PHY 712 Electrodynamics
10-10:50 AM MWF in Olin 103
Class notes for Lecture 6:

Reading: Chapter 1 - 3 in JDJ

Introduction to numerical methods

1. **Finite difference**
2. **Finite element**

Course schedule for Spring 2025

(Preliminary schedule -- subject to frequent adjustment.)

	Lecture date	JDJ Reading	Topic	HW	Due date
1	Mon: 01/13/2025	Chap. 1 & Appen.	Introduction, units and Poisson equation	#1	01/15/2025
2	Wed: 01/15/2025	Chap. 1	Electrostatic energy calculations	#2	01/17/2025
3	Fri: 01/17/2025	Chap. 1	Electrostatic energy calculations	#3	01/22/2025
	Mon: 01/20/2025	No Class	Martin Luther King Jr. Holiday		
4	Wed: 01/22/2025	Chap. 1	Electrostatic potentials and fields	#4	01/24/2025
5	Fri: 01/24/2025	Chap. 1 - 3	Poisson's equation in multiple dimensions		
6	Mon: 01/27/2025	Chap. 1 - 3	Brief introduction to numerical methods	#5	01/29/2025

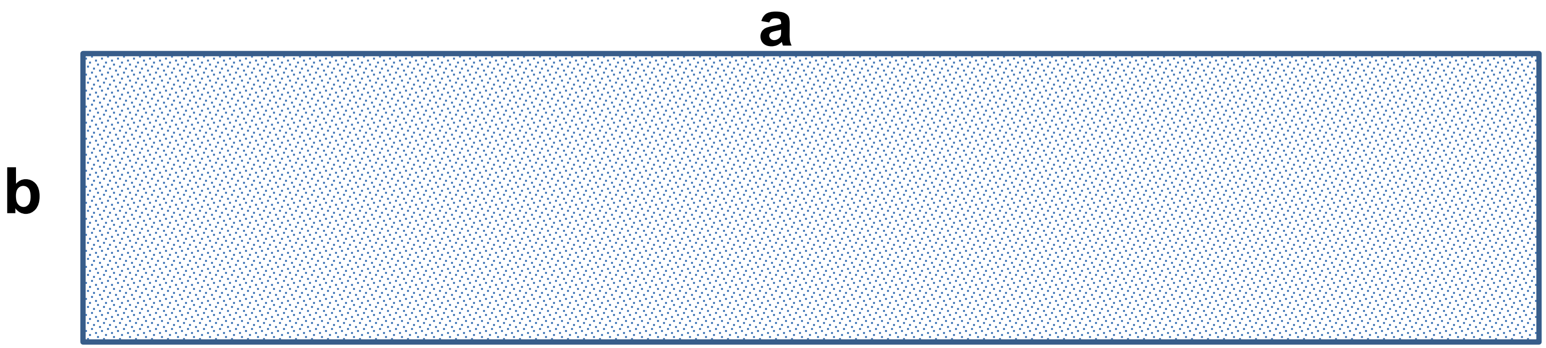
PHY 712 -- Assignment #5

Assigned: 1/27/2025 Due: 1/29/2025

Continue reading Chaps. 1-3 in **Jackson**.

1. Consider a square of length a which has a uniform charge density ρ_0. In Lecture 5 we found an "exact" expression for the electrostatic potential of this system $\Phi(x,y)$ for the case that the potential vanishes on the 4 boundary lines $\Phi(x,0)=\Phi(x,a)=\Phi(0,y)=\Phi(a,y)$.
   a. For the 5x5x5 grid discussed in Lecture 6 with $h=a/4$, evaluate the unique values of $\Phi(x,y)$ on the grid points by summing the series in the exact expressions.
   b. Using the finite difference method, find the approximate unique values of $\Phi(x,y)$ on the grid points and compare with the exact values.
   c. Using the finite element method, find the approximate unique values of $\Phi(x,y)$ on the grid points and compare with the exact values.

Recall example from Lecture 5:



$$\rho(x,y)=\rho_0 \qquad \text{for } 0\le x\le a \text{ and } 0\le y\le b$$

With boundary values $\Phi(0,y)=0, \Phi(a,y)=0, \Phi(x,0)=0, \Phi(x,b)=0$

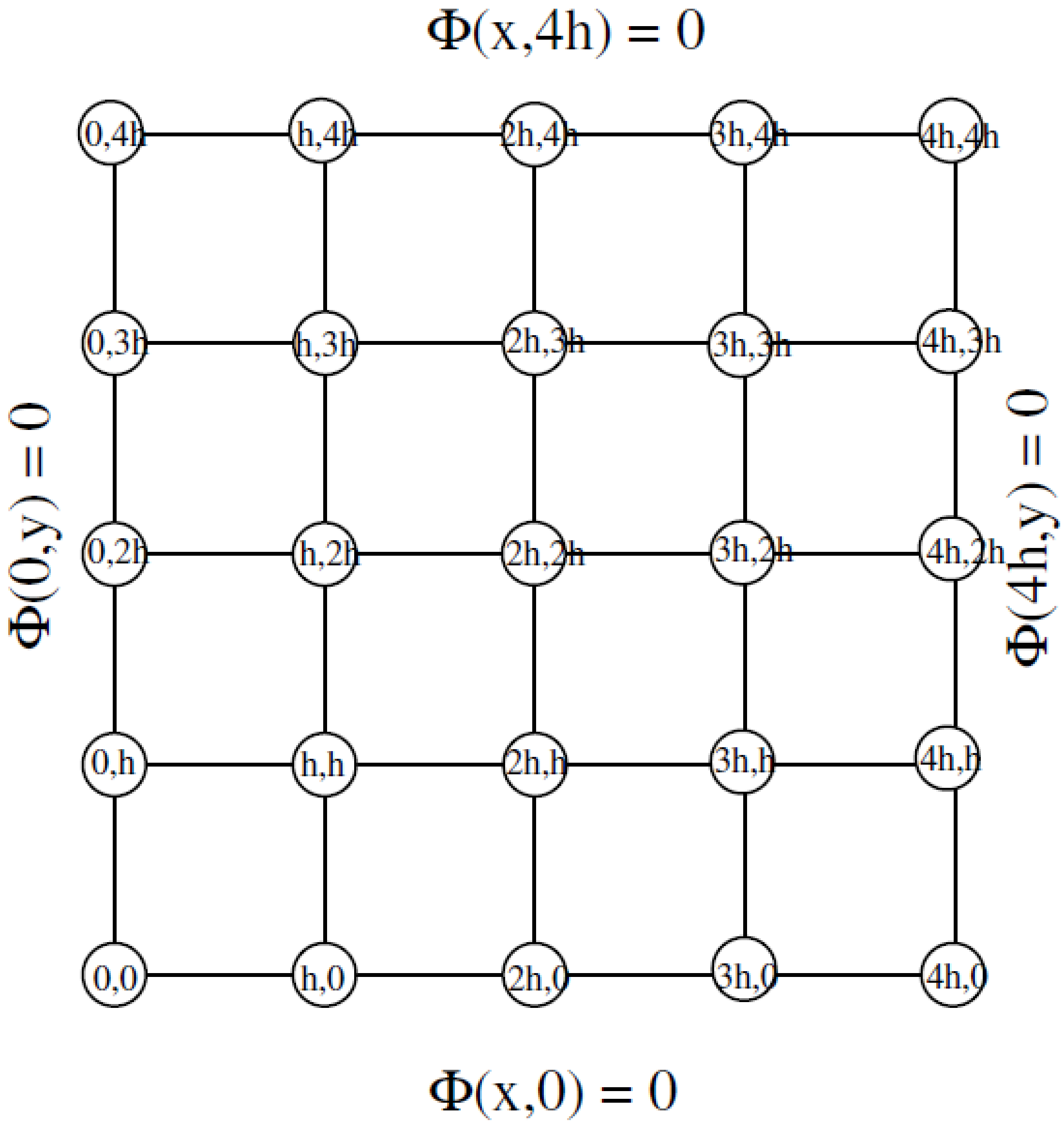
Solution: $\Phi(x,y)=$

$$\frac{16\rho_0 a}{4\pi^2\epsilon_0}\sum_{n\text{ (odd)}}\frac{\sin\left(\frac{n\pi x}{a}\right)}{n^2\sinh\left(\frac{n\pi b}{a}\right)}\left(\sinh\left(\frac{n\pi(b-y)}{a}\right)\int_0^y dy'\sinh\left(\frac{n\pi y'}{a}\right)+\sinh\left(\frac{n\pi y}{a}\right)\int_y^b dy'\sinh\left(\frac{n\pi(b-y')}{a}\right)\right)$$

$$=\frac{16\rho_0 a^2}{4\pi^3\epsilon_0}\sum_{n\text{ (odd)}}\frac{\sin\left(\frac{n\pi x}{a}\right)}{n^3\sinh\left(\frac{n\pi b}{a}\right)}\left(\sinh\left(\frac{n\pi b}{a}\right)-\sinh\left(\frac{n\pi y}{a}\right)-\sinh\left(\frac{n\pi(b-y)}{a}\right)\right)$$

$=$

Some details for fine grid --



9 interior grid points

➔reduced to 6 evaluation points using symmetry

Comment on finite difference methods

➔Based on Taylor's expansion

For 1-dimension: $$f(x+u) = f(x) + u\left(\frac{df}{dx}\right) + \frac{1}{2}u^2\left(\frac{d^2 f}{dx^2}\right) + \frac{1}{3!}u^3\left(\frac{d^3 f}{dx^3}\right)......$$

Note that $$f(x+h) + f(x-h) = 2f(x) + h^2\left(\frac{d^2 f}{dx^2}\right) + O\left(h^4\right)$$

Can determine this

If we know these

Detailed notes -- lecture6slides.pdf