Measuring the electric charge: Millikan’s experiment

Introduction.

This laboratory duplicates the beautiful experiments of Robert Millikan at the University of Chicago during 1909-1913. Everybody who has taken first-year college physics knows that $e=1.602 \times 10^{-19}$ coulombs. The currently accepted value is close to the value Millikan determined with apparatus quite similar to the one you’ll use.

The apparatus is shown in Fig. 1 below. Two parallel horizontal metal plates, A and B, are separated from one another by a distance $d$. Fine oil drops with radius around 1 μm, generated by an atomizer, enter the space between the plates through a small hole in the top plate. A beam of light is directed horizontally between the plates, and a telescope is set up to view the drops. When viewed through the telescope, the oil drops look like tiny stars. A scale (reticle) in the telescope permits precise measurements of the vertical position of the drops, so their velocities can be measured.

Some of the oil drops become charged during the atomization process, or they acquire a charge from the surrounding air, or they are ionized by an alpha-particle from a small radioactive source. Most of the drops are neutral, some are negatively charged, and occasionally one is positively charged.

Suppose a drop has a negative charge with absolute value $q$ and the plates are maintained at an electrical potential difference $V_{AB}$ such that there is a large downward electric field $E$ between the plates and the particle rises. The forces on the drop are its weight $mg$ and the upward force $qE$, and a drag force $F_d$ if it is moving (Fig. 2.)
By adjusting the field $E$, we can make $qE$ equal to $mg$. The oil drop is then in static equilibrium, its velocity is zero, $F_d = 0$, and

$$q = \frac{mg}{E} \quad (1)$$

The electric field $E$ is easily determined from $E = V_{AB}/d$, where $d$ is the distance between the plates. We can find $m$ if we know the drop volume $\frac{4}{3}\pi r^3$ and the mass density of the oil, $\sigma$. Then

$$q = \frac{4\pi\sigma r^3 gd}{3V_{AB}} \quad (2)$$

Everything on the right hand side of this equation is easy to measure except for the drop radius $r$, which is small. But here comes an example of Millikan’s genius. He determined the drop’s radius by measuring its terminal velocity (rate of fall $v_{\text{fall}}$) when $E = 0$. At the terminal velocity the weight $mg$ is just balanced by the viscous air-resistance drag force $F_d$. For slowly moving particles like tiny oil drops, the drag force is given by Stokes’ Law:

$$F_d = 6\pi \eta rv \quad (3)$$

where $\eta$ is the viscosity of the fluid in which the drop is moving and $v$ is the velocity of the particle relative to the fluid. Remember that the drag force acts in the opposite direction from the particle velocity. For a falling oil drop

$$\frac{4}{3}\pi r^3 \sigma g = 6\pi \eta rv_{\text{fall}} \quad (4)$$

and therefore
Thus \( q \) can be determined from Eqs. (2) and (5).

It turns out to be more accurate to make \( E \) large enough so that the particles go up, rather than just stop falling, when \( E \) is applied. At the terminal velocity, force \( qE \) balances the sum of \( F_{\text{drag,\,rise}} \) and \( mg \) (Fig. 2):

\[
qE = F_{\text{drag,\,rise}} + mg
\]

and \( q \) can again be determined, since \( F_{\text{drag,\,rise}} \) can be found from \( v_{\text{rise}} \) and Stokes’ Law.

The actual experiment was a little more complicated. Millikan corrected for the buoyant force of the air by replacing the density \( \sigma \) of the oil by \( \sigma - \rho \), where \( \rho \) is the density of air. This is a small correction so we will ignore it. He also used a more elaborate version of Stokes’ Law that takes into account the molecular nature of air, because the intermolecular distances in air are in the same range as the drop radius.

Millikan and his co-workers measured \( q \) for thousands of drops. Within the limits of their experimental error, every drop had a charge equal to some small integer multiple of a basic charge \( e \). That is, they found drops with \( q = e, \, q = 2e, \, q = 3e, \ldots \) and corresponding negative values, but never a drop with \( q = 0.65e \) or \( q = 2.37e \). A drop with \( q = -e \) has acquired one extra electron; a drop with \( q = -5e \) has 5 extra electrons, and so on.

**SUGGESTED PROCEDURE**

1. Set up a spreadsheet as follows:

<table>
<thead>
<tr>
<th>formula</th>
<th>Drop 1</th>
<th>Drop2</th>
<th>Drop3</th>
<th>Drop4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( dx_{\text{down}}(m) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( dt_{\text{down}}(s) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( v_{\text{fall}} )</td>
<td>( dx_{\text{down}}/dt_{\text{down}} )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( r )</td>
<td>( 3 \left( \sqrt{\frac{\eta v_{\text{fall}}}{2\sigma g}} \right) )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( mg )</td>
<td>( \frac{4}{3} \pi r^3 \sigma g )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( F_{\text{drag,,fall}} )</td>
<td>( \ldots )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( V_{AB} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( E(V/m) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( dx_{\text{up}} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( dt_{\text{up}} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
v_{rise} \\
F_{drag\_rise} \\
q(C) \\
n \\
q/n

Note constants you will need:
Viscosity of air $\eta$ in Pa·s = Ns/m². The value depends on temperature. See Appendix B in Pasco writeup. You will need to measure $T$.

Density of oil $\sigma = 886$ kg/m³.
Plate separation $d = 7.62 \times 10^{-3}$ m
Correction factor for Stokes’ Law: $b=8.2 \times 10^{-3}$ Pa·s

2. Set up apparatus:
level it 
turn on light source and check telescope focus on test wire
check telescope focus on internal scale(reticle). **Lines are separated by 0.10 mm. So a big division is 0.5 mm.**
Set up 400 Volts DC powersupply to generate $E$. Be careful! A 400 V power supply can give you a nasty shock.

3. Measure $q$ for a particular drop.
Using the atomizer, spray some oil drops into the apparatus. In spraying oil into the top of the chamber, first squeeze the bulb several times beforehand until you can see a small amount of oil spray onto a paper towel. Then spray once into the top of the chamber, being careful not to inject too much oil. Before spraying oil in move the Ionization Source lever from the Off to the Spray Droplet Position. After squeezing the bulb, do not let go of the squeezed position of the bulb until after you have moved the lever back to the OFF position. Then let go of the bulb after you remove the nozzle from the chamber entrance. The best condition is when you only have a few droplets in the field of view. This can take some practice.
Select a drop
Small drops are better, although they are harder to see. They are more likely to have 1,2,or 3 charges rather than 15 or 20 charges.
Measure $dt_{fall}$ for drop to fall distance $dx_{fall} = 5$ or 10 spaces. 
apply $V_{AB}$ to plates so droplet goes up instead of down. Measure $dt_{rise}$ to rise a distance $dx_{rise} = 5$ to 10 spaces. Again, drops that move slowly for $V_{AB} = 300-400$ V are better, because $q$ is less.
If you can make several measurements on the same drop, your accuracy will increase. Also, you can sometimes change $q$ on the drop by exposing it to the source of alpha particles.

4. Process the data in the spreadsheet
from the measured falling velocity, with $E=0$, compute the drop radius
compute the downward force on the falling drop
compute the drag force on the falling drop.

From the measured rising velocity, with E on,
compute the drag force on the rising drop, \( F_{\text{drag, rise}} \)
compute the charge from Eq. 6 in the form

\[
q = \frac{F_{\text{drag, rise}} + mg}{E}
\]

Repeat for the same drop over several up-down cycles, if possible. It is especially interesting to observe the same drop when it changes charge, for example from 3e to 2e, by turning on the radioactive source.

5. Apply the correction factor that compensates for the limitation of Stokes’ Law for small particles:

\[
q_{\text{corrected}} = \left[ \frac{1}{1 + \frac{b}{pr}} \right]^{\frac{3}{2}} q
\]

where \( p \) is the atmospheric pressure in Pa. Note that the correction factor is close to 1.

6. Finding the charge on the electron from your data. You should analyze your data by first finding the charge differences of all your measurements. Form a long column of all the differences. For example, suppose you measured \( n \) droplets. Then form \( q_1-q_2, q_1-q_3, q_1-q_4, \ldots q_1-q_n, q_2-q_3, q_2-q_4, q_2-q_5, \ldots q_2-q_n, q_3-q_4, q_3-q_5, \ldots \) etc. Then rank order the differences in descending order. In your sorted column, you should notice ‘clustering’ of values, e.g. you may have a list like

<table>
<thead>
<tr>
<th>Charge Differences (C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2 x 10(^{-19})</td>
</tr>
<tr>
<td>1.33 x 10(^{-19})</td>
</tr>
<tr>
<td>1.53 x 10(^{-19})</td>
</tr>
<tr>
<td>1.73 x 10(^{-19})</td>
</tr>
<tr>
<td>1.91 x 10(^{-19})</td>
</tr>
<tr>
<td>2.2 x 10(^{-19})</td>
</tr>
<tr>
<td>2.9 x 10(^{-19})</td>
</tr>
<tr>
<td>3.01 x 10(^{-19})</td>
</tr>
<tr>
<td>3.2 x 10(^{-19})</td>
</tr>
<tr>
<td>3.5 x 10(^{-19})</td>
</tr>
<tr>
<td>3.7 x 10(^{-19})</td>
</tr>
<tr>
<td>4.5 x 10(^{-19})</td>
</tr>
</tbody>
</table>
Notice there is a larger gap between the values 2.2x10^{-19} C and 2.9x10^{-19} C than between the other points that form the first five gaps. So we will cluster the first six values together by averaging that group and we will assign that group to the integer 1 (1 unit of charge). The next significant gap occurs between 3.7x10^{-19} C and 4.5x10^{-19} C, so we will average the values between 2.9x10^{-19} C and 3.7 \times 10^{-19} C into the second cluster and assign the integer 2 (2 units of charge). We continue doing this for the whole column. Note that some of the charge quanta may be missing. For example, there may be no differences of charges that corresponds to seven charges or 9 charges. After clustering, you need to plot Delta q versus n (the charge) number and do a linear least-squares fit to the line in Excel. What is the meaning of the slope of this line?