Bragg’s Law with Microwaves

In this lab you will do experiments on the scattering of microwaves from an array of steel balls to explore Bragg’s Law. However, the most important application of Bragg’s Law is to the scattering of x-rays from a crystal such as NaCl, which you’ll do in one of your next labs. So it’s useful to first read about x-ray scattering to motivate the microwave experiment.

(From Young and Freedman, University Physics, 9th ed., Addison-Wesley.

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**X-Ray Diffraction**

X rays were discovered by Wilhelm Röntgen (1845–1923) in 1895, and early experiments suggested that they were electromagnetic waves with wavelengths of the order of $10^{-10}$ m. At about the same time, the idea began to emerge that in a crystalline solid the atoms are arranged in a regular repeating pattern, with spacing between adjacent atoms also of the order of $10^{-10}$ m. Putting these two ideas together, Max von Laue (1879–1960) proposed in 1912 that a crystal might serve as a kind of three-dimensional diffraction grating for x rays. That is, a beam of x rays might be scattered (that is, absorbed and re-emitted) by the individual atoms in a crystal, and the scattered waves might interfere just like waves from a diffraction grating.

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38–16 (a) In an x-ray diffraction experiment, most x rays pass straight through the crystal. But some x rays are scattered and form an interference pattern that exposes the film in a pattern related to the atomic arrangement in the crystal. (b) Diffraction pattern (or Laue pattern) formed by directing a beam of x rays at a thin section of quartz crystal.
The first x-ray diffraction experiments were performed in 1912 by Friederich, Knipping, and von Laue, using the experimental setup sketched in Fig. 38–16a (page 1180). The scattered x rays did form an interference pattern, which they recorded on photographic film. Figure 38–16b is a photograph of such a pattern. These experiments verified that x rays are waves, or at least have wavelike properties, and also that the atoms in a crystal are arranged in a regular pattern (Fig. 38–17). Since that time, x-ray diffraction has proved to be an invaluable research tool, both for measuring x-ray wavelengths and for the study of crystal structure.

To introduce the basic ideas, we consider first a two-dimensional scattering situation, as shown in Fig. 38–18a, in which a plane wave is incident on a rectangular array of scattering centers. The situation might be a ripple tank with an array of small posts, 3-cm microwaves striking an array of small conducting spheres, or x rays incident on an array of atoms. In the case of electromagnetic waves, the wave induces an oscillating electric dipole moment in each scatterer. These dipoles act like little antennas, emitting scattered waves. The resulting interference pattern is the superposition of all these scattered waves. The situation is different from that with a diffraction grating, in which the waves from all the slits are emitted in phase (for a plane wave at normal incidence). Here the scattered waves are not all in phase because their distances from the source are different. To compute the interference pattern, we have to consider the total path differences for the scattered waves, including the distances from source to scatterer and from scatterer to observer.
38–18 (a) Scattering of waves from a rectangular array. (b) Interference of waves scattered from adjacent atoms in a row is constructive when $a \cos \theta_i = a \cos \theta_r$, that is, when the angle of incidence $\theta_i$ equals the angle of reflection $\theta_r$. Both angles are measured from the surface of the crystal, not from its normal. (c) Interference from adjacent rows is also constructive when the path difference $2d \sin \theta$ equals an integral number of wavelengths, as in Eq. (38–16).

As Fig. 38–18b shows, the path length from source to observer is the same for all the scatterers in a single row if the two angles $\theta_i$ and $\theta_r$ are equal. Scattered radiation from adjacent rows is also in phase if the path difference for adjacent rows is an integer number of wavelengths. Figure 38–18c shows that this path difference is $2d \sin \theta$, where
θ is the common value of θ₁ and θ₂. Therefore the conditions for radiation from the entire array to reach the observer in phase are (1) the angle of incidence must equal the angle of scattering and (2) the path difference for adjacent rows must equal mλ, where m is an integer. We can express the second condition as

\[ 2d \sin \theta = m\lambda \quad (m = 1, 2, 3, \ldots) \]  \hspace{1cm} \text{(Bragg condition for constructive interference from an array).}

**CAUTION** In this equation the angle θ is measured with respect to the surface of the crystal, rather than with respect to the normal to the plane of an array of slits or a grating. Also, note that the path difference in Eq. (38–16) is 2d sin θ, not d sin θ as in Eq. (38–13) for a diffraction grating.

In directions for which Eq. (38–16) is satisfied, we see a strong maximum in the interference pattern. We can describe this interference in terms of reflections of the wave from the horizontal rows of scatterers in Fig. 38–18. Strong reflection (constructive interference) occurs at angles such that the incident and scattered angles are equal and Eq. (38–16) is satisfied.

We can extend this discussion to a three-dimensional array by considering planes of scatterers instead of rows. Figure 38–19 shows two different sets of parallel planes that pass through all the scatterers. Waves from all the scatterers in a given plane interfere constructively if the angles of incidence and scattering are equal. There is also constructive interference between planes when Eq. (38–16) is satisfied, where d is now the distance between adjacent planes. Because there are many different sets of parallel planes, there are also many values of d and many sets of angles that give constructive interference for the whole crystal lattice. This phenomenon is called Bragg reflection, and Eq. (38–16) is called the **Bragg condition**, in honor of Sir William Bragg and his son Laurence Bragg, two pioneers in x-ray analysis.

**CAUTION** While we are using the term reflection, remember that we are dealing with an interference effect. In fact, the reflections from various planes are closely analogous to interference effects in thin films (Section 37–5).

As Fig. 38–16b shows, in x-ray diffraction there is nearly complete cancellation in all but certain very specific directions in which constructive interference occurs and forms bright spots. Such a pattern is usually called an x-ray diffraction pattern, although interference pattern might be more appropriate.

We can determine the wavelength of x rays by examining the diffraction pattern for a crystal of known structure and known spacing between atoms, just as we determined wavelengths of visible light by measuring patterns from slits or gratings. (The spacing between atoms in simple crystals such as sodium chloride can be found from the density of the crystal and Avogadro's number.) Then, once we know the x-ray wavelength, we can use x-ray diffraction to explore the structure and determine the spacing between atoms in crystals with unknown structure.

X-ray diffraction is by far the most important experimental tool in the investigation of crystal structure of solids. X-ray diffraction also plays an important role in studies of the structures of liquids and of organic molecules. It has been one of the chief experimental techniques in working out the double-helix structure of DNA and subsequent advances in molecular genetics.
38-19 A cubic crystal and two different families of crystal planes. The spacing of the planes in (a) is \( d = ah/2 \); that of the planes in (b) is \( a/\sqrt{3} \). There are also three sets of planes parallel to the cube faces, with spacing \( a \).
To summarize, two conditions must be met for constructive interference to occur:

First condition: the angle of incidence must equal the angle of scattering from a single plane of atoms. Experimentally, this means that if the angle between the incident beam and the reflecting beam is $\theta$, the angle between the incident beam and the scattered beam must be $2\theta$. We call this the $\theta - 2\theta$ condition.

Second condition: $m\lambda = 2d \sin \theta$. This results from the requirement that the path difference of the scattering from two adjacent planes of atoms be an integral number of waves.

**EXAMPLE 38-7**

You direct a beam of x rays with wavelength 0.154 nm at certain planes of a silicon crystal. As you increase the angle of incidence from zero, you find the first strong interference maximum from these planes when the beam makes an angle of 34.5° with the planes. a) How far apart are the planes? b) Will you find other interference maxima from these planes at larger angles?

**SOLUTION** a) To find the plane spacing $d$, we solve the Bragg equation, Eq. (38–16), for $d$ and set $m = 1$:

$$d = \frac{m\lambda}{2 \sin \theta} = \frac{(1)(0.154 \text{ nm})}{2 \sin 34.5^\circ} = 0.136 \text{ nm.}$$

This is the distance between adjacent planes.

b) To calculate other angles, we solve Eq. (38–16) for $\sin \theta$:

$$\sin \theta = \frac{m\lambda}{2d} = m \frac{0.154 \text{ nm}}{2(0.136 \text{ nm})} = m(0.566).$$

Values of $m$ of 2 or greater give values of $\sin \theta$ greater than unity, which is impossible. Hence there are no other angles for interference maxima for this particular set of crystal planes.

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Apparatus: We will test Bragg’s Law by measuring the scattering of 2.85 cm (10.52 GHz) microwaves from a “crystal” made up of a cubic lattice of steel balls held in place by polystyrene foam, as shown in Fig. 2 below.

Reflections can occur from planes AA, BB, CC, etc. The distance between these planes is different, as is their orientation within the crystal.

The apparatus is shown in Fig. 3. It consists of a microwave transmitter, microwave receiver, a goniometer for measuring the angles between the incident beam, the crystal planes, and the scattered beam. Use the ammeter located on the receiver to record the current values that correspond to signal.
Procedure

I. Carefully measure the distance $d$ between planes in the crystal of steel balls.

II. Predict the angles for constructive interference, using your measured value for $d$ and the wavelength of the microwaves, 2.85 cm.
   
   A. Calculate the angle $\theta_{A1}$ of constructive first-order scattering from AA planes.
   
   B. Calculate the angle $\theta_{A2}$ of constructive second-order scattering from the AA planes.
   
   C. Calculate the angle $\theta_{B1}$ of constructive first-order scattering from planes BB.

III. Using a ruler and protractor, check your predictions graphically by making accurate 1-to-1 drawings of each of the three scattering processes you have predicted. You may find it useful to use the three sheets of “crystal paper” provided in this writeup, which are just 1-to-1 copies of the lattice. Mark the scattering planes and trace two rays in each diagram. Verify that the measured path difference on each diagram is close to 2.85 cm (one wave) or 5.70 cm (2 waves)

IV. Test experimentally whether there is a peak in the scattered microwave intensity where predicted.
   
   A. Measure the scattered intensity of the microwaves in a zone around the predicted angle, that is, for $\theta_{a1} - 10^\circ < \theta < \theta_{a1} + 10^\circ$. This means you should make 21 measurements in each zone (every 1°). Plot the scattered intensities to test whether the peak occurs where predicted.
   
   B. Repeat for $\theta_{A2}$, second-order scattering from the AA planes.
   
   C. Repeat for $\theta_{B1}$, first-order scattering from planes BB.
Crystal Sheet