Determination of the wavelength $\lambda_e$ of a moving electron by diffraction from an atomic lattice

The purpose of this experiment is to test deBroglie’s hypothesis that particles of momentum $p$ will have a wavelength $\lambda$ given by

$$\lambda = \frac{h}{p} \quad (1)$$

where $h$ is Planck’s constant. The electron diffraction tube used to test Eq. 1 in this experiment is much like a TV or oscilloscope tube: it has a filament, a cathode, and an anode, which form an “electron gun”, and a fluorescent screen which emits light where the electrons hit. However, instead of deflection plates, the diffraction tube has a thin film of graphite, a crystalline form of carbon, near the anode. The graphite acts like a diffraction grating for the electrons; constructive interference occurs for a cone angle $\theta$ given by Bragg’s Law:

$$n\lambda = d \sin \theta \quad (2)$$

where $n = 1, 2, 3\ldots$ for first, second, and third order diffraction. Here $d$ is the spacing between atomic planes in the graphite. Because the graphite film has many small crystallites oriented in different directions, the electrons are scattered in a cone made visible as a circle where the conical beam strikes the screen. (Eq. 2 is not the standard form of Bragg’s law; a prefactor of 2 has been dropped. It is “recovered” by redefining the scattering angle $2\theta$ as $\theta$. This is satisfactory for small angles.)

The wavelength of the electrons is controlled by the potential difference $V_A$ between the cathode and anode, which gives them kinetic energy

$$K = eV_A \quad (3)$$

Provided $K$ is not too large ($v << c$), classical expressions for $K$ and $p$ apply,

$$K = \frac{1}{2}mv^2 \quad (4)$$

$$p = mv \quad (5)$$

From Eq 1 and 3-5, the expected value of $\lambda$ can be computed. In this experiment, you will compare measured values of $\lambda$ to the expected values.
Fig. 1 is a diagram of the diffraction tube and the external electrical connections it requires:

1. Filament voltage $V_f = 6.3$ V AC, applied to the two large, 4 mm diameter terminals at the base of the tube (usual banana sockets). This heats the cathode.
2. Anode voltage $V_A$, 2000 to 5000 V dc to accelerate the electrons. This controls the momentum, and thus the wavelength, of the electrons. The negative side of the HV supply is connected to the small (2 mm) socket at the base of the tube as well as to ground. If the ground connection is left out, the HV supply is “floating” and the experiment gives unpredictable results because the cathode-to-anode potential is undefined. The positive side of the HV is connected to a microammeter, which in turn connects to the anode.

NOTE THAT ANODE CURRENT SHOULD NOT EXCEED 0.17 mA (170 microamperes). Larger anode currents will destroy the thin graphite diffraction film.

![Diagram of the diffraction tube and the external electrical connections.](image)

Fig. 1 Diagram of the diffraction tube and the external electrical connections.

Procedure:

1. The apparatus consists of the Teltron 555 tube (Fig. 1), a TelAtomic 813 KV power supply, a microammeter, and calipers. The power supply has outputs for both 6.3 V AC and 5000 V DC. Set up the apparatus as shown in Fig. 1. Until one of the three HV terminals is connected to a known potential, such as ground, the high-voltage output is “floating”. Pay attention to the grounding of the high-voltage part of the circuit in Fig. 1. It ensures that the cathode and filament are at approximately ground potential and that the anode is at +2000 to +5000 V DC with respect to ground. Be careful to keep the anode current below 170 microamperes. NOTE: it takes about 5-10 MINUTES for the beam to TURN ON.
2. Determine $\lambda$ experimentally:
   A. Measure the diameter of the inner and outer luminescent rings for accelerating potential $V_A$ between 2500 and approximately 5000 V DC at intervals of 500 V.
   B. Use Bragg’s Law and the data from Step 2A to compute $\lambda_{\text{obs}}$, the observed value of $\lambda$. Take the distance $L$ between grating and screen to be $0.140 \pm 0.003 \ m$ and the grating spacings $d_{11} = 0.123 \ nm$ and $d_{10} = 0.213 \ nm$. See instructions below.

3. Compute the wavelength $\lambda_{\text{dB}}$ predicted by deBroglie’s relationship for each value of accelerating potential.

4. Compare the observed and predicted wavelengths to one another.

5. Justify the validity of the classical assumption used in part 2 by comparing relativistic and non-relativistic values of $\lambda_{\text{dB}}$ for the worst case (largest $V_A$). Hint: use the relativistic form for $p$.

6. For your measurements for the largest ring reported in Part 1, compute the correction required to properly consider the curvature of the screen. (See last page of writeup).

\textit{How to calculate the measured wavelength from your experimental data.}

The diffraction condition is

$$n\lambda = d \sin \theta$$  \hspace{1cm} (23.4)

We can find $\tan \theta$ from the measured $D$ and known $L$ if we neglect the curvature of the screen (Fig. 2):

$$\tan \theta = \frac{D}{2L}$$  \hspace{1cm} (23.5)

It is easy to solve Eq. 23.5 for $\theta$ and $\sin \theta$. But see “Correcting $\theta$ for the Curvature of the Screen” at the end of these notes.
Fig. 2. Envelope of diffraction of electrons by carbon scatterers.

Knowing $\sin \theta$ and the lattice spacing $d$, one can find $\lambda$. This is your measured or observed value of $\lambda$. It can be compared to the wavelength predicted by deBroglie’s hypothesis as given by Eq. 23.3.

We will take the lattice spacing $d$ as a known quantity. The target is a thin graphite film, in which carbon atoms are arranged in a “chicken wire” pattern, as shown below:

Two of the spacings, labelled $d_{11}$ and $d_{10}$, give the observed rings. Their values are $d_{11} = 0.123 \text{ nm}$ and $d_{10} = 0.213 \text{ nm}$.

Correcting eqn 23.5 for the curvature of the screen.

Consider the drawing below:

The lengths $L$, $R$, and $r$ are known. The goal is to find $\theta$ more precisely than given by 23.5 above. Since $L$ and $R$ are known, $L-R$ is known. If distance $b$ were known, then one could
find $\theta$ more precisely from $\tan \theta = r/(L-R+b)$, instead of $\tan \theta = r/L$. It’s not difficult to find $b$, given $R$ and $r$. 