Infrequent Assessments Distort Property Taxes: Theory and Evidence

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Abstract

Economists have long recognized that lags in property reassessment benefit infrequent movers because it reduces their property taxes. But in addition reassessment lags can influence the level of property taxes selected under majority rule. I show that short delays in community-wide reassessment increase property tax collections because it reduces the tax price for a majority of voters. However, longer delays reduce property tax collections because the aggregate assessed base (and thus the tax yield) declines so much. I formally characterize the cutoff between these regimes and show tax collections are generally above their socially optimal level. This theory can help explain why many people believe property taxes are excessive, and it also suggests that the American system of taxing capital gains at realization, rather than on accrual, might result in excessive rates. I test this theory on a sample of Pennsylvania municipalities in the Philadelphia suburbs. This is a suitable crucible for such an evaluation because community-wide reassessments are infrequently performed in Pennsylvania. It is not possible to reject the theory's basic predictions, and numerical estimates suggest that a five year delay in community-wide reassessment increases government revenues by six percent. However, reassessment delays do not impose statistically significant social losses because they benefit infrequent movers.

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1 Introduction

Despite predictions of their imminent demise, property taxes remain a significant feature of the American fiscal landscape. Between 1980 and 1994, property tax collections grew at a 12.5% annual rate with per capita payments reaching $760 in 1994. Property taxes also have the dubious distinction of remaining one of the most unpopular taxes in public opinion polls (Dearborn [8]) and are typically perceived as being set at an excessive rate (ACIR [2]). Many explanations for the tax's limited appeal involve its method of administration. A voter's property tax is determined by the product of his property assessment and the tax rate. At some regular frequency communities or states determine the assessed value of all properties, usually pegged to some fraction of market value. In all other years, sold or newly constructed homes are assigned assessed values which reflect their current market price while unsold homes maintain their initial value. Since property values tend to appreciate, owners of identical homes will face different tax burdens if they have moved at different times.

In this paper I evaluate how assessment practices shape voter demands for property taxes. I consider a model where residents vote over property taxes which fund a public good. A resident's property assessment is determined by his tenure, defined as the more recent of the time since his last move or the last community-wide reassessment. A community-wide reassessment sets all property at a common market value and resets everyone's tenure to zero. Between reassessments unsold homes maintain their previous assessed value (and increase in tenure) while sold and newly constructed homes are assessed at a higher, market value which reflects the inflation rate (and their tenure is set to zero). This means that assessed values, as well as property tax payments, are inversely proportional to home tenure.

I show that the tax revenue selected under majority rule is increasing in the ratio of the aggregate tax base to the median tenured voter's assessed value. Delays between community-wide reassessments influence both of the terms in this ratio. First, a delay reduces the real aggregate property tax base for the community: unsold homes maintain their original assessed value but prices are increasing due to inflation. Second, a delay reduces the median tenured voter's real assessed value if he is a non-mover, since his nominal assessed value

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remains fixed. Short delays between community-wide reassessments result in higher tax revenues, since the median's real assessed value falls faster than the real aggregate assessed base. This is because the median tenure surely increases (and thus the real median assessment surely falls) while only a fraction of the total housing stock, the unsold property, has a real reduction in assessed value. Long delays between reassessments lead to lower tax revenues because the median tenure eventually becomes fixed while the real aggregate tax base continues to shrink. I formally characterize the cutoff between the increasing and decreasing tax regimes.

The model is consistent with observed features of property taxes and has implications regarding capital gains taxation. First, the theory can help explain the common perception that property taxes are excessive. In the typical case where a majority of voters have not moved since the last reassessment, the tax rate selected is just optimal for the non-movers but is too high for everyone else. This is because residents who have moved at least once face a higher property assessment and thus a higher tax bill. Second, the theory is a potential explanation for why property taxes tend to spike up in the year of a community-wide reassessment. Previous authors who have documented this fact (Bloom and Ladd [4], Ladd [16]) attribute it to voter fiscal illusion. However if the delay between community reassessments is long enough, the majority-preferred tax level will fall below its initial level due to the reduction in the aggregate assessed base. Fully rational voters should therefore select higher taxes upon reassessment. Finally, the theory suggests that the American system of taxing capital gains only at realization encourages higher tax rates. Individuals with accrued gains (or with no capital assets) are like non-moving voters in the model: they face a low tax price and so will prefer a relatively high tax rate. When more than half of the voters are in this class, the capital gains rate selected under majority rule will be too high. Shifting to a system which taxes on accrual is likely to alleviate this bias, an argument which previous advocates of accrual taxation have failed to raise (see Auerbach [3]).

I evaluate the theory using a sample of Pennsylvania municipalities in the Philadelphia suburbs. This is an excellent crucible for examining the role of community-wide reassessments since Pennsylvania is perhaps the most lax state in enforcing its revaluation laws.

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With long lags between reassessments (typically exceeding 20 years), both periods of increasing and decreasing property tax revenues should be observed. To test the model's basic predictions, I estimate a flexible, non-structural equation relating community-wide reassessment delays to property tax revenues. It is not possible to reject the null that property taxes reach a peak at the particular reassessment delay predicted by the theory. Then I make a functional form assumption and estimate a structural equation relating community-wide reassessment delays to total revenues. It is possible to uncover parameters of an underlying utility function from these estimates and thus to perform social welfare calculations. I find that a five year delay in community-wide reassessment increases government revenues by 6%, and that even larger revenue increases result from delays up to fifteen years. However, the social welfare loss from delaying community-wide reassessment is not statistically different from zero. The reason the social loss is so small is that reassessment delays actually benefit non-moving voters, and this utility gain largely counteracts the losses of recent movers. Finally, the structural estimates are consistent with the observed timing of community-wide reassessments in the sample. In total these results suggest that assessment practices strongly influence property taxes and are further evidence of the important role institutions play in determining policy outcomes.

No previous research has linked assessment practices to voter demands for property taxes. Inman and Rubinfeld [14] note that preferential assessments can be used to bribe affluent voters into not moving, but this practice seems limited to large cities. O'Sullivan, Sexton and Sheffrin [18] evaluate how California's acquisition value system, in which homes are reassessed only when sold, influences property tax incidence and reduces mobility. Aaron [1] shows that high tax rates result in more uniform assessment and hence a more equitable distribution of tax burdens.

2 The Model

2.1 Setup

Consider a community in which individuals own and occupy identical homes with the aggregate housing stock initially normalized to unity. For simplicity assume that time is discrete and that an exogenous proportion, $\lambda > 0$, of the housing stock is sold each year (the case where sale probabilities vary across homes is discussed in Section 2.2). While homes never depreciate or wear out, a fraction, $\theta > 0$, of new homes are constructed each year. This means the total housing stock increases by a factor $(1 + \theta)$ each year. Define the tenure of a home, t, as the number of years since it was last sold or first constructed. Using the convention that the initial period is $Y = 0$, a simple induction argument verifies that the home tenure density in year Y is,

$$
f_Y(t|\lambda, \theta) \equiv \begin{cases} \dfrac{(\lambda + \theta)(1 - \lambda)^t}{(1 + \theta)^{t+1}} & t < Y \\[2mm] \dfrac{(1 - \lambda)^Y}{(1 + \theta)^Y} & t = Y \end{cases} \tag{1}
$$

Intuitively, higher λ and θ values result in a greater concentration of younger homes. Figure 1 plots the density for Y equal to 5, 25 and 100 years using representative property turnover and construction rates. Notice that the density is downward sloping everywhere except for the spike in the final year.

Initially each home is assessed at market value which is set equal to one. While property values appreciate at a constant annual rate, $\pi > 0$, assessed values remain constant unless a home is sold, constructed or there is a community-wide reassessment, any of which sets the home assessment at full market value and its tenure to $t = 0$. For every year in which a home is unsold and there is no community-wide reassessment, the home tenure increases by one year and its real assessed value (the ratio of assessed to market value) falls by the level of price increase, $1 + \pi$. This means that a t-tenure home has a real assessed value of $(1 + \pi)^{-t}$ which is decreasing in t (notice that freshly assessed, sold and constructed homes have a real assessed value of 1). Each year the local government may levy a common property tax rate

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("millage"), m_Y, on all homes. The real tax price for the owner of a t-tenure home equals this rate times his assessed value,

$$p_t(m_Y|\pi) = m_Y(1+\pi)^{-t} \tag{2}$$

The real aggregate tax base per home for a community which last reassessed Y years ago is $A_Y(\lambda, \theta, \pi) \equiv \sum_{t=0}^{Y} f_Y(t|\lambda, \theta)(1+\pi)^{-t}$ or,

$$A_Y(\lambda, \theta, \pi) = \frac{(\lambda+\theta)(1+\pi)}{(1+\theta)(1+\pi)-(1-\lambda)} + \frac{\pi}{(1+\theta)(1+\pi)-(1-\lambda)}\left[\frac{(1-\lambda)^{Y+1}}{(1+\theta)^Y(1+\pi)^Y}\right] \tag{3}$$

and the real tax revenue per home is,

$$R_Y(m_Y|\lambda, \theta, \pi) = m_Y A_Y(\lambda, \theta, \pi) \tag{4}$$

Because some homes are unsold each period, a delay between community-wide reassessments reduces the real aggregate tax base ($\partial A_Y/\partial Y < 0$) though at a decelerating rate ($\partial^2 A_Y/\partial Y^2 > 0$); even as Y approaches infinity, the real aggregate base remains positive.

To close the model I need to specify preferences, production possibilities and the method of community decision. While taxes are costly, government revenues are devoted to a public good which equally benefits all citizens. I will assume that the cost of government services increases at the same rate as property values (π) and that there is congestion in provision (the latter point is added for realism and is not central to any of the results which I prove). Then a linear utility specification for the owner of a t-tenure home is,[1]

$$U_Y(t, m_Y|\lambda, \theta, \pi) = B\left(R_Y(m_Y|\lambda, \theta, \pi)\right) - p_t(m_Y|\pi) \tag{5}$$

where $B(\cdot)$ maps real tax revenue per home into public good provision under the assumption that $B' > 0$ and $B'' < 0$. Notice that the benefit (the first term) is constant for all citizens while the tax price (the second term) varies based on home tenure.

The community uses majority rule to determine its tax rate. It is not difficult to show

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[1]This form is equivalent to the more conventional utility function in which private consumption enters in an additively separable fashion.

that citizens have single-peaked preferences over millage. Using (2), (4) and (5) the second derivative of the utility function is $\partial^2 U_Y(t, m_Y)/\partial m_Y^2 = B''(m_Y A_Y)A_Y^2 < 0$, and the single-peaked result follows so long as the first derivative $\partial U_Y(t, m_Y)/\partial m_Y$ is positive when revenue is small.[2] Due to Black's Law the unique majority rule outcome is the median tenured home-owner's ideal millage, and this rate is characterized by his first order condition,

$$\frac{\partial U_Y(t_{median}, m_Y)}{\partial m_Y} = B'(R_Y)A_Y - (1 + \pi)^{-t_{median}} \equiv 0 \tag{6}$$

where t_{median} is the median tenure. Because $B'' < 0$ the optimal revenue, R_Y, is increasing in the aggregate to median property assessment ratio, $A_Y(1 + \pi)^{t_{median}}$. This ratio is the marginal benefit to marginal cost of taxation for the median voter.

2.2 Results and Caveats

The first step is to identify the median tenured home in a community that last reassessed Y years ago. By definition the median satisfies,[3]

$$t_{median} \equiv \min \left\{ s : \sum_{t=0}^{s} f_Y(t | \lambda, \theta) \geq 0.5 \right\} \tag{7}$$

There are two possible relationships between t_{median} and Y. If there was a recent community-wide reassessment then the median owner has not moved, $t_{median} = Y$. A visual depiction of this may be found in the first panel of Figure 1. Formally this case requires $\sum_{t=0}^{Y-1} f_Y(t | \lambda, \theta) < 0.5$ or using (1), $1 - [(1 - \lambda)/(1 + \theta)]^Y < 0.5$. This implies that Y is less than or equal to,

$$Y^*(\theta, \lambda) \equiv \text{trunc} \left(\frac{\ln 2}{\ln(1 + \theta) - \ln(1 - \lambda)} \right) \tag{8}$$

Evaluated at typical parameter values (e.g. Langhoff [17] Table 2.17, Figure 2.5), $Y^*(\theta, \lambda)$ tends to fall in the interval of 5 to 25 years. Notice that the threshold is smaller when owners are more mobile ($\partial Y^*/\partial \lambda < 0$) or homes are constructed more rapidly ($\partial Y^*/\partial \theta < 0$) but is

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[2] A formal assumption which suffices for this point is $\lim_{R \to 0} B'(R) = \infty$.

[3] In the knife-edge case where *exactly* half of the homes have a tenure less than or equal to t, I call t (rather than $t + 1$) the median. This is an innocuous convention.

independent of the inflation rate ($\partial Y^*/\partial\pi = 0$) which does not influence the tenure density.

The second case occurs when Y exceeds the threshold Y^* at which point the median is fixed at Y^*. To see this recall that when Y equals Y^*, at least half of the homes have been sold or constructed since the last community-wide reassessment. Because the top row of (1) is independent of Y, the density of these young tenure homes will remain constant over time. These two facts imply that the threshold Y^* must also be the median for any later year. The median will therefore satisfy the relationship $t_{median} = \min(Y, Y^*)$.

I can now present the substantive results which relate delays in community-wide reassessment to property taxes. Recall that in a non-reassessment year all unsold property increase in tenure by one year ($t \to t+1$), sold and newly constructed property have their tenure reset at zero ($t = 0$), and the reassessment variable increases by one year ($Y \to Y+1$); in a reassessment year both the tenure of all property and the reassessment variable are set to zero ($t, Y = 0$).

Proposition 1 *In the years immediately following a community-wide reassessment, reassessment delays increase tax revenues. However, if the delay is long enough then collections begin to fall. The threshold for the regime change occurs when the median age becomes fixed, at delay $Y = Y^*(\theta, \lambda)$.*

PROOF: See Appendix A.1.

The intuition follows from the two main dynamics here, the shrinking real aggregate tax base and the shrinking real median assessment (recall from (6) that taxes are set by the product of these terms). The first effect, noted following (3), encourages lower tax revenues since a given millage raises less revenues when the aggregate base is small. The second effect, however, induces higher tax revenues since the median's tax price falls along with his assessment. For short reassessment delays, the median effect dominates resulting in a tax revenue increase. The reason is that the delay *surely* increases the median tenure by one year (since $t_{median} = Y$) and thus surely reduces the real median assessment. But only some of the overall housing stock increases in tenure (the proportion of unsold homes, $(1 - \lambda)/(1 + \theta) < 1$), and so the aggregate tax base shrinks more slowly than the median assessment. For longer reassessment delays, however, the median tenure and thus the real

median assessment becomes fixed (since $t_{median} = Y^*$), and so the shrinking real aggregate assessed base results in a tax revenue decrease. The proposition is illustrated in Figure 2 which plots the optimal trajectory of tax revenues. The path seems insensitive to the particular functional form of $B(R)$ which is used.[4] In contrast the millage dynamics are quite sensitive to functional form assumptions, and so I will not focus on them here.[5]

It is also interesting to see whether property taxes are socially optimal (the role of non-assessment distortions is discussed at the end of the section). Not surprisingly, in a non-reassessment year property taxes are generally non-optimal. Under a utilitarian criterion, the welfare maximizing millage is,

$$m_Y^* = \arg\max_{m_Y} \sum_{t=0}^{Y} f_Y(t) \left[B(m_Y A_Y) - m_Y (1+\pi)^{-t} \right] \tag{9}$$

The socially optimal level of tax revenue, $R_Y^* \equiv m_Y^* A_Y$, is characterized by,

$$B'(R_Y^*) = 1 \tag{10}$$

The left-hand side of this equation represents the marginal social benefit of raising taxes, the increase in public services, while the right-hand side is the marginal social cost. Notice that the socially optimal revenue is time invariant.

Now in the revaluation year all homes are assessed at true market value (one) and the

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[4]It is possible to prove that the *slope* of the optimal revenue stream is insensitive to the choice of functional form. Using the results in Appendix A.1,

$$\frac{\partial R_Y}{\partial Y} = \frac{\partial \left[A_Y (1+\pi)^{t_{median}} \right]^{-1} / \partial Y}{B''[B'^{-1}((1+\pi)^{-t_{median}} A_Y^{-1})]}$$

where B'^{-1} is the inverse of B' and $R_Y = B'^{-1}((1+\pi)^{-t_{median}} A_Y^{-1})$. The functional form of $B(\cdot)$ only enters the right-hand side through the denominator, and all functions in the class $B' > 0$, $B'' < 0$ will impart similar values.

[5]To see this, differentiate $m_Y \equiv R_Y / A_Y$ to get,

$$\frac{\partial m_Y}{\partial Y} = \frac{A_Y \partial R_Y / \partial Y - B'^{-1}((1+\pi)^{-t_{median}} A_Y^{-1}) \partial A_Y / \partial Y}{A_Y^2}$$

where B'^{-1} is the inverse of B'. The shape of the right-hand side is more sensitive to the choice of $B(\cdot)$ than is $\partial R_Y / \partial Y$ because $B(\cdot)$ enters the second term in the numerator. Nonetheless, it is possible to show that reassessment delays increase millage for $Y \leq Y^*$ and for all functional forms which I examined weakly decrease millage for $Y > Y^*$.

size of the tax base is unity. This means the median's first order condition (6) satisfies,

$$B'(R_0) = 1 \qquad (11)$$

and so the socially optimal revenue is selected. The intuition is that a community-wide reassessment both equalizes all tax prices and also sets all assessed values at their true market value. This same result does not hold in other years since the first order condition in (6) may be written as,

$$B'(R_Y) = [A_Y(1+\pi)^{t_{median}}]^{-1} \qquad (12)$$

I show in the proof of Proposition 1 that when $Y \leq Y^*(\theta, \lambda)$ the median is a non-mover, the right-hand side of (12) is below one, and so tax collections are super-optimal (since $B'' < 0$). Reassessment delays beyond $Y^*(\theta, \lambda)$ increase the right-hand side of (12) since the median tenure is fixed and the aggregate assessed base is shrinking. It turns out that eventually tax revenues become sub-optimal, so long as the parameters are sufficiently close to zero.[6]

Proposition 2 *Property taxes generally differ from the social optimum in non-reassessment years. Immediately following a revaluation collections are too high though with a long enough wait (and presuming λ, θ, π are close to zero) the level becomes too low. The cutoff between these regimes, which must occur in finite time, is the first delay $Y > Y^*(\theta, \lambda)$ which meets,*

$$A_Y(1+\pi)^{Y^*(\theta, \lambda)} < 1 \qquad (13)$$

PROOF: See Appendix A.2.

Figure 2 illustrates this point: tax revenues eventually fall below their reassessment year level, the social optimum.[7]

The final result characterizes the condition for majority rule approval of a community-wide reassessment. Because this sets everyone's property at full market value, it increases

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[6]The parameter assumption is needed to justify Taylor expansions in the proof. When the parameters approach one, tax revenues need not ever fall below their initial level (e.g. when $\lambda = 0$ and $\theta = 1$ then the top line of (23) is negative). In reality property turnover (λ), construction (θ) and inflation (π) rates tend to be close to zero, so this is a reasonable assumption.

[7]The inefficiency result is related to majority rule's inability to represent preference intensity. Individuals face different tax prices based on how recently they have moved, but each gets a single vote. Initially the non-movers determine the rate, so new residents' intense opposition to high taxes goes unheeded. Eventually taxes become sub-optimal since non-movers' preference for high taxes is ignored.

the aggregate tax base and hence the collections from any fixed tax rate (in addition, there is a non-trivial cost associated with appraising each home's value). Community-wide reassessment is more costly for those who have not moved in many years, so if the median tenured home-owner prefers a reassessment, then a strict majority will also.[8] Recall that for short reassessment delays the median is a non-mover who has little desire for a new community-wide reassessment. However, eventually the median tenure becomes fixed though the aggregate tax base continues to shrink. This means the median is getting less services for any given tax rate and suggests that if enough time elapses he will eventually prefer a reassessment.

Proposition 3 *There can be majority support for a community-wide reassessment only when there has been a long wait since the last one. If the physical cost of appraising property is low and λ, θ, π are close to zero the minimum wait is finite, and when reassessment is costless it is the smallest $Y > Y^*(\theta, \lambda)$ which satisfies (13).*

PROOF: See Appendix A.3.

So when reassessment timing is endogenous, I can indirectly test the theory. In particular, the model predicts the number of voters lobbying for a community-wide reassessment should increase with the lag since the last one.

Before turning to the empirical results, it is important to discuss various assumptions in the model. First, while the model assumes that all home-owners move with a common probability in reality mobility is likely to be inversely related to tenure length. This reflects the fact that long-time residents have a lower home assessment and thus a lower tax burden, a benefit they give up if they move. I show in an extension, available upon request, when property turnover decreases in tenure ($\partial \lambda / \partial t < 0$) that delays in community-wide reassessment increase tax revenues almost everywhere for realistic parameter values (in contrast to the constant turnover case, the median tenure continues to increase here). While it is not possible to formally test whether property turnover varies with tenure length using the data discussed in the next section,[9] I will test whether the tax revenue implications of the varying

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[8]Those who have moved or purchased more recently than the median see an even smaller increase in assessed value and by definition this group has a mass of at least one half.

[9]There is one indirect piece of evidence consistent with a constant turnover rate and counter to a variable rate: I cannot reject the null that community-wide reassessment lags have no effect on the mean community rate of property turnover in the sample (regression omitted). If property turnover declined with tenure, then there would be a significant negative relationship.

turnover case hold.

Two other assumptions should be noted. While the model presumes all distortions work through the assessment process, property taxes may distort the capital market or may be too high because of politician agenda setting power (Romer and Rosenthal [22]). The omission of these factors means the social welfare calculations should be interpreted cautiously.[10] The model also ignores capitalization of taxes. Infrequent reassessment places a large tax burden on new home-owners and so drives down the equilibrium market value of all property.[11] This would lower the aggregate tax base but have only a limited affect on the median assessment, reducing equilibrium property tax revenues. This suggests that the model overstates the distortion due to reassessment delays.

3 Application: Property Taxes in Pennsylvania

3.1 Background and Sample

I will examine how property reassessment influences taxes in a sample of Pennsylvania communities. There are three types of local governments in Pennsylvania: the largest, counties, contain several municipalities and school districts which in turn are geographically coterminous. All three governmental forms levy property taxes with the selection of rates delegated to elected politicians. In this paper I will focus only on municipalities since they collect more property revenues than counties and are more numerous than school districts. In addition, as the most visible and scrutinized local government[12] municipalities are more likely to approximate the direct democracy assumption at the center of the theoretical framework. In future work it would be interesting to consider the interaction between these local governments.

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[10]Still I will focus on social welfare differences and such relative calculations are unlikely to be greatly altered by a constant, unmodeled distortion.

[11]There cannot be differential capitalization when property is identical. Because a prospective buyer values all homes equally (since they will be assessed at full value upon purchase), homes of every tenure length will have an identical market value.

[12]Public awareness of a local government is largely determined by the visibility of its spending responsibilities. Counties administer a court and prison system whereas municipalities are responsible for higher profile services such as police, fire protection and sanitation. School districts run primary and elementary schools. A full discussion of Pennsylvania local governments may be found in *Citizen's Guide to Pennsylvania Local Government* [19].

Property assessment is a county responsibility in Pennsylvania. During a non-reassessment year parcels which are not sold maintain their prior assessed value[13] while sold properties (and all homes in a reassessment year) are pegged at some fraction of market value;[14] this fractional assessment does not change any of the predictions of the model so long as practices are constant over time. While counties are legally obligated to reassess all property each year, for various reasons Pennsylvania does not enforce this rule. Table 1 lists the most recent reassessments for the four counties in the Philadelphia suburbs: none have revalued in the last fifteen years while one has not acted since the Depression.[15] Not surprisingly these delays significantly deteriorate the taxable property base: in 1960 the mean assessed to market value ratio for these counties was 0.331 while in 1992 the ratio was 0.053. These counties are not isolated examples: the most recent survey of the International Association of Assessing Officers [17] identifies Pennsylvania as the state which allows the most time between reassessments.

Pennsylvania will be an ideal crucible for testing my theory. First, there should be enough time between county reassessments to evidence or disprove the predicted tax dynamics. This is because reassessment lags both less than and greater than the critical threshold Y^* will be observed. Second, there will be sufficient heterogeneity to identify the key relationships. Not only do the counties listed in Table 1 revalue property in different years, but each municipality has a unique threshold, Y^*, due to variation in property turnover and construction rates. This means I will be able to distinguish between possible time trends in tax collections and reassessment lag effects.

My sample will include annual observations from 1960 to 1992 of the 237 municipalities which are contained in exactly one of the four suburban Philadelphia counties. The goal will be to explain variation across these observational units in property tax revenues. The central explanatory variables in the model are the rates of property turnover (λ), construction

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[13]Homes improvements are differentially assessed with the original portion maintaining its initial value.

[14]While these facts are not formally codified, the chief assessing officers of several counties confirmed that they represent accepted practices in Pennsylvania.

[15]These dates were obtained from each county's chief assessing officer and cross-checked against State Tax Equalization Board records whenever possible. Unfortunately, due to the extensive lags between the reassessments there is some uncertainty with regard to the earlier years.

(θ)[16] and inflation (π). For the inflation rate I use the Philadelphia standard metropolitan statistical area (SMSA) consumer price index (CPI) though the results are quite similar when I substitute the Philadelphia SMSA house price index. The other two variables are only available for a limited period at the end of the sample (the last 5 and 13 years for λ and θ respectively), so I use the average value for each community; due to potential year-specific anomalies in the raw data, I decided it was not warranted to use the annual values.

An advantage of using panel data is that I can control for demographic and fiscal characteristics which may influence a community's property taxes but were omitted from the stylized model. These controls include median household income, total population, the population growth rate, the percentage of senior citizens, the home ownership rate, the number of jobs, the percentage of land devoted to business and residential use, as well as indicators for governmental form. A full discussion of these variables may be found in a related paper, Strumpf [23]. Another explanatory variable I consider is an index based on government overhead spending since I show in Strumpf [24] that overhead is correlated with expenditure levels. Finally I control for non-property tax revenues. First, Pennsylvania municipalities may levy an earned income tax (EIT) at a rate of one percent. I have shown elsewhere (Strumpf [24]) that EIT collections can be treated as exogenous lump sums: a community will levy only if neighboring communities already have the tax.[17] Second, intergovernmental grants such as state highway aid can be an important and exogenous[18] revenue source. Descriptive statistics and sources for all variables may be found in Table 2.

3.2 Estimation Strategy

The main objective of the empirical estimates is to test the model in Section 2 and to determine whether county reassessment delays have significant effects in practice. The model

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[16]The construction rate is based on the number of housing units authorized by building permits. While this potentially is a leading rather than current indicator, it is unlikely to be a significant problem since I average values over an extended period.

[17]To control for differences in citizen tax burdens, I also include as explanatory variables the percentage of workers who commute to Philadelphia, another state or pay the EIT at work.

[18]State highway aid, for example, is distributed based on a weighted average of population and local road miles.

predicts that short reassessment delays increase property tax revenues, longer delays decrease property tax revenues and that the cutoff between these regimes occurs at a delay Y^* which varies across communities. Also, according to (3), (6) and (8) the dynamic path of taxes will depend on the values of π, λ and θ with the exact relationship depending upon the functional form of the government production function. While it is not possible to consider every possible alternative theory linking reassessment delays to tax revenues, several leading hypotheses are listed in the first four rows of Table 3. The common feature of the first three alternatives is that they assume reassessment lags have an identical influence in all communities whether it is no effect (H1), a monotone effect (H2), or inducing a common extremum (H3). The fourth alternative (H4) presumes that reassessment lags always increase tax revenues though the rate of increase may vary across communities and time; this is the prediction of the variable property turnover rate model mentioned at the end of Section 2.2. The model proposed in this paper (H5: Varying Single Peak) is mutually incompatible with these alternatives because it picks a distinct maximum for each community.

To help choose between the various hypotheses, I will estimate a reduced form equation relating county reassessment delays to municipality property tax revenues. It would be preferable not to impose any functional relationship in this estimation, but unfortunately current semi- and non-parametric techniques rely on having continuous explanatory variables while this problem is inherently discrete (since fiscal decisions are made once a year). Instead I approximate the relationship between reassessment lags Y and property tax revenues R_Y using a K degree polynomial of reassessment lags for each community i,

$$R_{Yi} = \sum_{n=0}^{K} (\rho_{in} + Z_i \omega_n) Y_i^n + X_i \nu + u_i \tag{14}$$

The reassessment lags are interacted with a community-specific parameter ρ_{in} and the three variables from the model, $Z_i \equiv \pi_i | \lambda_i | \theta_i$;[19] X_i is some additional set of covariates which are independent of Y;[20] ρ_{in}, ω_n and ν are parameters to be estimated and u_i is a normally dis-

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[19]The observed inflation rate is constant across communities but varies with time. So in any year t, $\pi_i = \pi_t \; \forall i$.

[20]These additional variables are included to capture community-specific factors which are omitted from the theoretical model. The assessed base A_Y is explicitly excluded from X_i because it trends with Y. Its

tributed error term. This specification captures much of the model's non-linearity through the interaction terms and the community-specific parameters. The hypotheses in Table 3 are special cases of (14), and the parameter restrictions they impose are listed in the table's second column. Hypotheses H1–H3 impose a zero parameter on the interaction between reassessment lags and community characteristics ($\omega_n = 0$) while Hypothesis H4 imposes an inequality restriction on the parameters ($\rho_{in} + Z_i\omega_n \geq 0$). My hypothesis H5 identifies a unique polynomial peak for each community which can be computed from observed data. This imposes a set of linear equality constraints (first order conditions) and inequality constraints (second order conditions) on the parameters.[21] Hypothesis H5 also requires that any other extrema fall outside of the observed sample for each community.[22] I will refer to (14) as the *non-structural* estimate because it is not directly connected to any underlying utility function.

While the non-structural estimate provides a framework for evaluating the model, it does not take full advantage of the theoretical restrictions. An alternative approach is to estimate a *structural* equation which is derived from an utility function.[23] Underlying the structural estimates are four assumptions, none of which alter the theoretical predictions of the model: (i) the functional form for the government production function is $B(S) = \alpha S^\gamma$ where S is total government revenue per home (see below), $\alpha > 0$ is a scaling factor and $\gamma \in (0, 1)$ measures government efficiency (I also consider $B(S) = -\alpha \exp(-\gamma S)$ but omit the results because of space constraints); (ii) allow community factors X to pre-multiply the

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influence is captured in a reduced form manner by Z_i.

[21] The first order condition requires that $\partial R_Y(Y = Y_i^*)/\partial Y = 0$ or $\sum_n n(\rho_{in} + Z_i\omega_n)(Y_i^*)^{n-1} = 0 \; \forall i$. The second order condition requires that $\partial^2 R_Y(Y = Y_i^*)/\partial Y^2 < 0$ or $\sum_{n>0} n(n-1)(\rho_{in} + Z_i\omega_n)(Y_i^*)^{n-2} < 0 \; \forall i$.

[22] A polynomial which has non-sample extrema should more realistically fit the data. If there is only a single maximum, a reassessment delay will have a larger effect on taxes the farther it is from the peak. The theory rules out this possibility: because the assessed base asymptotes to a finite value (see (3)) so must the tax revenues (take limits of (6) with $t_{median} = Y^*$). This means longer delays should actually have a smaller influence on taxes. A polynomial which has a maximum within sample and a minimum beyond sample will better match the theory: reassessment delays beyond the peak no longer have an accelerating effect on tax revenues and the function need not be symmetric about the peak.

[23] The main difficulty with the structural approach is that it assumes a functional form for the government production function $B(\cdot)$ which in turn imposes a particular relationship between property taxes and reassessment delays. The structural approach also assumes that the median age satisfies $t_{median} = \min(Y, Y^*)$ where Y^* is calculated from the observed data using (8). This condition is important because the estimated equation imposes that revenues are maximized when $Y = Y^*$. Fortunately this assumption is tested in the non-structural estimate and so is less problematic than the functional form assumption.

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production function using the form $X^\beta \equiv \prod_j X_j^{\beta_j}$ where β is some parameter; (iii) allow each community to receive exogenous, non-property tax revenue G (see Section 3.1); (iv) allow freshly assessed homes to be valued at a fraction $c < 1$ of true market value M, and for the inflation rate to be time-varying, π_s where s is time. Some algebra shows the majority rule level of spending $S_Y \equiv R_Y + G$ is,

$$\ln(S_Y) = \kappa_0 + \kappa_1 \ln(A_Y^{obs}) + \kappa_2 \ln(M) + \kappa_3 \sum_{s=1}^{t_{median}} \ln(1 + \pi_s) + \sum_j \kappa_{4j} \ln(X_j) + u \qquad (15)$$

where $A_Y^{obs} \equiv cMA_Y$ is the observed assessed base per home, $t_{median} \equiv \min(Y, Y^*)$ is calculated from the observed data using (8), $\kappa_1 = -\kappa_2 = \kappa_3 \equiv (1 - \gamma)^{-1} > 1$, and u is a normally distributed error term. This is the structural equation which can be estimated using ordinary least squares.[24]

The parameter estimates from (15), denoted with a "$\wedge$," can be used used to calculate two measures of social loss due to county reassessment delays. First, $\hat{S}_Y / \hat{S}_0 = A_Y^{\hat{\kappa}_1} \prod_{s=1}^{t_{median}} (1 + \pi_s)^{\hat{\kappa}_3}$ measures the relative spending distortion from a Y year county reassessment delay because spending following a reassessment, S_0, is socially optimal. Second, the social welfare loss under a utilitarian criterion, $W_Y \equiv \sum_{t=0}^{Y} f_Y(t) U_Y(t)$, can be written as $\Delta \hat{W}_Y \equiv \hat{W}_0 - \hat{W}_Y = (\hat{S}_0^{\hat{\gamma}} - \hat{S}_Y^{\hat{\gamma}}) X^{\hat{\beta}} \hat{\alpha} - (\hat{S}_0 - \hat{S}_Y)$, where $\hat{S}_Y$, $\hat{\gamma}$, $\hat{\beta}$ and $\hat{\alpha}$ are determined from (15).[25] In addition the estimates can be used to make inferences about the median voter's utility function which can then be used to indirectly evaluate Proposition 3, a theory of the timing of county-wide reassessments.

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[24]In the interest of improving fit, I will also allow for year- and community-specific constant terms. I will also perform a rough test of the functional form assumption by including higher order versions of the regressors; if these terms are significant, (15) is likely to be misspecified.

[25]In deriving (15) one can show $\hat{\gamma} = 1 - \hat{\kappa}^{-1}$; $\hat{\alpha} = (c\hat{\kappa}/(\hat{\kappa} - 1)) \exp(\hat{\kappa}_0/\hat{\kappa})$; $\hat{\beta}_j = \hat{\kappa}_{4j} \hat{\kappa}$ for $\hat{\kappa} = \hat{\kappa}_1$, $-\hat{\kappa}_2$, $\hat{\kappa}_3$. These relationships presume that the parameter restrictions listed following (15) hold.

4 Estimates

Using the approach outlined in Section 3.2 I can investigate the influence of county reassessment lags on property tax revenues.[26] The first step is to estimate a third order[27] polynomial version of the non-structural equation in (14), and the results are reported in Table 4.[28] Column 1 contains the unconstrained specification while column 2 contains the constrained specification where the reassessment lag-community characteristic interactions have zero parameters. Using the F-statistic in the second to last row, it is possible to reject the null that the constrained specification is correct even at the 99% confidence level. This result suggests rejecting hypotheses H1–H3 from Table 3 (relative to an unrestricted alternative) because the parameter restrictions in column 2 are a necessary condition for those alternatives. Similarly, I reject hypothesis H4 because the Wald statistic from the relevant restricted specification exceeds the critical value (these estimates are omitted and computed using the technique mentioned in the next paragraph). The latter result is important since hypothesis H4 is the restriction imposed by the variable property turnover rate model discussed at the end of Section 2.2.

Testing hypothesis H5: Varying Single Peak, the theory proposed in this paper, is more difficult due to the presence of inequality constraints. As a first step, column 3 presents results when the equality constraints (from each community's first order condition) are imposed.[29] Using the F-statistic in the second to last row, it is not possible to reject these restrictions at the 95th percentile. In this restricted specification the second order condition from the theory is satisfied for 205 of the 237 communities in the sample (an 86% success rate). To more formally investigate this point, the inequalities from the second order condi-

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[26]Land use variables, *residential* and *biz*, are not included as regressors in any of the specifications since they are only available going back to 1970. These factors have insignificant parameters when included in a truncated sample regression.

[27]I initially considered a first degree polynomial and continued to add higher order Y terms until the most recent addition was insignificant. As a robustness check I considered specifications which included various combinations of terms up to order Y^5. None of the additional Y^n terms were significant in the supplemental regressions.

[28]The non-structural estimates imply a relationship between tax revenues and reassessment lags which is quite similar to that found in the structural estimates. To conserve space I will only interpret the structural parameters.

[29]In constructing the equality constraints discussed in footnote 21, for Z_i I use the mean π_t value and each community's θ_i and λ_i value. A similar convention is used for the inequality constraints.

tions are imposed along with the first order conditions. The resulting quadratic programming problem is solved using the primal active set method discussed in Fletcher [13] Chapter 10.3 and the results are reported in column 4. Of primary interest is the Wald statistic listed in the last row which tests the imposed restrictions. It is not possible to reject at the 95th percentile the null that the first and second order conditions both hold.[30] The final restriction of hypothesis H5 is that there should be no other extremum in the observed sample period. For the restricted specification in column 4, 211 communities have an unconstrained extremum[31] exceeding their sample maximum Y and 26 have an unconstrained extremum within sample (an 89% success rate). Unfortunately, it is possible to reject this restriction with 95% confidence when it is imposed on the parameters (estimates omitted). However, all of the unwanted extrema fall at the end of the sample for communities in Delaware County which has not reassessed in over fifty years. Overall, it is difficult to reject the restrictions imposed by hypothesis H5 relative to the completely unrestricted alternative.

With the non-structural estimates lending support to the main predictions of the theory, it is reasonable to consider the structural approach. Table 5 presents estimates for the government revenue specification in (15).[32] These parameter estimates[33] can be used to

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[30]While Wolak [25] shows that it is infeasible to calculate the exact critical value for this test when there are more than eight restrictions, Kodde and Palm [15] provide upper and lower bounds akin to the bounds for the Durbin-Watson statistic. The critical lower bound is the c which solves, $\alpha = 0.5 \Pr[\chi^2(q) \geq c] + 0.5 \Pr[\chi^2(q + 1) \geq c]$ where each right-hand side term is a chi-squared distribution with q degrees of freedom (the number of equality restrictions) and α significance level. The approximate solution when $\alpha = 0.05$ and $q = 237$ is $c = 274.5$, and the null hypothesis cannot be rejected if the Wald statistic is less than c.

[31]Because a third order polynomial is used, there will be no more than two extrema per community and one of them is already determined by the parameter restrictions.

[32]The structural estimates are consistent with four tests of the underlying theory mentioned following (15). First, it is not possible to reject the hypothesis that the assessed base, the negative of market value and the summed inflation term all have identical parameters; formally the test statistics listed below the restricted specifications in columns two and four of Table 5 do not exceed the critical F value at the 95th percentile. Second, it is not possible to reject the hypothesis that this common parameter value exceeds one using a one-sided t-test. Third, higher order versions of these regressors are insignificant– either singly or jointly– when included in the specification (estimates omitted). Fourth, it is not possible to the reject the null that the presumed assessment relationship $A_Y^{obs} \equiv cMA_Y$ holds (estimates omitted). In addition I re-estimated the structural equation using the alternative functional form for the government production function, $B(S) = -\alpha \exp(-\gamma S)$. I found similar revenue distortions and welfare losses as those reported below (results omitted).

[33]To conserve space, I will not discuss the parameter estimates for the control variables. These parameters have a similar sign and magnitude as those typically found in studies of local revenue or expenditure setting behavior (see Fisher [12]). One interesting parameter to consider is the efficiency measure for the government production function. Using the fixed effects, restricted specification in column 4 of Table 5 and the formula

calculate a measure of the revenue distortion from county reassessment delays. The first row of Table 6 shows the revenue per home for various reassessment lags relative to the revenue per home in a reassessment year (calculated at sample mean values). Recall that this ratio can be interpreted as an efficiency wedge because revenues are socially optimal in a reassessment year. Delays of even five years, typical in virtually all states (Langhoff [17]), increase revenues by 6%; given the small standard error, it is possible to reject the hypothesis that this value is zero. Further revaluation delays have an accelerating affect with a 15 year delay increasing revenues by 42%.[34] However, lags beyond the sample mean $Y^* = 16$ result in lower revenues because at this point the median tenure is fixed while the assessed base continues to shrink. This means the value in the last column represents the maximal distortion.

While the revenue distortion from county reassessment delays is large, the associated welfare loss is quite small. The second row of Table 6 shows the mean welfare loss per home for various reassessment lags. A five year lag imposes a loss of only $0.89 per home and the value is not statistically different from zero. The value in the last row shows that this loss is less than one percent of the revenue per home in a reassessment year. Even a 15 year delay imposes a loss of only $19.74 per home or 7% of the revenue per home in a reassessment year (the latter figure is not statistically different from zero). The reason the social loss is so small is that reassessment delays are redistributive, benefiting infrequent movers due to their low tax price. Because over half of the residents have not moved since the last county reassessment ($t_{median} = Y \leq Y^*$), such gains go a long way towards offsetting the losses of recent movers.

The structural estimates can also be used to predict the timing of county reassessments.[35]

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in footnote 25, the estimated value is $\hat{\gamma} = 0.130$ (an application of the delta method gives the asymptotically valid standard error, $se(\hat{\gamma}) = 0.056$). While it is difficult to compare this to the literature, the closeness to zero suggests a significant degree of decreasing returns in government services.

[34]The reason distortions accelerate is that reassessment delays increase the median tenure at a one-for-one rate (because $t_{median} = \min(Y, Y^*)$) but decrease the assessed base at a slower rate (because $\partial^2 A_Y / \partial Y^2 > 0$). Due to this, the median will demand higher revenues at an increasing rate so long as $Y \leq Y^*$.

[35]The estimates to this point have presumed that county reassessments are exogenous events. However, the endogeneity presumed here takes a simple form: a county does not reassess until some well-defined delay is reached. Because the actual delays are quite long (see Table 1), the parameters in Tables 4 and 5 would be largely unchanged if instead I estimated a system which includes a reassessment timing equation. Still the results in this paragraph are not based on a full statistical framework and should be interpreted cautiously.

A reassessment should occur when it benefits the median tenured home-owner, and Proposition 3 shows this requires an assessment delay exceeding some threshold. This threshold is uniquely determined by the median's utility function: reassessment is beneficial after Y years if $U_0(t_{median}) - U_Y(t_{median}) - C > 0$ where C is the physical cost per home of reassessment (in practice this cost is about \$37 per home).[36] This formula can be evaluated using the utility parameters backed out of the structural estimates. The median's net gain from reassessment is presented in Table 7 for two of the four counties in the sample. The predicted median break-even year is in fact quite close to the observed lag between reassessments listed in Table 1. For example Delaware County, which last reassessed 60 years ago, has a break-even year of 55 while Chester County, which has been reassessing roughly every 20 years, has a break-even year of 23. The reason these counties have such disparate break-evens is that Delaware County is much more established and so has a much older median.[37] Long reassessment delays are relatively less costly to the Delaware County median since he enjoys a low tax price.

A final issue to consider is the revenue change when a county reassesses. The structural estimates imply that revenues in the year prior to a reassessment are below their initial value in each of the four counties (this calculation is omitted and uses the reassessment dates listed in Table 1). This suggests that voters will select a tax revenue increase upon reassessment confirming the prediction of Proposition 3.[38] Bloom and Ladd [4] and Ladd [16] also find evidence that revaluations induce higher taxes which they attribute to voter fiscal illusion. The results here suggest another, fully rational explanation. Reassessments are associated with tax hikes because they increase the size of the assessed base and thus the median's demand for taxes.[39]

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[36] I estimate reassessment costs from the current experience of Delaware County. Its ongoing reassessment will cost \$7.8 million (*Philadelphia Inquirer*, 22 February 1996) for its 211,024 homes (1990 Census of Housing) or \$36.96 per home.

[37] Formally, a lower construction rate θ will imply a higher Y^* value which is the median tenure for the long reassessment lags considered here.

[38] According to Proposition 3, when reassessment is costless the median will prefer to reassess when tax revenues just reach their initial value. However reassessments actually entail non-trivial physical costs, and so the median will not actually prefer reassessment until taxes have fallen *below* their initial value.

[39] One potential problem with using this theory to explain Bloom and Ladd's observation is that it presumes reassessment timing is endogenous. Given the lack of state enforcement and few observed revaluations in their sample, this seems entirely plausible (the communities which they observe have an average reassessment

5 Conclusion

In this paper I develop a model which draws a connection between community-wide re-assessment lags and property tax revenues. Short lags result in higher tax collections since the median voter faces a reduced tax price while longer lags decrease revenues due to the diminished aggregate property base. Estimates based on a sample of communities in the Philadelphia suburbs confirm the basic predictions of the theory. While reassessment lags can significantly increase the level of revenues, they tend not to impose large social losses because they benefit long-time residents. Still it is important to be cautious in interpreting these results due to the presence of simplifying assumptions which can only be indirectly tested in the sample here. Further tests using other data will help evaluate the appropriateness of these assumptions.

The distortionary effect of community-wide reassessment lags has potential policy implications. For example it provides another rationale, beyond the usual horizontal equity argument, for more frequent revaluations: delays result in taxes which exceed the social optimum. Of course one reason that revaluations are not conducted every year is that they are costly. Given this, one practical solution would be to increase the assessed value of unsold properties at the same rate as the region's home price index (which the Bureau of Labor Statistics calculates) in the intermediate years. If homes uniformly appreciate at this rate, such a reform would completely eliminate the distortion since both tax prices and the assessed base would reflect true market value. Unfortunately such a change is politically infeasible since the current system actually benefits the majority (at the expense of frequent movers), and the reform would particularly hurt long time residents, who wield disproportionate power in local politics.

This theory also has testable implications regarding popular perceptions of property

---

delay of 25 years, a figure which is comparable to three of the four counties in my sample). It is more difficult to apply this argument to Ladd because the counties in her study are required to reassess every eight years. Nonetheless, her sample likely involves higher rates of property construction and inflation than those found in this paper, and these differences could lead the median voter to prefer a tax increase upon reassessment (a higher construction rate lowers the maximum median age Y^* while higher inflation increases the rate at which reassessment delays diminish the assessed base; these two forces work together to lower the minimum reassessment lag at which tax collections fall below their initial level).

taxes. Infrequent community-wide reassessments may lead fully informed voters to believe that property taxes are too high as well as unfair. This is because in nearly all states the median home-owner will not have moved since the last reassessment, and he will select a tax rate which is just optimal for his property assessment but is excessive for those who have moved since they face a higher property assessment. Hence, a random survey of residents will find that a significant minority (the movers) consider property taxes to be too high while no one believes taxes are too low. This explanation is testable. If an opinion survey includes an individual's home tenure, then it is possible to test whether recent movers disproportionately consider property taxes to be excessive. Even without such micro-data, indirect tests are possible if the survey includes geographic markers. The proportion of residents who find property taxes excessive should be larger in municipalities where more time has elapsed since the last reassessment. Investigating this possibility would be an interesting topic for future work.

A Proofs

A.1 Proof of Proposition 1

Each community's tax rate is the optimal choice of the median aged home-owner which is characterized by (6). When $Y \leq Y^*(\theta, \lambda)$ then $t_{median} = Y$ and (6) can be written as,

$$
\begin{aligned}
B'(R_Y) &= \left[A_Y (1 + \pi)^Y \right]^{-1} \\
&= \left[\frac{(\lambda + \theta)(1 + \pi)^{Y+1} + \pi(1 - \lambda)^{Y+1}/(1 + \theta)^Y}{(1 + \theta)(1 + \pi) - (1 - \lambda)} \right]^{-1}
\end{aligned}
\tag{16}
$$

Since this is an identity I can differentiate it with respect to Y to get,

$$
\frac{\partial R_Y}{\partial Y} = \frac{\partial \left[A_Y (1 + \pi)^Y \right]^{-1}/\partial Y}{B''(R_Y)}
\tag{17}
$$

Because $B'' < 0$, the left-hand side will have the same sign as the derivative of the square bracketed term in the numerator,

$$
\frac{\partial A_Y (1 + \pi)^Y}{\partial Y} = \frac{(\lambda + \theta)\ln(1 + \pi)(1 + \pi)^{Y+1} + \pi\left[\ln(1 - \lambda) - \ln(1 + \theta)\right](1 - \lambda)^{Y+1}/(1 + \theta)^Y}{(1 + \theta)(1 + \pi) - (1 - \lambda)}
\tag{18}
$$

To show (18) is positive, it will be sufficient to prove the numerator is positive. Now the numerator is increasing in Y so it will be sufficient to consider the case $Y = 0$. Some tedious algebra shows that the resulting term,

$$
(\lambda + \theta)\ln(1 + \pi)(1 + \pi) + \pi\left[\ln(1 - \lambda) - \ln(1 + \theta)\right](1 - \lambda)
\tag{19}
$$

is strictly increasing in π so long as $\lambda, \theta > 0$. This means that (19) is strictly positive when $\pi > 0$ (since (19) equals zero when $\pi = 0$), which thus proves that (17) is strictly positive.

Now when $Y > Y^*(\theta, \lambda)$ the median age is fixed at $Y^*(\theta, \lambda)$. Full differentiation of the first order condition (6) and rearrangement yields,

$$
\frac{\partial R_Y}{\partial Y} = \frac{-B'(R_Y)\partial A_Y/\partial Y}{A_Y B''(R_Y)}
\tag{20}
$$

which is strictly negative since $\partial A_Y/\partial Y < 0$. This completes the proof.

Q.E.D.

A.2   Proof of Proposition 2

In the proof of Proposition 1 I showed that R_Y increases in Y when $Y \leq Y^*(\theta, \lambda)$. This means in particular that $R_Y > R_0$, so tax collections exceed the socially optimal level in this interval. All that remains is to consider $Y > Y^*(\theta, \lambda)$. In analogy to (16), the first order condition may be written as,

$$
\begin{aligned}
B'(R_Y) &= \left[A_Y(1+\pi)^{Y^*(\theta,\lambda)}\right]^{-1} \\
&= \left[\frac{(\lambda+\theta)(1+\pi)^{Y^*(\theta,\lambda)+1} + \pi(1-\lambda)^{Y+1}/(1+\theta)^Y}{(1+\theta)(1+\pi)-(1-\lambda)}\right]^{-1}
\end{aligned}
\tag{21}
$$

Given the social optimality condition (10), taxes are sub-optimal when the right-hand side is greater than unity or $A_Y(1+\pi)^{Y^*(\theta,\lambda)} < 1$. Since the right-hand side of (21) is increasing in Y, a sufficient condition to complete the proof is to find some Y at which this inequality holds. As Y grows unbounded, the second term in the numerator of (21) goes to zero,

$$
\lim_{Y \to \infty} B'(R_Y) = \left[\frac{(\lambda+\theta)(1+\pi)^{Y^*(\theta,\lambda)+1}}{(1+\theta)(1+\pi)-(1-\lambda)}\right]^{-1}
\tag{22}
$$

Now take logarithms of the right-hand side,

$$
\begin{aligned}
\ln\left[\frac{(\lambda+\theta)(1+\pi)^{Y^*(\theta,\lambda)+1}}{(1+\theta)(1+\pi)-(1-\lambda)}\right]^{-1} &= \ln\left[1+\frac{\pi(1+\theta)}{\lambda+\theta}\right] - [Y^*(\theta,\lambda)+1]\ln(1+\pi) \\
&\approx [(1-\lambda)-\ln 2]\frac{\pi}{\lambda+\theta}
\end{aligned}
\tag{23}
$$

where I employ the definition of $Y^*(\theta, \lambda)$ and two first order Taylor expansions (Taylor expansions are valid for the purposes of signing when θ, λ and π are suitably close to zero). So long as $1-\lambda > \ln 2$, which must hold for reasonable rates of property turnover, this value is positive and hence the right-hand side of (21) must exceed one. This shows that for Y large enough tax revenue will be sub-optimal. The threshold for this event is the smallest Y value for which the right-hand side of (21) exceeds one. Due to the continuity of that expression and the strict inequality from (23), this must occur in finite time.

Q.E.D.

24

A.3 Proof of Proposition 3

Presume that a community-wide reassessment costs some amount C per home. Since a voter's net benefit from reassessment is decreasing in t, the age of his property (proof omitted), there will be majority support for reassessing only when the median aged voter prefers it. For $Y \leq Y^*(\theta, \lambda)$, it will be sufficient to show that assessment delays increase the median's welfare (this implies that a reassessment will lower the median's welfare in addition to imposing the cost C). Differentiating the median's utility given in (5) yields,

$$\frac{\partial U_Y(t_{median})}{\partial Y} = \left[\frac{\partial A_Y/\partial Y}{A_Y} + \ln(1 + \pi) \right] m_Y (1 + \pi)^{-Y} \tag{24}$$

where I have used (2), (4), the first order condition (6), and the fact that $t_{median} = Y$ when $Y \leq Y^*(\theta, \lambda)$. Substituting $\partial A_Y/\partial Y$ from (3) gives,

$$\begin{aligned}
\frac{\partial U_Y(t_{median})}{\partial Y} &= \left[\frac{(\lambda + \theta)(1 + \pi)}{(1 + \theta)(1 + \pi) - (1 - \lambda)} \frac{\ln(1 + \pi)}{A_Y} \right. \\
&\left. + \left(1 - \frac{(\lambda + \theta)(1 + \pi)}{((1 + \theta)(1 + \pi) - (1 - \lambda))A_Y} \right) \ln \left(\frac{1 - \lambda}{1 + \theta} \right) \right] m_Y (1 + \pi)^{-Y}
\end{aligned} \tag{25}$$

The sign of $\partial U_Y(t_{median})/\partial Y$ will be identical to the sign of the square bracketed term which is itself increasing in Y. It will thus be sufficient to consider this term evaluated at $Y = 0$,

$$sign \left(\frac{\partial U_Y(t_{median})}{\partial Y} \right) = sign \left[\frac{(\lambda + \theta)(1 + \pi)\ln(1 + \pi) + (1 - \lambda)\pi \ln((1 - \lambda)/(1 + \theta))}{(1 + \theta)(1 + \pi) - (1 - \lambda)} \right] \tag{26}$$

Because the right-hand side denominator is positive and (some tedious algebra shows that) the numerator is increasing in π, it will be sufficient to consider the case $\pi = 0$. Since the right-hand side of (26) is zero at $\pi = 0$, this means $\partial U_Y(t_{median})/\partial Y > 0$ for $\pi > 0$ which completes this part of the proof.

When $Y > Y^*(\theta, \lambda)$ assume $\exists \tilde{Y} : m_{\tilde{Y}} A_{\tilde{Y}} = m_0 A_0$ (by Proposition 2, $\tilde{Y}$ exists and is finite when the parameters are sufficiently near zero). Some algebra shows that $m_{\tilde{Y}}(1 + \pi)^{-Y^*} =$

$m_0 A_0$ and therefore that,

$$U_{\tilde{Y}}(t_{median}) \equiv B(m_{\tilde{Y}} A_{\tilde{Y}}) - m_{\tilde{Y}}(1+\pi)^{-Y^*} = B(m_0 A_0) - m_0 A_0 \equiv U_0(t_{median}) \qquad (27)$$

Some algebra shows that for $Y > Y^*(\theta, \lambda)$ the median's welfare is decreasing in Y, $\partial U_Y(t_{median})/\partial Y < 0$. Combined with (27) this means that the median (and thus a majority of voters) will strictly prefer community-wide reassessment only when $Y > \tilde{Y}$ so long as reassessing is costless ($C = 0$). Notice from the proof of Proposition 2 that $\tilde{Y}$ is exactly the year at which $A_Y(1+\pi)^{Y^*(\theta, \lambda)} = 1$, so when reassessment is costless the median first prefers reassessing when (13) is satisfied. Notice also that so long as C is not too large, there will always be some finite $Y > \tilde{Y}$ at which the median prefers reassessment.

$Q.E.D.$

References

[1] H. Aaron, "Who Pays the Property Tax? A New View," The Brookings Institution, Washington D.C. (1975).

[2] Advisory Commission on Intergovernmental Relations, "Tax and Expenditure Limits On Local Governments," Washington D.C. (1995).

[3] A. Auerbach, On the design and reform of capital-gains taxation, *American Economic Review*, 82, 263–267 (May 1992).

[4] H. Bloom and H. Ladd, Property tax revaluation and tax levy growth, *Journal of Urban Economics*, 11, 73–84 (1982).

[5] Bureau of the Census, "Census of Population and Housing: Summary Characteristics for Pennsylvania," Washington D.C. (1960-1990).

[6] Bureau of Labor Statistics, "Consumer Price Index-All Urban Consumers. Area: Philadelphia-Wilmington-Trenton. Series ID: CUURA102SA0," `http://stats.bls.gov/` (1997).

[7] Bureau of Transportation Statistics, "Census Transportation Planning Package," Department of Transportation, Washington D.C. (1980-1990).

[8] P. Dearborn, ACIR 1993 poll takes pulse on taxes, *State Tax Notes*, 5, 781 (1993).

[9] Delaware Valley Regional Planning Commission, "Employment Data," Philadelphia (1979, 1984, 1993).

[10] Delaware Valley Regional Planning Commission, "Housing Units Authorized by Building Permits," Philadelphia (1979-1992).

[11] Delaware Valley Regional Planning Commission, "Land Use in the Delaware Valley," Philadelphia (1984, 1994).

[12] R. C. Fisher, "State and Local Public Finance," Irwin, Chicago (1996).

[13] R. Fletcher, "Practical Methods of Optimization," John Wiley & Sons, Chichester (1987).

[14] R. Inman and D. Rubinfeld, The judicial pursuit of local fiscal equity, *Harvard Law Review*, 192, 1662–1750 (1979).

[15] D. Kodde and F. Palm, Wald criteria for jointly testing equality and inequality constraints, *Econometrica*, 54, 1243–1248 (1986).

[16] H. Ladd, Property tax revaluation and tax levy growth revisited, *Journal of Urban Economics*, 30, 83–99 (1991).

[17] G. Langhoff, "Assessment Jurisdictions and Agency Resources in the United States: Summary of the 1986 Survey," International Association of Assessing Officers, Chicago (1988).

[18] A. O'Sullivan, T. Sexton and S. Sheffrin, "Property Taxes and Tax Revolts: The Legacy of Proposition 13," Cambridge University Press, Cambridge (1995).

[19] Pennsylvania Department of Community Affairs, "Citizen's Guide to Pennsylvania Local Government," Bureau of Local Government Services, Harrisburg (1991).

[20] Pennsylvania Department of Community Affairs, "Local Government Financial Statistics," Harrisburg (1960-1992).

[21] Pennsylvania State Tax Equalization Board, "Municipality Sales Data (unpublished)," Harrisburg (1990-1995).

[22] T. Romer and H. Rosenthal, Bureaucrats versus voters: on the political economy of resource allocation by direct democracy, *Quarterly Journal of Economics*, 93, 563–587 (1979).

[23] K. Strumpf, Government credibility and policy choice: evidence from the pennsylvania earned income tax, UNC Chapel Hill Economics Department Typescript (1998).

[24] K. Strumpf, A predictive index for the flypaper effect, *Journal of Public Economics*, forthcoming (1998).

[25] F. Wolak, An exact test for multiple inequality and equality constraints in the linear regression model, *The Journal of the American Statistical Association*, 82, 782–793 (1987).

County	Most Recent Reassessments
Bucks	1973, 1960
Chester	1975, 1958
Delaware	1935
Montgomery	1978, 1955

Table 1: Recent Reassessments in the Philadelphia Suburbs (through 1996)

Variable (Source)	Symbol	Mean	Std Dev	Max	Min
Assessment delays (*PSTEB*)	Y	15.740	14.430	57.000	0.000
Assessment cutoff (*PSTEB, DVRPC*)	Y^*	16.675	7.647	71.000	3.000
Property sale rate (*PSTEB*)	λ	0.033	0.013	0.098	0.006
Property construction rate (*DVRPC*)	θ	0.017	0.016	0.095	0.000
Inflation rate (*BLS*)	π	0.050	0.030	0.130	0.008
Property tax PH (*PDCA*)	R_Y	211.933	152.508	1451.107	0.000
Revenue PH (*PDCA*)	S_Y	386.082	170.396	2428.570	44.346
Millage (*PDCA*)	m_Y	19.413	22.777	252.400	0.000
Assessment PH [$\times 10^3$\$] (*PDCA*)	A_Y	15.261	8.383	76.554	1.810
Market value PH [$\times 10^3$\$] (*PDCA*)	M	76.718	43.682	406.741	8.099
Number homes [$\times 10^3$] (*Census*)	*homes*	2.763	4.081	34.162	0.127
Median HH income [$\times 10^3$\$] (*Census*)	y	40.484	10.798	114.890	10.855
Population [$\times 10^3$] (*Census*)	*pop*	8.180	11.731	95.910	0.420
% Population growth (*Census*)	*popg*	1.435	2.222	13.889	-6.422
% Seniors (*Census*)	*%Senior*	10.213	3.965	32.100	1.800
Housing tenure (*Census*)	*owner*	3.970	2.717	25.360	0.380
Jobs PC (*DVRPC, Census*)	*jobs*	0.465	0.409	4.810	0.000
% commute to Philadelphia (*Census*)	*%Phil*	11.576	13.047	69.160	0.000
% commute out of state (*Census*)	*%OutState*	8.122	11.186	72.372	0.000
% pay EIT at work (*Census*)	*%PayEIT*	27.162	28.183	96.778	0.000
Overhead index (*PDCA*)	*overhead index*	-0.000	0.470	1.610	-2.465
% Business Property (*DVRPC*)	*biz*	30.953	19.165	81.200	0.000
% Residential Property (*DVRPC*)	*residential*	26.280	17.763	92.300	0.600

Table 2: Descriptive Statistics

Notes:

Sample: 1960-1992, 237 Philadelphia SMSA municipalities. 'PH' = per home, 'HH' = household, 'PC' = per capita. All money denominated terms are normalized to 1992 values using the Philadelphia general CPI (the choice of base year does not influence the theoretical model).

Sources:

BLS = Bureau of Labor Statistics [6]
Census = Department of Census [5], [7]
DVRPC = Delaware Valley Regional Planning Commission [9], [10], [11]
PDCA = Pennsylvania Department of Community Affairs [20]
PSTEB = Pennsylvania State Tax Equalization Board [21]

Hypothesis	Parameter Restrictions
H1: Independence $\frac{\partial R_Y(Y)}{\partial Y} = 0 \ \forall Y$	$\omega_n, \ \rho_{in} = 0 \ \forall n > 0$
H2: Monotonicity I $\left(\frac{\partial R_Y(Y)}{\partial Y} \right) = \text{constant} \ \forall Y$	$\omega_n = 0 \ \forall n > 0, \ \ \rho_{in} = 0 \ \forall n > 1$
H3: Common Single Extremum $\frac{\partial R_Y(Y = \hat{Y})}{\partial Y} = 0$	$\omega_n = 0 \ \forall n > 0, \ Y_i(\rho_i, \omega) = \hat{Y} \ \forall i$
H4: Monotonicity II sign $\left(\frac{\partial R_Y(Y)}{\partial Y} \right) > 0 \ \forall Y$	$\rho_{in} + Z_i \omega_n \geq 0 \ \forall i, n > 0$
H5: Varying Single Peak $\frac{\partial R_Y(Y)}{\partial Y} \gtreqless 0 \leftrightarrow Y \gtreqless Y_i^*$	$\overline{Y}_i(\rho_i, \omega)^{sample} = Y_i(\rho_i, \omega)^{sample} = Y_i^*$

Table 3: Leading Hypotheses and Their Restrictions on (14)

Notes:
For H3 and H5: $Y_i(\rho_i, \omega) \equiv \{Y_i : \partial R_Y / \partial Y = 0\}$. For H5: "sample" indicates a domain restricted to the observed sample; $\overline{Y}_i(\rho_i, \omega) \equiv \{Y_i : \partial R_Y / \partial Y = 0, \ \partial^2 R_Y / \partial Y^2 < 0\}$; Y_i^* is defined in (8).

Regressors	Dependent variable: R_Y			
	Unconstrained	(H1–H3) Constrained I	(H5 FOC) Constrained II	(H5 FOC, SOC) Constrained III
constant	-35.826	96.607	-47.451	-50.488
	(30.963)	(38.885)	(33.441)	(23.254)
π_t	-413.790	-675.529	-107.856	-190.568
	(602.967)	(670.238)	(585.864)	(509.235)
λ_i	-856.352	-1639.570	-743.879	-799.865
	(306.139)	(396.522)	(330.567)	(287.012)
θ_i	1015.528	942.525	1309.578	1397.800
	(267.876)	(342.619)	(304.233)	(290.456)
Y	8.022	12.088	8.746	8.811
	(2.425)	(3.410)	(2.633)	(2.308)
$Y \times \pi_t$	7.181	—	6.999	6.879
	(27.528)		(21.832)	(20.074)
$Y \times \lambda_i$	11.105	—	11.003	11.009
	(57.313)		(55.124)	(53.127)
$Y \times \theta_i$	209.475	—	185.741	189.324
	(49.418)		(47.578)	(39.897)
Y^2	-0.540	-0.634	-0.479	-0.501
	(0.120)	(0.160)	(0.137)	(0.111)
$Y^2 \times \pi_t$	-0.177	—	-0.205	-0.287
	(1.486)		(1.388)	(1.323)
$Y^2 \times \lambda_i$	-3.026	—	-2.742	-2.799
	(2.953)		(2.847)	(2.711)
$Y^2 \times \theta_i$	-2.782	—	-4.117	-3.948
	(2.558)		(2.755)	(2.323)
Y^3	8.013×10^{-3}	8.867×10^{-3}	8.144×10^{-3}	8.193×10^{-3}
	(1.61×10^{-3})	(2.09×10^{-3})	(1.53×10^{-3})	(1.37×10^{-3})
$Y^3 \times \pi_t$	-0.017	—	-0.010	-0.012
	(0.021)		(0.022)	(0.017)
$Y^3 \times \lambda_i$	0.023	—	0.029	0.045
	(0.040)		(0.042)	(0.034)
$Y^3 \times \theta_i$	0.024	—	0.028	0.023
	(0.034)		(0.040)	(0.49)
N	7486	7486	7486	7486
R^2	0.700	0.677	0.691	—
F-statistic	—	55.21	0.82	—
Wald statistic	—	—	—	211.4

Table 4: Non-Structural Estimates: Equation (14)

Notes:

(standard errors). Sample: 1960-1992, 237 Philadelphia SMSA municipalities. The Unconstrained and Constrained I-II specifications are estimated using OLS (the results are similar when seemingly unrelated regressions are used); the Constrained III specification is estimated using the primal active set method discussed in Fletcher [13] Chapter 10.3. The constant and non-interacted Y^n terms have community-specific parameters; the table presents the average value. Constrained I imposes a zero parameter on the reassessment lag - community characteristic interactions; Constrained II imposes on the parameters the first order condition (FOC) for each community at $Y = Y_i^*$; Constrained III imposes on the parameters the first and second order conditions (SOC) at $Y = Y_i^*$. The F and Wald statistics are used to test these restrictions (see text): the 95% critical values are 1.88 (for Constrained I), 1.00 (for Constrained II), and 274.5 (for Constrained III). Additional regressors included in all specifications: M, y, pop, $popg$, $\%Senior$, $owner$, $jobs$, $\%Phil$, $\%OutState$, $\%PayEIT$, $overhead\ index$, $overhead\ index \times EIT$, $overhead\ index \times HighwayAid$ and period dummies.

| Regressors | Dependent variable: $\ln(S_Y)$ | | | |
	(OLS) Unrestricted	(OLS) Restricted	(FE) Unrestricted	(FE) Restricted
constant	2.742	3.124	—	—
	(0.235)	(0.250)		
$\ln(A_Y^{obs})$	1.194	—	1.103	—
	(0.084)		(0.074)	
$\ln(M)$	-1.149	—	-1.184	—
	(0.099)		(0.098)	
$\sum_{s=1}^{t_{median}} \ln(1+\pi_s)$	1.184	—	1.177	—
	(0.082)		(0.066)	
$\ln(A_Y^{obs}) - \ln(M) + \sum_{s=1}^{t_{median}} \ln(1+\pi_s)$	—	1.157	—	1.149
		(0.077)		(0.074)
$\ln(y)$	0.382	0.303	0.346	0.306
	(0.023)	(0.033)	(0.026)	(0.025)
$\ln(pop)$	0.114	0.080	0.056	0.051
	(0.004)	(0.004)	(0.014)	(0.014)
$popg$	-0.024	-0.025	-0.013	-0.015
	(0.002)	(0.002)	(0.002)	(0.002)
$\%Senior$	0.002	0.003	0.001	0.001
	(0.001)	(0.001)	(0.001)	(0.001)
$\ln(owner)$	0.028	0.025	0.013	0.043
	(0.007)	(0.009)	(0.010)	(0.010)
$\ln(jobs)$	0.057	0.164	0.054	0.044
	(0.005)	(0.006)	(0.007)	(0.007)
$\%Phil$	0.006	0.004	0.006	0.006
	(0.001)	(0.001)	(0.001)	(0.001)
$\%OutState$	-0.001	-0.000	-0.001	-0.001
	(0.001)	(0.001)	(0.001)	(0.001)
$\%PayEIT$	0.002	0.002	0.003	0.004
	(0.001)	(0.001)	(0.001)	(0.001)
$overhead\ index$	0.041	0.048	0.040	0.036
	(0.015)	(0.017)	(0.006)	(0.007)
$\mathcal{I}(city)$	0.528	0.340	—	—
	(0.040)	(0.045)		
$\mathcal{I}(borough)$	0.245	0.279	—	—
	(0.010)	(0.011)		
Municipality Fixed Effect?	No	No	Yes	Yes
Period Fixed Effect?	Yes	Yes	Yes	Yes
N	7486	7486	7486	7486
R^2	0.799	0.761	0.844	0.817
F-statistic	—	1.794	—	1.941

Table 5: Structural Estimates: Equation (15)

Notes:

(standard errors). Sample: 1960-1992, 237 Philadelphia SMSA municipalities. FE = Municipality fixed effects regression. Key regressors are in bold. Mean of dependent variable = 5.875. F-statistic is for testing H_0: restricted specification in columns 2 and 4; the 95% critical values are $F(2,7438) \approx F(2,7204) = 3.00$.

Statistic	$Y = 5$	$Y = 10$	$Y = 15$
$\hat{S}_Y/\hat{S}_0$	1.060	1.195	1.419
	(0.024)	(0.051)	(0.086)
$\Delta\hat{W}_Y$	$0.886	$5.561	$19.744
	(1.292)	(1.750)	(4.236)
$100 \times \Delta\hat{W}_Y/\hat{S}_0$	0.320	2.008	7.130
	(3.026)	(4.087)	(9.813)

Table 6: Reassessment Lags: Revenue Distortion and Welfare Loss Per Home

Notes:

Numbers in parentheses are asymptotically valid standard errors calculated using the delta method. Values are based on parameter estimates in columns 3 and 4 of Table 5 and mean community characteristics. The formulae for $\hat{S}_Y/\hat{S}_0$ and $\Delta\hat{W}_Y$ are given in Section 3.2. $\hat{S}_0 = 276.9$ is calculated from (15).

Chester County	$Y = 15$	$Y = 20$	$Y = 25$
$U_0(t_{median}) - U_Y(t_{median})$	-$66.76	$15.84	$58.13
C	$36.96	$36.96	$36.96
Net Benefit	$-103.72	-$21.12	$21.17

Delaware County	$Y = 45$	$Y = 50$	$Y = 55$
$U_0(t_{median}) - U_Y(t_{median})$	$19.34	$30.79	$37.25
C	$36.96	$36.96	$36.96
Net Benefit	-$17.62	-$6.17	$0.29

Table 7: The Median's Net Benefit from Reassessment

Notes:

See text for a discussion of how these figures are calculated. For each county, the mean θ, λ and A_Y values of the component municipalities are used to estimate S_Y and m_Y from the structural estimates. There is no county-specific π_s, so the geometric mean of the SMSA measure is used for each county. No standard errors are calculated because there is no variance measure for C (see footnote 36).