Physics 712
Chapter 11 Problems

1. We want to consider the effect of two boosts along the $x$-axis. The following hyperbolic identities will prove useful:
\[
\cosh(\phi_1 \pm \phi_2) = \cosh \phi_1 \cosh \phi_2 \pm \sinh \phi_1 \sinh \phi_2, \quad \text{and} \quad \tanh(\phi_1 \pm \phi_2) = \frac{\tanh \phi_1 \pm \tanh \phi_2}{1 \pm \tanh \phi_1 \tanh \phi_2}.
\]
(a) For two successive boosts with rapidity $\phi_1$ and $\phi_2$ find the equivalent rapidity $\phi_{\text{tot}}$.
(b) For two successive boosts with velocity $v_1$ and $v_2$ find the equivalent velocity $v_{\text{tot}}$.

2. Consider a particle moving along the $x$-axis whose 4-velocity is given at proper time $\tau$ by $U^\mu = c(\cosh \phi, \sinh \phi, 0, 0)$, where $\phi$ is an unknown function of time.
(a) Check that $U \cdot U = c^2$. Find the proper acceleration $a(\tau)$ at time $\tau$ for an arbitrary function $\phi(\tau)$.
(b) Suppose $a(\tau) = g$, a constant. Assuming the particle starts at the origin at $\tau = 0$ and is initially at rest, find $\phi(\tau)$, $U(\tau)$ and $x(\tau)$.
(c) How much proper time (in years) would it take to get to Alpha Centauri ($4.3 \, c \cdot y$), the center of our galaxy ($2.6 \times 10^4 \, c \cdot y$), or the edge of the visible universe ($4.5 \times 10^{10} \, c \cdot y$) if you start at rest and accelerate in a straight line at proper acceleration $g = 9.8 \, \text{m/s}^2$?

3. A pion (mass $m_\pi$) at rest decays to a muon (mass $m_\mu$) and a neutrino (mass 0). Find the energies of the two final particles.

4. A particle of mass $m$ and charge $q$ is in the presence of constant electric and magnetic fields $\mathbf{E} = E \hat{x}$ and $\mathbf{B} = B \hat{z}$.
(a) Write out explicitly all four components of the equation for $\dot{U}^\mu$, where dot stands for $d/d\tau$. Find an equation for $\dot{U}^1$.
(b) What is the general solution for $U^1(\tau)$ if $E < cB$? Argue that it will exhibit periodic behavior (in $\tau$), and find the period.
(c) Repeat part (b) if $E > cB$. Will it be periodic in this case?

5. Consider a line of charge with linear charge density $\lambda$ arranged, in a primed frame, along the $y'$-axis at rest. Write the electric field at all points in Cartesian coordinates in the primed frame. Now, consider a line of charge with the same linear charge density, parallel to the $y$-axis, but this time moving in the $+x$ direction at velocity $v$. Find the electric and magnetic fields everywhere in the unprimed frame.