1. In Richard Williams’ lab, a laser can (briefly) produce 50 GW of power and be focused on a region of size 1 μm². How large are the maximum electric and magnetic fields?

The intensity of the beam is the power over the area, or

\[ I = \frac{5.0 \times 10^{10} \text{ W}}{(1.0 \times 10^{-6} \text{ m})^2} = 5.0 \times 10^{22} \text{ W/m}^2. \]

We then equate this to the magnitude of the Poynting vector. We have

\[ I = \frac{1}{2} \sqrt{\frac{\varepsilon_0}{\mu_0}} E_0 \cdot E_0^*, \]

\[ E_0 \cdot E_0^* = 2 \sqrt{\frac{\mu_0}{\varepsilon_0}} I = \frac{2 \mu_0}{\sqrt{\varepsilon_0 \mu_0}} I = 2 \varepsilon_0 c_0 I = 2 \left( 2.998 \times 10^8 \text{ m/s} \right) \left( 4 \pi \times 10^{-7} \text{ N/A}^2 \right) \left( 5.0 \times 10^{22} \text{ W/m}^2 \right) \]

\[ = 3.76 \times 10^{25} \text{ N}^2 \cdot \text{A}^{-2} \cdot \text{s}^{-2} = 3.76 \times 10^{25} \text{ N}^2 \cdot \text{C}^{-2}, \]

\[ |E_0| = \sqrt{3.76 \times 10^{25} \text{ N}^2 \cdot \text{C}^{-2}} = 6.14 \times 10^{12} \text{ N/C}. \]

The magnetic fields are given by

\[ B_0 = \sqrt{\mu_0 \varepsilon_0} \hat{k} \times E_0 = \frac{1}{c} \hat{k} \times E_0, \]

\[ |B_0| = \frac{1}{c} |E_0| = \frac{6.14 \times 10^{12} \text{ N/C}}{2.998 \times 10^8 \text{ m/s}} = 20,500 \text{ T}. \]

By comparison, a very strong static electric field would be about $10^8$ N/m and a big magnetic field would be 100 T.
2. Suppose a perfect polarizer extracts from a pure wave in the \( z \)-direction just the polarization \( \hat{e}_x = \hat{x}, \hat{e}_y = \hat{y}, \hat{e}_z = \frac{1}{\sqrt{2}}(\hat{x} + \hat{y}), \) or \( \hat{e}_r = \frac{1}{\sqrt{2}}(\hat{x} - \hat{y}). \) In each case, write the resulting intensity in terms of just the Stokes parameters. Find a relationship between the four intensities \( I_x, I_y, I_I, I_I. \)

The most general wave we can have is of the form \( E_0 = E_x \hat{x} + E_y \hat{y}. \) It is obvious that if we use the first two cases, the resulting electric fields will be just \( E_0 = E_x \hat{x} \) or \( E_0 = E_y \hat{y}, \) and the resulting intensities will be

\[
I_x = \frac{1}{2} \sqrt{\frac{\varepsilon}{\mu}} E_x E_x^*, \quad I_y = \frac{1}{2} \sqrt{\frac{\varepsilon}{\mu}} E_y E_y^*
\]

On the other hand, we can also write the same wave in the form

\[
E_0 = E_x \hat{x} + E_y \hat{y} = \frac{1}{\sqrt{2}}(E_1 + E_2)(\hat{x} + \hat{y}) + \frac{1}{\sqrt{2}}(E_1 - E_2)(\hat{x} - \hat{y}) = \frac{1}{\sqrt{2}}(E_1 + E_2)\hat{e}_x + \frac{1}{\sqrt{2}}(E_1 - E_2)\hat{e}_y.
\]

If we extract out just one of these two polarizations, it is evidence that we have

\[
I_I = \frac{1}{2} \sqrt{\frac{\varepsilon}{\mu}} \frac{1}{2}(E_1 + E_2)(E_1^* + E_2^*) = \frac{1}{4} \sqrt{\frac{\varepsilon}{\mu}} (E_1 E_1^* + E_2 E_2^* + E_1 E_2^* + E_2 E_1^*)
\]

\[
= \frac{1}{4} \sqrt{\frac{\varepsilon}{\mu}} \left[ E_1 E_1^* + E_2 E_2^* + 2 \text{Re}(E_1 E_2^*) \right],
\]

\[
I_\parallel = \frac{1}{2} \sqrt{\frac{\varepsilon}{\mu}} \frac{1}{2}(E_1 - E_2)(E_1^* - E_2^*) = \frac{1}{4} \sqrt{\frac{\varepsilon}{\mu}} (E_1 E_1^* + E_2 E_2^* - E_1 E_2^* - E_2 E_1^*)
\]

\[
= \frac{1}{4} \sqrt{\frac{\varepsilon}{\mu}} \left[ E_1 E_1^* + E_2 E_2^* - 2 \text{Re}(E_1 E_2^*) \right].
\]

Compare each of these with some of the Stokes’ parameters:

\[
s_0 = E_1 E_1^* + E_2 E_2^*, \quad s_1 = E_1 E_1^* - E_2 E_2^*, \quad s_2 = 2 \text{Re}(E_1 E_2^*).
\]

We see that we can write the intensities as

\[
I_x = \frac{1}{4} \sqrt{\frac{\varepsilon}{\mu}} (s_0 + s_1), \quad I_y = \frac{1}{4} \sqrt{\frac{\varepsilon}{\mu}} (s_0 - s_1), \quad I_I = \frac{1}{4} \sqrt{\frac{\varepsilon}{\mu}} (s_0 + s_2), \quad I_\parallel = \frac{1}{4} \sqrt{\frac{\varepsilon}{\mu}} (s_0 - s_2).
\]

It is then trivial to see that \( I_x + I_y = I_I + I_\parallel. \)