Firm Taxation and the Endogenous Distribution of Markups*

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Abstract

Melitz and Ottaviano (2008) predicts a monotonic relation between productivity and markups. When including revenue taxes, however, this relation is non-monotonic and depends on taxes. Even without taxes, productivity and markups can be non-monotonic depending on how non-homotheticity is modeled.

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1 Introduction

Firm microstructure matters for macroeconomic models. Hopenhayn (2011) argues this methodological point in a recent review about the impact of industrial organization on macroeconomics by examining the link between firm microstructure and economic aggregates, especially aggregate productivity. In this note, we contribute to the understanding of this class of models by exploring the effects of firm taxation. The effects of firm taxation on output and productivity have already been analyzed in papers such as Guner, Ventura, and Xu (2008) and Restuccia and Rogerson (2008). However, by abstracting away from endogenous distributions of markups, these papers remain silent about the impact of taxation on markups and, thus, miss an important mechanism through which taxation affects firm-specific and aggregate outcomes. This note fills the void by considering the role played by firm taxation on the endogenous distribution of markups.

In particular, we consider the closed economy model in Melitz and Ottaviano (2008), which has heterogeneous firms and non-homothetic (quadratic) preferences. The important difference between this note and Melitz and Ottaviano (2008) is that we include firm taxation and study its impact on markups. We only consider taxes on total revenue, but our model is easily adapted to include other forms of taxation. The tractability of our model allows us to completely solve the equilibrium outcomes as closed-form solutions under reasonable assumptions.

The key feature of our model is endogenous variable markups resulting from the interaction between the heterogeneous firm structure and non-homothetic preferences. Melitz and Ottaviano (2008) shows more productive firms charge higher markups. This result, however, is not robust. First, including taxes on total revenue in the model generates a non-monotonic relation between productivity and markups, and the relation depends on taxes. We also show increasing the tax rate increases the markup, but more productive firms increase their markups less than low productivity firms. Second, the type of non-homotheticity also matters for the markup result in Melitz and Ottaviano (2008). We show a model with different non-homothetic preferences generates a non-monotonic relation between productivity and markups, even in the case when there are no taxes.

Researchers interested in estimating markups will find the tractability of our model and
its sharp testable implications useful. For example, our model implies variation in firm taxation should explain variation in the distribution of markups across countries. De Loecker and Warzynski (2012) provides a simple method for estimating markups from plant-level data which could be used to compare firm taxation and markups across countries. The quantitative findings would improve our understanding of the role played by firm taxation on markups and provide motivation for or against the inclusion of endogenous distributions of markups in macroeconomic models.\footnote{Endogenous distributions of markups have already been shown to be important for measuring the gains from international trade. See, for example, Edmond, Midrigan, and Xu (2012).}

## 2 Model

Our description of the model follows that of Melitz and Ottaviano (2008). The economy contains \( L \) consumers. Each consumer is endowed with one unit of labor, which is inelastically supplied. Consumers demand differentiated products. There is a continuum of these differentiated products indexed by \( z \in \Omega \), where \( \Omega \) represents the set of potential differentiated products. \( \Omega^* \subset \Omega \) denotes the subset of products consumed, which is endogenously determined in equilibrium. Consumers share the same non-homothetic (quadratic) utility:

\[
U = \int \left[ \alpha c(z) - \frac{1}{2} \gamma (c(z))^2 \right]dz ,
\]

where \( c(z) \) represents the quantity of product \( z \) demanded and \( \alpha \) and \( \gamma \) are both positive. \( \alpha \) shifts the demand, and \( \gamma \) measures the degree of differentiation across products. The marginal utility, \( \alpha - \gamma c(z) \), is bounded for any product, so a consumer may not have positive demand for all products. A representative consumer has the following budget constraint:

\[
\int_{z \in \Omega} p(z)c(z)dz = w ,
\]

where \( p(z) \) is the price of product \( z \) and \( w \) denotes the nominal wage.

Taking the first order condition of the utility maximization problem with respect to \( c(z) \) and solving for \( c(z) \) yields the demand function for product \( z \) when \( c(z) > 0 \):
\[ c(z) = \frac{\alpha - \lambda p(z)}{\gamma}, \]  

(3)

where \( \lambda \) is the Lagrange multiplier on the budget constraint. \( p^{\text{max}} \) represents the upper threshold price, below which, \( p(z) < p^{\text{max}} \), a consumer is still willing to pay for product \( z \). There is no demand for product \( z \) if \( p(z) \geq p^{\text{max}} \). Setting \( c(z) = 0 \) in the demand function (3), we define the upper threshold as

\[ p^{\text{max}} = \frac{\alpha}{\lambda}. \]  

(4)

The total market demand for product \( z \) is given by the demand for product \( z \) multiplied by the total number of consumers \( L \).

Each differentiated product \( z \) is produced by a single firm. Every firm has access to the same production technology, a constant returns to scale production function with productivity \( \phi \), which drives the marginal cost. The only factor of production is labor. Firms differ ex-ante only by their productivity \( \phi \), which they pay a sunk entry cost \( s \) (in terms of labor) in order to realize. New entrants draw productivities from the same known Pareto distribution \( F(\phi) \) with support \([b, \infty)\) and c.d.f.

\[ F(\phi) = 1 - \frac{b^k}{\phi^k}, \]  

(5)

where \( k \geq 1 \) is the shape parameter. We assume \( b \geq 1 \). After drawing a productivity, those firms able to make positive profits stay in the market and begin producing; those firms unable to make positive profits shut down without producing and exit the market entirely. The remaining firms in the market maximize profits subject to demand.

We introduce taxes on firms to examine their impact on the endogenous distribution of markups. For ease of exposition, we only consider taxes on total revenue, which can be thought of as sales taxes, for example. However, our model is easily adaptable to other forms of firm taxation, such as taxes on variable labor costs or the sunk costs of entry. We assume government expenditures are paid from the revenues raised from the taxes, but we do not model government explicitly, abstracting away from how government expenditures might enter the representative consumer’s utility, for instance. This allows us to isolate the effects of the tax rate in the model.
in a sharper way.

A firm’s profit function is

$$(1 - \tau)p(\phi)Lc(\phi) - \frac{w}{\phi}Lc(\phi),$$

where $0 < \tau < 1$ denotes the tax rate on total firm revenue. We drop the $z$ notation and denote both a firm and a product by the productivity level $\phi$. Let $\phi^*$ represent the marginal firm earning zero profits, because the marginal revenue $(1 - \tau)p(\phi^*)$ equals marginal cost $\frac{w}{\phi}$. In this case, the demand of the marginal firm $c(\phi^*)$ declines to zero. Note, $p(\phi^*) = p^{\text{max}}$, because the marginal firm sets the highest possible price. Using these facts and equation (4), we rewrite demand as

$$c(\phi) = \frac{\alpha}{\gamma} \left(1 - \frac{p(\phi)}{p^{\text{max}}}\right).$$

(7)

Taking the first order condition of the profit maximization problem of a firm with productivity $\phi$, i.e. (6) subject to (7), with respect to $p(\phi)$ and solving for $p(\phi)$ yields:

$$p(\phi) = \frac{1}{2} \left[ p^{\text{max}} + \frac{w}{\phi} \frac{1}{1 - \tau} \right].$$

(8)

Since demand goes to zero for products priced higher than $p^{\text{max}}$, firms with $\phi \geq \phi^*$ will stay in the market, while firms with $\phi < \phi^*$ will exit. $\phi^*$ is, thus, the well-known threshold productivity in heterogeneous firm models.

Combining the zero profit condition for the marginal firm and $p(\phi^*) = p^{\text{max}}$ gives $p^{\text{max}}$ in terms of the threshold productivity $\phi^*$:

$$p^{\text{max}} = \frac{w}{\phi^*(1 - \tau)}.$$
We define the markup as firm $\phi$’s price minus marginal cost, $M(\phi) = p(\phi) - \frac{w}{\phi}$. The markup can be rewritten in terms of the threshold productivity level $\phi^*$:

$$M(\phi) = \frac{w}{2} \frac{1}{1 - \tau} \left[ \frac{1}{\phi^*} + \frac{2\tau - 1}{\phi} \right].$$  \hspace{1cm} (11)$$

The variable markups generated in our model capture a major difference between the findings in our paper and Melitz and Ottaviano (2008). Melitz and Ottaviano (2008) finds a monotonic relation between productivity and markups: more productive firms charge higher markups. This result, however, is not robust. The actual level of the tax rate on total revenue $\tau$ now gives a non-monotonic relation between productivity and markups. We establish this result in Proposition 1 below.

In order to fully characterize the equilibrium, we consider the free entry condition, which implies a firm’s expected profits conditional on entry must be equal to the entry costs, and the equilibrium condition stating total income in the economy must equal total spending. The market clearing condition in the labor market then holds by Walras’ Law. We normalize the nominal wage $w = 1$.

The free entry condition allows us to characterize the threshold productivity in equilibrium:

$$\phi^* = \left[ \frac{Lb^k}{2\gamma} \frac{1}{(k+2)(k+1)} \right]^{\frac{1}{k+1}},$$ \hspace{1cm} (12)$$

which pins down equilibrium prices and quantities. Firms with productivity equal to or higher than the threshold remain in the market, while those with lower productivity exit. The threshold depends on underlying parameters of the economy. For example, an increase in both market size $L$ and the minimum productivity level $b$ increases the threshold productivity level.

We now consider the impact taxes on total revenue have on the endogenous distribution of markups in the economy:

**Proposition 1** The model with revenue taxes $\tau$ introduces a non-monotonic relation between productivity $\phi$ and markups $M(\phi)$.

**Proof** Taking the partial derivative of the markup equation (11) with respect to the productivity level $\phi$ yields
\[
\frac{\partial M(\phi)}{\partial \phi} = -\frac{1}{2} \frac{1}{1 - \tau} (2\tau - 1)\phi^{-2} \geq 0, \quad (13)
\]

the sign of which depends on the value of the tax on total revenue \(\tau\):

\[
\frac{\partial M(\phi)}{\partial \phi} > 0 \quad \text{if} \quad \tau < \frac{1}{2}
\]

\[
\frac{\partial M(\phi)}{\partial \phi} = 0 \quad \text{if} \quad \tau = \frac{1}{2}
\]

\[
\frac{\partial M(\phi)}{\partial \phi} < 0 \quad \text{if} \quad \tau > \frac{1}{2} \quad (14)
\]

□

In order to understand Proposition 1, consider the effect of increasing productivity \(\phi\) on the two components of the markup \(M(\phi)\), price \(p(\phi)\) and marginal cost \(\frac{1}{\phi}\): \(\frac{\partial p(\phi)}{\partial \phi} = -\frac{1}{2} \frac{1}{1 - \tau} \phi^{-2} < 0\) and \(\frac{\partial MC(\phi)}{\partial \phi} = -\phi^{-2} < 0\). If \(\tau = \frac{1}{2}\), then the decline in price and marginal cost are the same, which means the markup does not change for this case. If \(\tau > \frac{1}{2}\), then the decrease in price is greater than the decrease in marginal cost, which drives down the markup. If \(\tau < \frac{1}{2}\), then the decrease in price is less than the decrease in marginal cost, so the markup increases. This case coincides with the markup result in Melitz and Ottaviano (2008).

**Proposition 2** Increasing the tax rate on total revenue \(\tau\) increases the markup \(M(\phi)\), but more productive firms increase \(M(\phi)\) less than low productivity firms.

**Proof** Taking the partial derivative of (11) with respect to the tax rate \(\tau\) yields

\[
\frac{\partial M(\phi)}{\partial \tau} = \frac{1}{2} \frac{1}{(1 - \tau)^2} \left[ \frac{1}{\phi^*} + \frac{1}{\phi} \right] > 0 \quad (15)
\]

By inspection, with increases in \(\tau\), more productive firms increase their markup \(M(\phi)\) less than low productivity firms.

□
Proposition 1 establishes the monotonic relation between productivity and markups in Melitz and Ottaviano (2008) is not robust to the inclusion of taxes on total revenue. However, the difference between the model in Melitz and Ottaviano (2008) and the model with taxes only occurs in the region $\tau > \frac{1}{2}$, which may suggest tax rates too high to be empirically relevant. It turns out the monotonic relation between productivity and markups in Melitz and Ottaviano (2008) is also not robust to the type of non-homotheticity. Moreover, a non-monotonic relation between productivity and markups can be generated in the region $\tau < \frac{1}{2}$.

Consider the non-homothetic utility function

$$U = \int_{z \in \Omega} \log(c(z) + 1)dz. \quad (16)$$

Solving the model as outlined above yields the following markup in terms of the threshold productivity level $\phi^*$:

$$M(\phi) = \frac{w}{1 - \tau} \left( \phi \phi^* \right)^{-\frac{1}{2}} - \frac{w}{\phi}. \quad (17)$$

Normalizing $w = 1$ again, the analogous version of Proposition 1 reads:

**Proposition 3** In the case with utility function (16), there is a non-monotonic relation between productivity $\phi$ and markups $M(\phi)$, even without taxes.

**Proof** Taking the partial derivative of the markup equation (17) with respect to the productivity level $\phi$ yields

$$\frac{\partial M(\phi)}{\partial \phi} = -\frac{1}{2} \frac{1}{1 - \tau} \phi^{-\frac{3}{2}} (\phi^*)^{-\frac{1}{2}} + \phi^{-2} > 0, \quad (18)$$

the sign of which depends on the value of the tax on total revenue $\tau$. Equating (18) to zero and rewriting yields

$$4(1 - \tau)^2 = \frac{\phi}{\phi^*}. \quad (19)$$

If $\tau \geq \frac{1}{2}$, then the LHS $\leq$ RHS of (19), because $\phi \geq \phi^*$ for all $\phi$ conditional on entry. In this
case, $\frac{\partial M(\phi)}{\partial \phi} \leq 0$. However, in the case when $\tau < \frac{1}{2}$, which is more realistic and interesting, there is a non-monotonic relation between productivity and the markup:

$$\frac{\partial M(\phi)}{\partial \phi} > 0 \text{ if } \phi < 4(1 - \tau)^2 \phi^*$$

$$\frac{\partial M(\phi)}{\partial \phi} = 0 \text{ if } \phi = 4(1 - \tau)^2 \phi^*$$

$$\frac{\partial M(\phi)}{\partial \phi} < 0 \text{ if } \phi > 4(1 - \tau)^2 \phi^*$$

(20)

The logic used to understand Proposition 3 is the same used for Proposition 1, so we skip this discussion because of space considerations.\(^2\) Notice, even when $\tau = 0$, the relationship between the markup and firm productivity remains non-monotonic, which establishes the markup result in Melitz and Ottaviano (2008) is also not robust to the type of non-homotheticity.

Moreover, Proposition 2 holds in the case with utility function (16).

\section{Conclusion}

We study the impact of firm taxation in a closed economy with heterogeneous firms and non-homothetic preferences. Specifically, taxes on total revenue change the endogenous distribution of markups. The relation between productivity and markups is non-monotonic and depends on the tax rate on total revenue. Even in an economy without taxes, productivity and markups can have a non-monotonic relation depending on the way non-homotheticity is modeled. Although we focus only on taxes on total revenue in this note, the model can easily be extended to incorporate different types of fiscal policy and richer assumptions regarding government behavior. The tractability of our model should prove useful for researchers estimating markups.

\(^2\)For reference, however, consider the effect of increasing productivity $\phi$ on the two components of the markup $M(\phi)$, price $p(\phi)$ and marginal cost $\frac{1}{\phi}$:

$$\frac{\partial p(\phi)}{\partial \phi} = -\frac{1}{2 \phi^2} < 0 \text{ and } \frac{\partial MC(\phi)}{\partial \phi} = -\phi^2 < 0.$$
References


