Physics 712
Chapter 8 Problems

1. Consider a wave guide consisting of two long parallel flat conductors, separated by a distance \( d \) in the \( y \)-direction, and having width \( w \) in the \( x \)-direction, with \( w \gg d \) (ignore edge effects). Find general formulas for the electric and magnetic field for waves moving in the \( +z \) direction, if the electric field has magnitude \( E_0 \). Find the time-averaged Poynting vector \( \langle S \rangle \) and integrate it over the space between the conductors to find the rate of energy transmission.

2. Electrical power is transmitted by the use of wave guides using TEM modes, with one conductor being the wire and the other being the Earth. We will model this as a coaxial cable with wire radius \( a = 1.0 \text{ cm} \) and “Earth distance” \( b = 10.0 \text{ m} \), with air (vacuum) in between.
   (a) For TEM modes of the form given in class, if it is travelling in the \( +z \) direction, find the time-averaged Poynting vector \( \langle S \rangle \). Integrate it over the whole area between the conductors to get the rate of power transmission.
   (b) If the power transmitted is 1.00 MW, determine the voltage difference between the inner and outer conductors.

3. Show that TEM modes for wave guides with vacuum always have phase velocity \( v_p = c \), while TE and TM modes always have phase velocity \( v_p > c \). Does this imply you can transmit information faster than light? Perform the appropriate calculation, and show that it never leads to superluminal velocities.

4. Consider a conducting cavity of radius \( a \) and length \( 2a \) with nothing (vacuum) inside. Find the frequencies of the five lowest frequencies \( \omega \) for this cavity as multiples of \( c/a \).

5. For a box-shaped conducting cavity of dimensions \( a \times b \times d \), with \( d \) along the conventional \( z \)-direction, work out explicitly every non-zero component for the TM_{1,1,0} mode. Then calculate the electric energy and magnetic energy in the cavity as a function of time. As a check, show the total is independent of time.