Answer each of the following questions. Points for each question are indicated in red with the amount being spread equally among parts (a,b,c etc). Be sure to show all your work. Use the back of the pages if necessary.
Question 1. **(10 points)** Consider a constant electric field that points directly downward towards the ground.

(a) Compare the electric potential at a point P located at a height of \( y = 5 \text{ m} \) to that of a point Q located at a height of \( y = 1 \text{ m} \). Is point P at a higher, lower, or the same potential?

(b) Compare the electric potential energy of a proton located at these same points (P and Q). When is the electric potential energy of the system higher, when the proton is at P, at Q, or is the same for both?

(c) Compare the electric potential energy of an electron located at these same points (P and Q). When is the electric potential energy of the system higher, when the electron is at P, at Q, or is the same for both?

(d) If the electron is moved from point P to Q, describe the work done by the electric field. Is it positive, negative, or zero?

Solutions

(a) The electric field points from high to low potential so the higher point is at a higher potential. P is at a higher potential.

(b) The potential energy is \( qV \), and \( q \) is positive, so the system has more potential energy at the higher position. So it is higher at P. Note, also, that you have to do work to lift the proton.

(c) Now things are opposite. Q is negative. The potential energy is highest with the electron on the ground, so it is higher at Q.

(d) The displacement is downward. The force by the field on the electron is upward. Thus \( W = F \cdot s \) is negative (theta is 180 degrees).
Question 2. (10 points) Consider a parallel plate capacitor with an area A and plate-separation of d with air between the plates. The capacitor is initially hooked up to an ideal 9 V battery?

(a) What happens to the capacitance of the capacitor if it is now hooked up to an ideal 18 V battery?

(b) What happens to the charge on the capacitor after it is moved to the 18 V battery (compared to when it was hooked up to 9 V)?

(c) How about the energy stored in the electric field of the capacitor as it is moved from the 9V to the 18V battery?

While the capacitor is still hooked up (and remains hooked up) to the 18V battery, you insert a dielectric (that has a much higher dielectric constant than air)

(d) What happens to the capacitance of the capacitor that now has the dielectric in it compared to before the dielectric was inserted?

(e) What happens to the charge stored on the capacitor after you insert the dielectric compared to before it is inserted.

(f) What happens to the energy stored in the capacitor after you put the dielectric in compared to before it is put in.

Solutions

(a) The capacitance is independent of the voltage across it. It remains the same.

(b) \( Q = CV \). V doubled so \( Q \) doubles.

(c) \( C \) stays the same so think about \( U = \frac{1}{2} CV^2 \). V doubles so \( U \) quadruples.

(d) The capacitance increases (by a factor of \( \kappa \)).

(e) \( Q = CV \). V is constant and \( C \) increases so \( Q \) increases.

(f) V stays the same so think about \( U = \frac{1}{2} CV^2 \). \( C \) increases so \( U \) increases as well.
Question 3. (10 points) Consider the circuit below in which there are 4 identical light bulbs, labeled A, B, C, D.

In each of the following considerations, assume only the bulb mentioned in each part is unscrewed (so in part (a) only A is unscrewed and in part (b) only C is unscrewed).

(a) When A is unscrewed, what happens to the brightness of bulbs B, C, D?
(b) When C is unscrewed, what happens to the brightness of bulb D?
(c) When C is unscrewed, what happens to the brightness of bulbs A and B?

Solutions
(a) When A is unscrewed, all the lights go out since the combination of B, C, D are in series with A. There is no complete circuit anymore.
(b) Current cannot go through that path anymore (R goes way up) so D goes out. This is like Christmas lights and as demonstrated in class.
(c) When C is unscrewed the resistance of the whole circuit increases (there are fewer paths for the current). Thus the current going through A decreases and so $P = I^2R$ for A decreases: A gets dimmer. The voltage drop across A also gets smaller since I gets smaller. This means the voltage drop across B increases. Thus $P = \frac{V^2}{R}$ increases for B and B gets brighter.
Problem 1. (15 points) A spherical conductor has a radius of 14.0 cm and a charge of 26.0µC. Calculate the electric potential at the following distances from the center: (a) \(r = 10\) cm, (b) \(r = 20\) cm, (c) \(r = 14\) cm.

(d) If a particle of mass \(3 \times 10^{-9}\) Kg and charge of 2 nC is accelerated from rest from \(r = 14\) cm to \(r = 20\) cm, what will the final speed of the particle be?

Solutions

\[
V = \begin{cases} 
\frac{kq}{r} & \text{for } r > R \\
\frac{kq}{R} & \text{for } r < R 
\end{cases}
\]

(a) \(V = \frac{9 \times 10^9 \times 26 \times 10^{-6}}{0.14} = 1.67\) MV

(b) \(V = \frac{9 \times 10^9 \times 26 \times 10^{-6}}{0.2} = 1.17\) MV

(c) \(V = \frac{9 \times 10^9 \times 26 \times 10^{-6}}{0.14} = 1.67\) MV

(d) We want to employ conservation of energy. The initial mechanical energy = the final mechanical energy.

\[K_i + U_i = K_f + U_f\]

\[K_i = 0.\]

\[K_f = -(U_f - U_i) = -\Delta U.\] The change in potential energy will be \(q\Delta V.\)

\[K_f = \frac{1}{2}mv^2\]

\[\Delta V = 1.67 - 1.17 = 0.5\] MV

\[v^2 = 2 \times \frac{2 \times 10^{-9} \times (5 \times 10^5)}{3 \times 10^{-9}} = 6.67 \times 10^5\]

\[v = 817\] m/s
Problem 2. (15 points) Consider the circuit below where are $C_1 = 3 \mu F$, $C_2 = 1 \mu F$, and $C_3 = 5 \mu F$.

(a) What is the equivalent capacitance if all three capacitors were reduced to a single equivalent one?

If the battery supplied an emf of 9 V, find

(b) the charge and

(c) the voltage across each of the three capacitors.

\[ C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2 + C_3}} = \frac{1}{\frac{1}{3} + \frac{1}{6}} = \frac{1}{\frac{1}{2}} = 2 \mu F \]

The charge on the equivalent capacitor is the same as that on $C_1$.

\[ Q_1 = Q_{eq} = C_{eq}V_1 = (2)(9) = 18 \mu C. \]

The voltage across this capacitor is then $V_1 = \frac{Q_1}{C_1} = \frac{18}{3} = 6 V$.

We can get the voltages $V_2$ and $V_3$ by subtracting that across $C_1$ from the 9V so we get $V_2 = V_3 = 3 V$.

Now we can get the charges on these

\[ Q_2 = C_2V_2 = 3 \mu C \]
\[ Q_3 = C_3V_3 = 15 \mu C \]
Problem 3. (15 points) The ammeter shown in the figure below reads 2.00A. (a) Which of the resistors shown are in parallel with each other? (b) Which of the resistors shown are in series with each other? Find (c) $I_1$, (d) $I_2$, (e) $\varepsilon$.

(a) None are in parallel.

(b) None are in series

(c) Applying Kirchoff’s 2nd law to the top loop we have

$$15 - 7I_1 - (5)(2) = 0,$$

so $I_1 = \frac{5}{7} = 0.714$ A

(d) Applying Kirchoff’s first law to the left hand junction we have $I_1 + I_2 = 2$ A, so

$$0.714 + I_2 = 2,$$

and $I_2 = 1.29$ A

(e) Applying Kirchoff’s 2nd law to the top loop we have

$$\varepsilon - (2)(1.29) - (5)(2) = 0.$$ Thus, $\varepsilon = 12.6$ V
Possibly Useful Information

\[ F = \frac{1}{4\pi\varepsilon_0} \frac{|q_1||q_2|}{r^2} \]

\[ e = 1.6 \times 10^{-19} \text{ C} \]

\[ E = \frac{|q|}{4\pi\varepsilon_0 r^2}, \quad E = \sigma/\varepsilon_0 \]

\[ \Delta x = x_2 - x_1, \quad \Delta t = t_2 - t_1 \]

\[ \bar{s} = (\text{total distance}) / \Delta t \]

\[ \bar{a} = \Delta v / \Delta t \]

\[ v = v_0 + at \]

\[ x-x_0 = v_0 t + \left(\frac{1}{2}\right)at^2 \]

\[ v^2 = v_0^2 + 2a(x-x_0) \]

\[ x-x_0 = \frac{1}{2}(v_0+ v)t \]

\[ x-x_0 = vt - \frac{1}{2}at^2 \]

\[ \ddot{a} = \frac{dv}{dt} \]

\[ \Delta U = U_f - U_i = -W \]

\[ \Delta V = V_f - V_i = -W/q_0 = \Delta U/q_0 \]

\[ V_f - V_i = -\int_i^f \vec{E}.d\vec{s} \]

\[ V = \frac{1}{4\pi\varepsilon_0} \frac{q}{r} \]

\[ U_f + K_f = U_i + K_i \]

\[ K = \frac{1}{2}mv^2 \]

\[ E_s = \frac{\partial V}{\partial s} \]

\[ E = \frac{\Delta V}{\Delta s} \]

\[ Q = CV \]

\[ C = 2\pi\varepsilon_0 \frac{l}{\ln(b/a)} \]

\[ C = 4\pi\varepsilon_0 R \]

\[ \varepsilon_0 = 8.85 \times 10^{-12} \text{ (C}^2/\text{N} - \text{m}^2\text{)} \]

\[ \vec{E} = \frac{\vec{F}}{q_0} \]

\[ \varepsilon_0\Phi = \varepsilon_0 \int \vec{E}.d\vec{A} = q_{\text{enc}} \]

\[ \nabla = \Delta x / \Delta t \]

\[ v = dx/dt \]

\[ a = dv/dt = d^2x/dt^2 \]

\[ g = 9.8 \text{ m/s}^2 \]

\[ \vec{r} = \hat{x} + y\hat{j} + z\hat{k} \]

\[ \Delta\vec{r} = \vec{r}_2 - \vec{r}_1 \]

\[ \Delta\vec{v} = \Delta\vec{r} / \Delta t, \quad \vec{v} = d\vec{r} / dt \]

\[ \ddot{a} = \Delta\vec{v} / \Delta t \]

\[ V = \int_i^f \vec{E}.d\vec{s} \]

\[ V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i} \]

\[ V = \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{r} \]

\[ E_x = \frac{\partial V}{\partial x}; \quad E_y = \frac{\partial V}{\partial y}; \quad E_z = \frac{\partial V}{\partial z} \]

\[ U = -W = \frac{1}{4\pi\varepsilon_0} \frac{q_1q_2}{r_1r_2} \]

\[ C = \frac{\varepsilon_0 A}{d} \]

\[ C = 4\pi\varepsilon_0 \frac{ab}{b-a} \]

\[ C_{\text{eq}} = \sum C_j \text{ (parallel)} \]
\[
\frac{1}{C_{eq}} = \sum \frac{1}{C_j} \text{ (series)}
\]
\[u = \frac{1}{2} \varepsilon_0 E^2\]
\[I = \frac{dQ}{dt}\]
\[\rho = \frac{1}{\sigma}\]
\[R = \frac{\rho L}{A}\]
\[P = IV\]

\[P_{emf} = IE\]

\[
\frac{1}{R_{eq}} = \sum \frac{1}{R_j} \text{ (parallel)}
\]
\[I = \left(\frac{\varepsilon}{R}\right)e^{t/RC}\]
\[I = \left(\frac{Q}{RC}\right)e^{t/RC}, \ I_0 = \left(\frac{Q}{RC}\right)\]
\[E = \sigma/\varepsilon_o\]

\[U = \frac{Q^2}{2C} = \frac{1}{2} CV^2\]
\[C = \kappa C_0\]

\[V = IR\]
\[P = I^2R = V^2/R\]
\[I = \frac{\varepsilon}{(R + r)}\]
\[R_{eq} = \sum R_j \text{ (series)}\]
\[q(t) = Q(1 - e^{-R/C})\]
\[q(t) = Qe^{t/RC}\]
\[\lambda = Q/L, \ \sigma = Q/A, \ \rho = Q/V\]