\partial u^2} + \frac{2}{u}\frac{\partial}{\partial u}\right)\frac{1}{\sqrt{u^2+a^2}} = -\frac{1}{\left(u^2+a^2\right)^{3/2}} + \frac{3u^2}{\left(u^2+a^2\right)^{5/2}} - \frac{2}{\left(u^2+a^2\right)^{3/2}}$$

$$= -\frac{3a^2}{\left(u^2+a^2\right)^{5/2}}$$

If the function $f(\mathbf{r})$ is continuous, we can make a Taylor expansion of it about the point $\mathbf{r} = \mathbf{r}'$, keeping only the first term. The integral over the small sphere about $\mathbf{r}'$ can be carried out analytically, by changing to a coordinate system centered at $\mathbf{r}'$;

so that

$$\int_{\substack{\text{small sphere}\\ \text{about } \mathbf{r}'}} d^3r\, \nabla^2 \left(\frac{1}{|\mathbf{r}-\mathbf{r}'|}\right) f(\mathbf{r}) \approx f(\mathbf{r}') \lim_{a\to 0} \int_{u<R} d^3u \frac{-3a^2}{(u^2+a^2)^{5/2}}.$$



$$\mathbf{r} = \mathbf{r}' + \mathbf{u} \qquad f(\mathbf{r}) \approx f(\mathbf{r}')$$

$$\int_{u<R} d^3u \frac{-3a^2}{(u^2+a^2)^{5/2}} = 4\pi \int_0^R du\, \frac{-3a^2u^2}{(u^2+a^2)^{5/2}} = 4\pi \frac{-R^3}{(R^2+a^2)^{3/2}}.$$

$$\int_{u<R} d^3u \frac{-3a^2}{(u^2+a^2)^{5/2}} = 4\pi \int_0^R du\, \frac{-3a^2u^2}{(u^2+a^2)^{5/2}} = 4\pi \frac{-R^3}{(R^2+a^2)^{3/2}}.$$

For $a \ll R$, $4\pi \frac{-R^3}{\left(R^2+a^2\right)^{3/2}} \approx -4\pi$

$$\rightarrow \int_{\substack{\text{small sphere} \\ \text{about } \mathbf{r}'}} d^3r\, \nabla^2 \left(\frac{1}{|\mathbf{r}-\mathbf{r}'|}\right) f(\mathbf{r}) \approx f(\mathbf{r}')(-4\pi),$$

$$\rightarrow \nabla^2 \left(\frac{1}{|\mathbf{r}-\mathbf{r}'|}\right) = -4\pi\delta^3(\mathbf{r}-\mathbf{r}')$$

Memorable identity

$$\nabla^2 \left(\frac{1}{|\mathbf{r} - \mathbf{r}'|} \right) = -4\pi\delta^3(\mathbf{r} - \mathbf{r}')$$