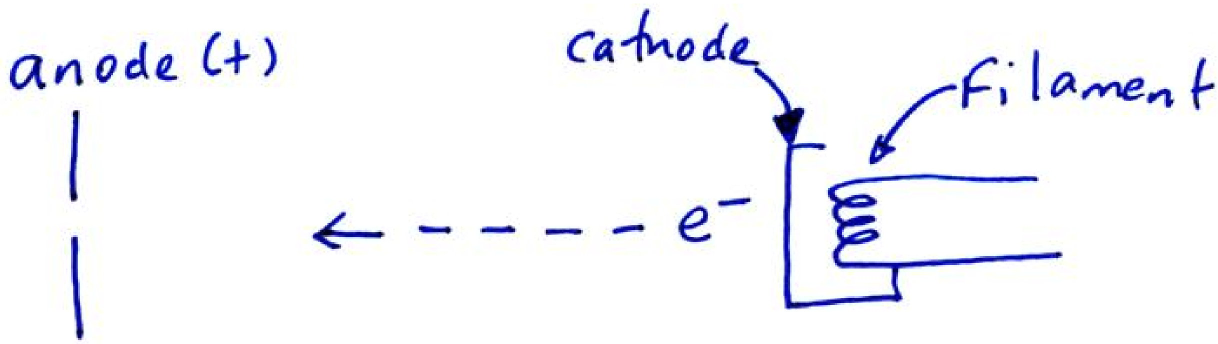
Physics 165 Laboratory

J. J. Thomson's Measurement of e/m for the electron

In 1897, the cathode ray tube or CRT, which is now a part of most TV sets, was the last word in advanced laboratory instrumentation and TV was still 40 to 50 years in the future. At Cambridge University in England, JJ Thomson studied the rays emanating from the cathode of the CRT.

Thomson measured the ratio of charge to mass (e/m) of these "corpuscles" of which the rays were composed. We now call them electrons. We'll do his experiment, with a somewhat modified apparatus. Main parts of the apparatus:

Electron gun: consists of (1) filament (2) cathode, and (3) anode.



Filament. The role of the filament is to heat the cathode. The filament is a thin tungsten wire with a low resistance. When 6.3 Volts is applied to it, a current I flows and the filament heats up; recall $P=IV=I^2R$. DO NOT EXCEED 6.3 Volts. Additional voltage means additional current means destruction of the filament.

Cathode. The hot filament heats a nearby cathode, which is coated with a material with an especially low work function. Because of the low work function, the cathode readily releases electrons into the vacuum surrounding it.

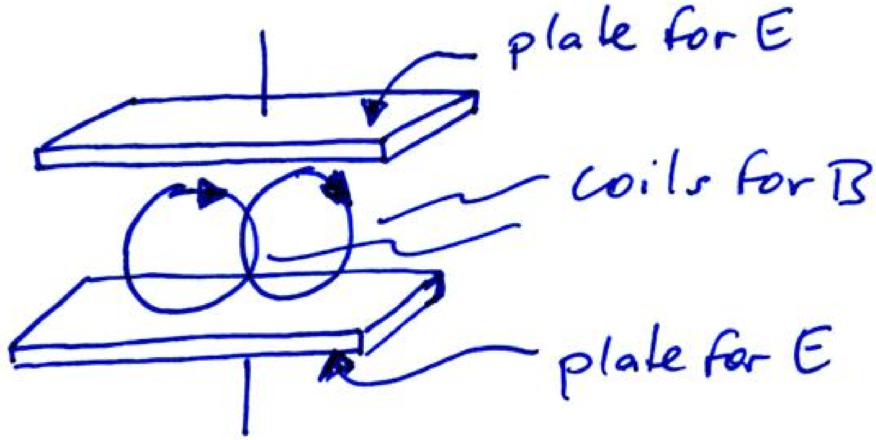
Anode. The anode is a metal structure with a slit in it. It is made positive with respect to the cathode by an *accelerating* voltage ΔV_a = 1000 to 3000 V, so electrons released by the cathode are rapidly accelerated from the cathode to the anode. When they reach the anode their energy is $e\Delta V$ = 1000 to 3000 eV. Many of the electrons crash into the anode, but a small number go through the slit and form a ribbon of corpuscles. If Thomson had known e and m, he could have calculated the velocity v of his corpuscles. However, he didn't know e and m.

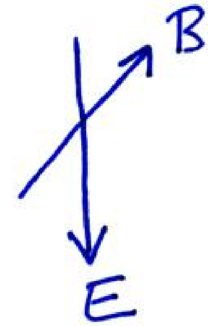
Deflection Zone.

The deflection zone is a space where you can apply a transverse electric or magnetic field to the moving corpuscles.

The transverse electric field E is provided by two metal plates separated by about 10 cm, on either side of the ribbon of corpuscles. You can apply a second potential difference V_d to deflect the beam.

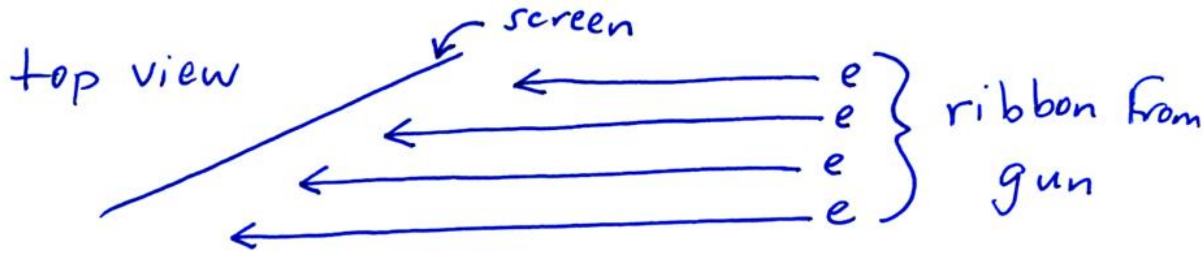
The transverse magnetic field B is provided by two coils, called a Helmholtz pair.



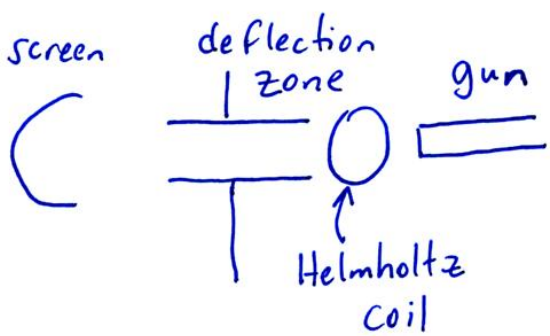


Observation Zone

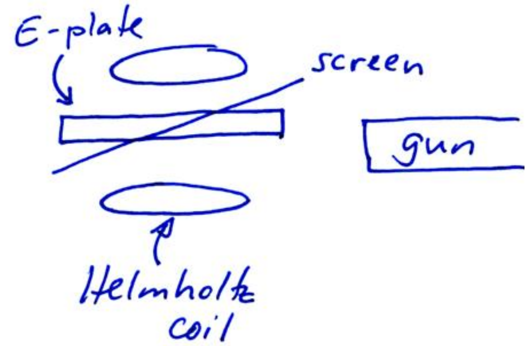
The effect of E and B on the beam is made visible when the corpuscles hit a fluorescent screen and release photons. In Thomson's tube, the screen was on the end. In our apparatus, the screen is cleverly set up at an angle within the observation zone. Different parts of the ribbon of corpuscles hit the screen at various places, giving an illusion that you are seeing the path of the corpuscle.



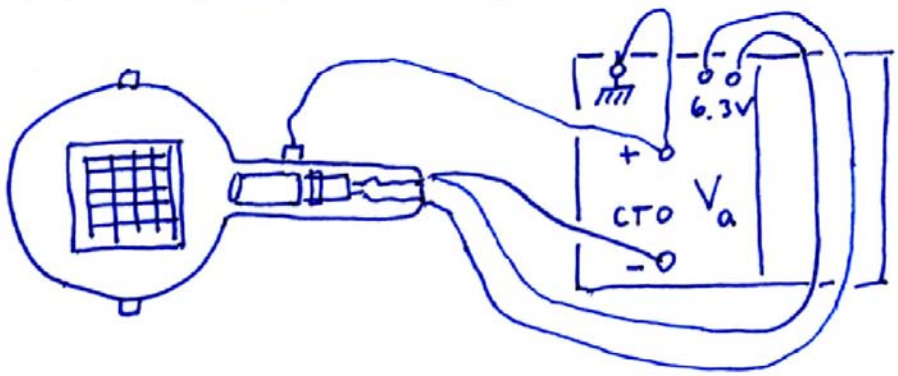
Thomson's scheme



Our scheme



Experiment 0. Set up the electron gun.

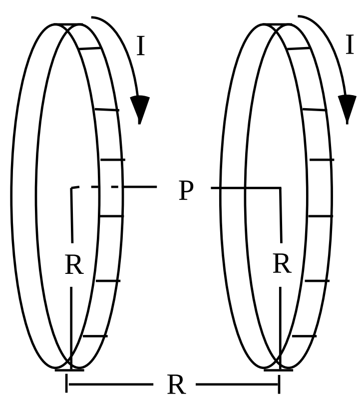


Apply 6.3 Volts DC or AC (rms)to the filament (two banana plug sockets at the plastic cap at the end of the neck of the tube). Apply a high-voltage DC potential difference V_a (0-4000 V) between the filament and the anode by connecting the positive terminal of the HV supply to the anode terminal located on the side of the neck of the tube. This terminal of the high-voltage supply should also be connected *to ground, so that the electrons are at zero potential when they leave the gun*. Connect the negative side of V_a to one of the filament connections, as in the drawing above. The electron beam should be visible on the screen and traveling in a straight line along the screen. Adjust the orientation of the tube if necessary, since the earth's B-field may deflect the beam slightly.

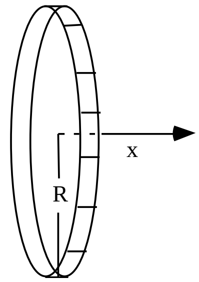
Experiment 1. Movement of Electrons in a uniform B-field.

Helmholtz Coil:

The setup uses a Helmholtz coil pair to get a relatively uniform B-field inside the tube. A Helmholtz coil consists of two circular coaxial coils each of N turns and radius R, separated by a distance R, and carrying equal current I in a direction which makes the fields of the two coils additive midway between the coils, as shown below:



For each coil, the field at distance x from the center of the coil, along its axis, is given by



$$B = \frac{\mu_0 NIR^2}{2(x^2 + R^2)^{3/2}} \qquad (1)$$

Helmholtz showed that, at the midpoint P between two coils, both dB/dx and d^2B/dx^2 are zero if the distance between the coils is equal to the radius R of the coils. Even d^3B/dx^3 is zero (by symmetry). Thus the field changes very little along the centerline near P. Under these conditions, the field at the midpoint P is

$$B = \frac{8\mu_0 NI}{5^{(3/2)} R} \qquad (2)$$

For the particular coils, N = 320 turns (each). Since I and R can be measured, one can calculate B. (You could check your results with a Hall probe if you wish.).

Suppose the electron has been accelerated in an electron gun through a potential difference V_a, acquiring a velocity v_x.

$$v_x = (2eV_a/m)^{½}$$

(3)

If this electron then moves into a region with uniform **B**, it will be subjected to a force $\mathbf{F}_B$ given by:

$$\mathbf{F}_B = q\mathbf{v} \times \mathbf{B} \qquad (4)$$

If B points in the +z direction, F_B will point in the y-direction and the electron will be accelerated in that direction:

$$a_y = F_B/m \qquad (5)$$

If the electron spends time t in the B field, its deflection in the y direction will be

$$y = a_y t^2/2 \qquad (6)$$

We can evaluate t from v_x and the length L of the path along x:

$$t = L/v_x. \qquad (7)$$

Eliminating t, we find that

$$y = qBL^2/2mv_x \qquad (8)$$

Procedure.

1. Set up the Helmholtz coils on a separate stand so you can check their operation. Apply 3.5 Vdc or less to one coil while measuring the current with an ammeter. Use the magnaprobe to check the direction of the resultant B field (on the magnaprobe magnet, red = North, blue = South).

2. Place the two coils a distance R apart and hook up the two coils in series (why?). Be sure to hook them up in such a way that the fields from the two coils add in the central region. Use 7.2 V dc max.

3. Check the field out with the magnaprobe.

4. Set the Helmholtz coils around the Thomson tube.

5. With acceleration voltage V_a = 3000 V, determine the Helmholtz coil current necessary to deflect the beam downward 2 cm (Hint: What do you need to record in order to be able to evaluate B?) Then reverse the current through the Helmholtz coils and determine the B field necessary to deflect the beam upward by 2 cm. Compare the observed deflection to the predicted deflection (Eq. 8)

6. Repeat step 5 for V_a=2000 V.
In your writeup, compare the measured deflection to that predicted by theory. If they don't agree, try to figure out why.

Experiment 2. MOVEMENT OF Charged Corpuscles IN A UNIFORM E-field.

If the corpuscle with charge e and velocity v_x acquired in a gun moves through a region with uniform electric field E, the corpuscle will experience a force

$$F_E = qE \tag{9}$$

If q=-e, then F = qE = -eE

If E points in the y direction, F_E will point in the -y direction. The value of E can be estimated from the potential difference V_d between the deflecting plates, and the separation d between the plates:

$$E = V_d/d \tag{10}$$

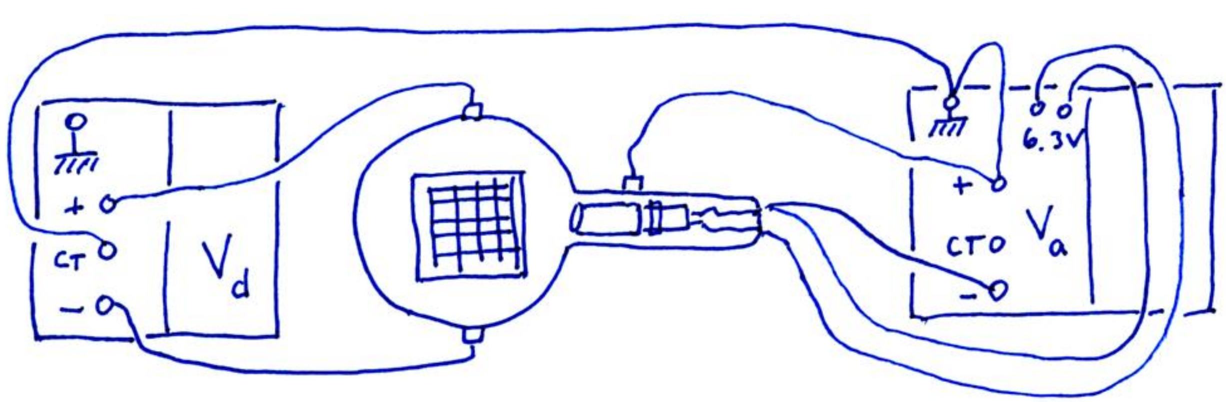
Following the same arguments as for F_B, the time t over which the force acts is again L/v_x. Since $a_y = -eE/m$, the net deflection y' will be

$$y' = (1/2)a_y t^2 = -eEL^2/2mv_x^2 \tag{11}$$

We can check this result by measuring the deflection y' arising from a field E perpendicular to v_x.

<u>Suggested procedure.</u>

1. Set up the e/m tube to obtain a visible beam without deflection fields.
2. Hook up a high-voltage supply for deflection with an E field. The center-tap of this high-voltage should be at ground, so that 1 plate is at $+V_d/2$ while the other is at $-V_d/2$. (Remember that the anode is at ground potential also).



3. Measure the value of V_d required to deflect the beam 2 cm when V_a=3000 V. Compare to predictions (Eq. 11)

4. Repeat for V_a = 2000 V.

In your writeup, compare the measured deflection to what you would expect from theory.

Experiment 3. The Thomson experiment.

Thomson did not know the value of e. Since he did not know e, he could not determine v_x, and thus could not use the deflections in B and E separately. However, he could eliminate v_x as a variable, thereby obtaining e/m for the electron, by measuring the value of B which just cancelled the deflection by E. Let's repeat his procedure:

a. Set V_a to 3000 V. This fixes v_x.
b. Set V_d to give a deflection of 2 cm, with B = 0..
c. Then adjust B so that the beam goes along the x axis without deflection, that is, from(2,0) to (10,0)(no deflection). This is enough for one measurement of e/m.
d. Repeat for V_a = 2000 and 4000V.

Process the data to obtain e/m. Derive a formula expressing e/m as a function of E,B, and V_a. Then determine e/m for your several data sets. Obtain mean, standard deviation, and standard deviation of the mean for your data.

Some items for discussion in your report:

1. We have discussed the uniformity of B along the axis of the Helmholtz coils, but the electrons travel perpendicular to this axis. What is the direction of the resultant systematic error?

2. In your evaluation of E, you have probably used the formula appropriate for infinite plates. If the plates are finite, what is the direction of the systematic error?

3. We have determined e/m in a way which is slightly differently from that of J.J.himself. Comment on the difference.

Appendix: Maple Program to calculate B

Helmholtz coil fields along coil axis (on centerline).
Maple file helmholtz
Plots the field of two coils separated by a distance equal to their radius.
The field at the center is remarkably uniform; the first, second,third derivatives of B are zero at the midpoint between the two coils.

```
>
>
>
```

Let B1 be the field of the first coil with N turns, current i, radius a;
Let x be the coordinat along the axis of the coil; Put coordinate x=0 at the center of this coil.

```
> B1:=N*i*a^2/(2*(x^2 + a^2)^(3/2));
```

$$B1 := \frac{1}{2}\frac{N\,i\!\sim a^2}{(x\!\sim^2 + a^2)^{(3/2)}}$$

Let B11 be B1 for a=1,N=1,i=1 for simplicity

```
>
B11:=subs(N=1,i=1,a=1,B1);
```

$$B11 := \frac{1}{2}\frac{1}{(x\!\sim^2 + 1)^{(3/2)}}$$

```
> p1:=plot(B11,x=0..1):
```

Let B2 be the field of the second coil, located at x=a:

```
> B2:=N*i*a^2/(2*((x-a)^2 + a^2)^(3/2));
```

$$B2 := \frac{1}{2}\frac{N\,i\!\sim a^2}{((x\!\sim - a)^2 + a^2)^{(3/2)}}$$

```
> B22:=subs(N=1,a=1,i=1,B2);
```

$$B22 := \frac{1}{2}\frac{1}{((x\!\sim - 1)^2 + 1)^{(3/2)}}$$

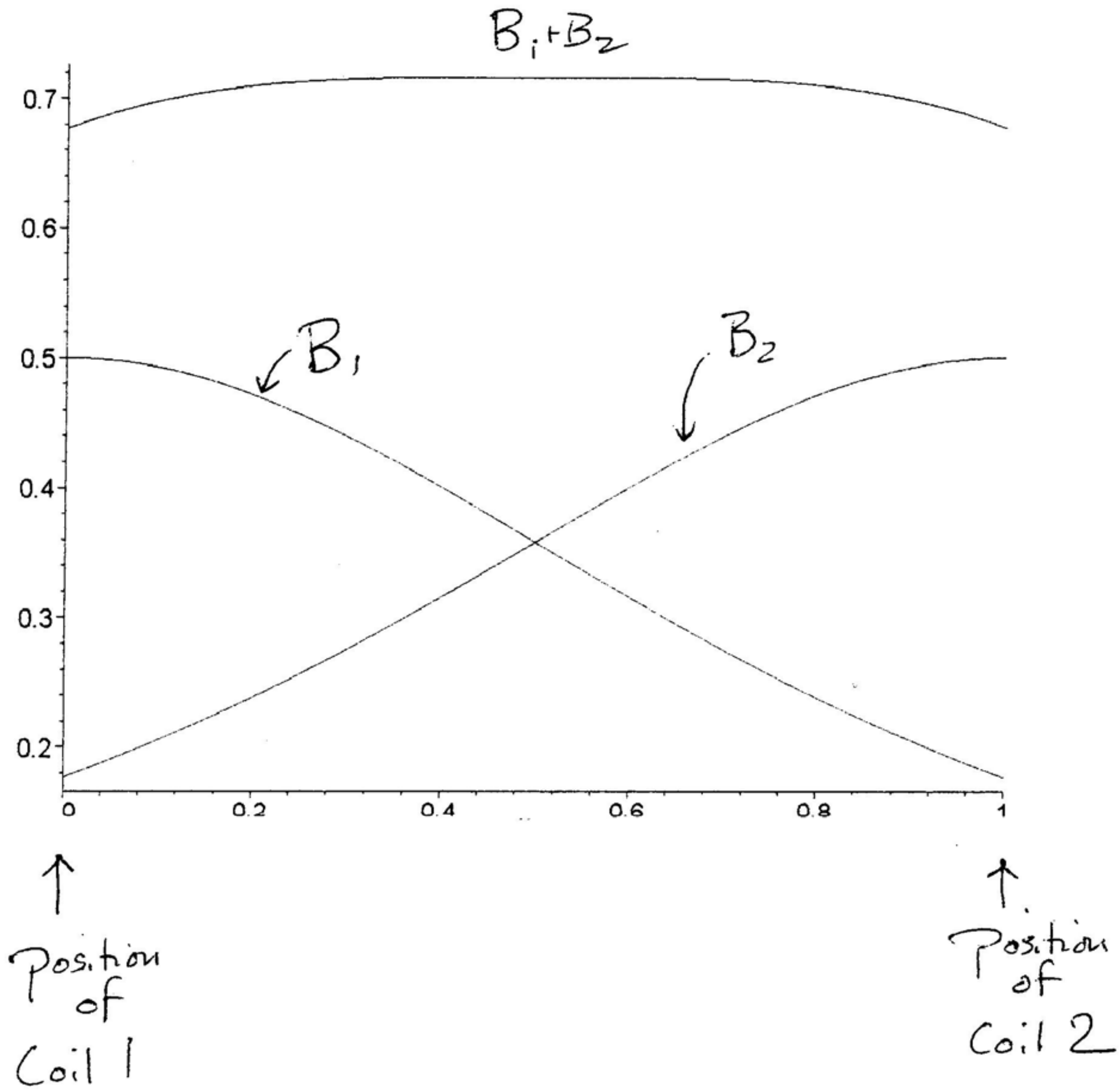
```
> p2:=plot(B22,x=0..1):
>
```

Let B be the sum of the two fields along the centerline.

```
> B:=B11+B22;
```

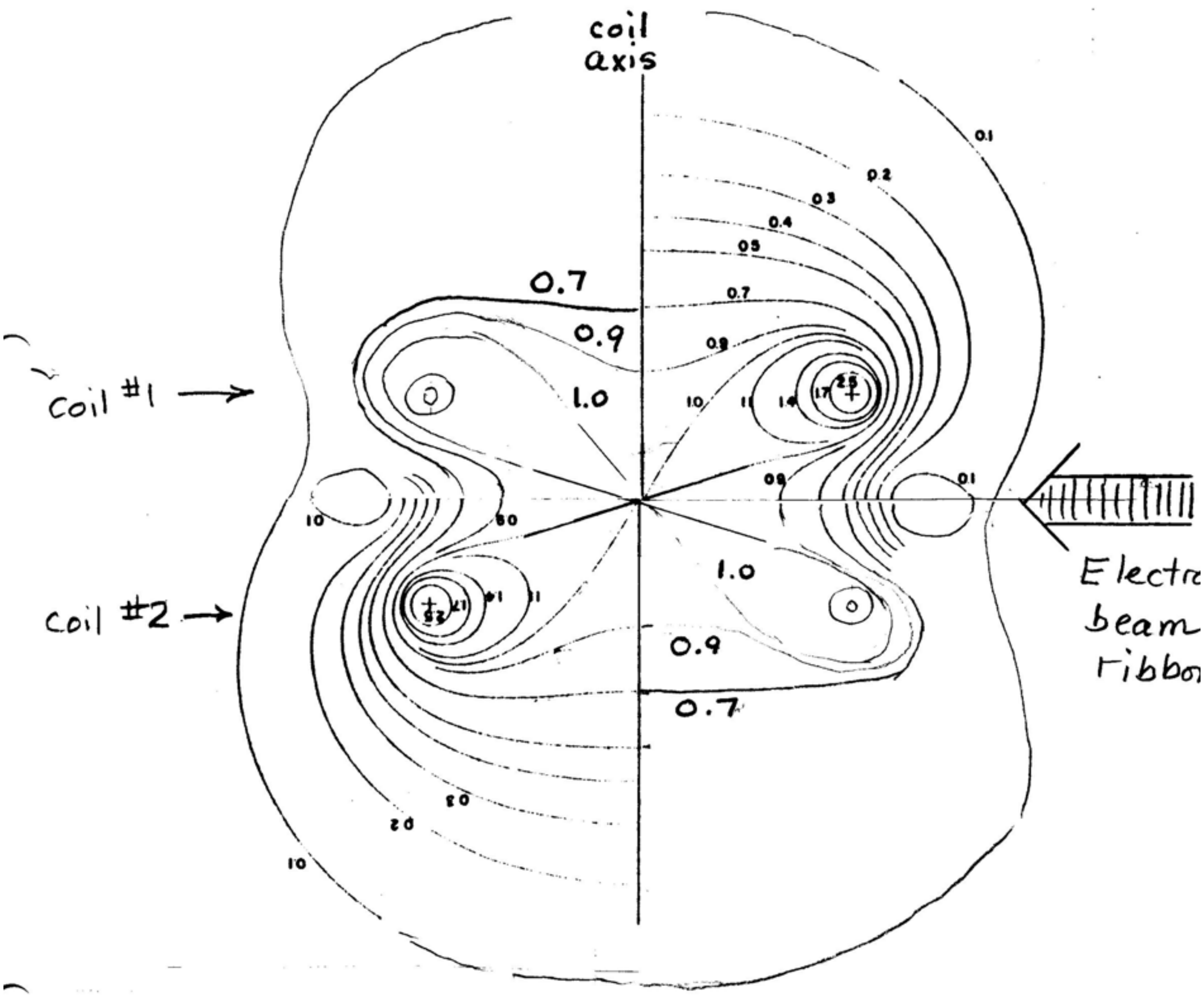
$$B := \frac{1}{2}\frac{1}{(x\!\sim^2 + 1)^{(3/2)}} + \frac{1}{2}\frac{1}{((x\!\sim - 1)^2 + 1)^{(3/2)}}$$

```
> p3:=plot(B,x=-0..1):
> display({p1,p2,p3});
```



B_1+B_2
0.7
0.6
0.5
0.4
0.3
0.2
B_1
B_2
0
0.2
0.4
0.6
0.8
1
Position of Coil 1
Position of Coil 2

Non-Uniformity of B in Helmholtz Coils
Normalized to B at center point midway between coils. Coil separation = Coil radius.



Approximately
½ scale

Ref: Cacak and Craig,
Rev. Sci. Instr. 40, 1468-1470 (1981)