Physics 712
Chapter 2 Problems

1. A point charge $q$ is at the position $(x, y, z) = (0, 0, h)$ above a grounded conducting plane at $z = 0$. Find the potential everywhere. Find the electric field on the surface $z = 0$, check that it is normal, and find the surface charge density $\sigma$ on the surface. Integrate the charge density over the entire plane.

2. A ring of charge of total charge $q$ forms a circle of radius $2a$ in the $xy$-plane around a conducting sphere of radius $a$. Find the potential and electric field everywhere along the $z$-axis for $z > a$ if the conducting sphere is (a) grounded (b) neutral.

3. Consider a square of side $a$ with $\Phi = 0$ on three sides and $\Phi = V$ on the surface $y = a$ in two dimensions. Our goal is to compute the potential everywhere, and particularly $\Phi\left(\frac{1}{4}a, \frac{1}{2}a\right)$. Write the potential in the form $\Phi(x, y) = \sum_{n=1}^{\infty} A_n(y) \sin\left(n\pi x/a\right)$. What is the form of the functions $A_n(y)$? By matching appropriate boundary conditions, determine any unknown coefficients, and find $\Phi(x, y)$ as an infinite sum. Sum it numerically to find $\Phi\left(\frac{1}{4}a, \frac{1}{2}a\right)$. Compare your results with the results of problem 1.7.