Name ____________________
Test 1
September 18, 2015

This test consists of three parts. Please note that in parts II and III, you can skip one question of those offered.

Possibly useful formulas:

$$\vec{F} = q\vec{E} + q\vec{u} \times \vec{B}$$
$$p = qRB$$
$$\vec{u} = \frac{\vec{p}c}{E}$$

$$f = \frac{f_0}{\gamma(1 - \nu \cos \theta/c)}$$

$$x' = \gamma(x - vt)$$
$$t' = \gamma(t - vx/c^2)$$
$$y' = y, \quad z' = z$$

$$E' = \gamma(E - vp_x)$$
$$p'_x = \gamma(p_x - vE/c^2)$$

$$u'_x = \frac{u_x - v}{1 - vu_x/c^2}$$

$$u'_y = \frac{u_y}{\gamma(1 - vu_x/c^2)}$$

$$u'_z = \frac{u_z}{\gamma(1 - vu_x/c^2)}$$

$$(1 + \epsilon)^n = 1 + n\epsilon + \frac{1}{2}n(n-1)\epsilon^2 + \cdots$$

Part I: Multiple Choice [20 points]
For each question, choose the best answer (2 points each)

1. A fast-moving clock will run _____ and be _____ than the same clock at rest
   A) Slower, longer
   B) Faster, longer
   C) Slower, shorter
   D) Faster, shorter
   E) None of the above

2. Which particles are massless?
   A) Protons  B) Neutrons  C) Electrons  D) Photons  E) None of these

3. Suppose I take some object, and add some energy to it. Which types of energy will cause a change in the object’s mass \(m\) (also known as rest-mass)
   A) Any type of energy, including potential or kinetic energy
   B) Potential energy, like nuclear, chemical or mechanical energy, but not kinetic energy
   C) Nuclear or chemical energy, but not mechanical energy
   D) Nuclear energy, but not chemical or mechanical energy
   E) No type of energy changes an object’s rest-mass
4. Which of the following formulas can be used for massless particles?
   A) \( E^2 - c^2 p^2 = \left( mc^2 \right)^2 \) (only)
   B) \( E = \gamma mc^2 \) (only)
   C) \( p = \gamma m\vec{v} \) (only)
   D) All of the above
   E) None of the above

5. As an electron approaches the speed of light, \( v \to c \), the momentum approaches ____ and the energy approaches ____.
   A) \( mc, \frac{1}{2}mc^2 \)
   B) \( mc, \infty \)
   C) \( \infty, \frac{1}{2}mc^2 \)
   D) \( \infty, mc^2 \)
   E) \( \infty, \infty \)

6. In the twin paradox, one twin stays at rest on Earth, while the other travels quickly and ages less. But from the perspective of the moving twin, the one on Earth should age less. Why are these two perspectives not truly interchangeable?
   A) The one on Earth is actually at rest, and the other is truly moving
   B) Lorentz contraction means that the distances travelled are not truly interchangeable
   C) Because all objects are not rigid in special relativity, time itself can be stretched, making such comparisons meaningless
   D) The traveling twin did not travel at constant velocity. He changed speed when he turned around to come back
   E) Actually, they are interchangeable. The moving twin will return to discover the stationary twin is younger

7. Which of the following is a pretty good approximation for \( \gamma \) for small velocities \( v \)?
   A) \( \gamma = 1 + \frac{v^2}{c^2} \)
   B) \( \gamma = 1 + \frac{v^2}{2c^2} \)
   C) \( \gamma = 1 - \frac{v^2}{c^2} \)
   D) \( \gamma = 1 - \frac{v^2}{2c^2} \)
   E) \( \gamma = 1 - \frac{c^2}{2v^2} \)

8. Which of the formulas below is still valid in special relativity?
   A) \( \vec{F} = \frac{d\vec{p}}{dt} \)
   B) \( F = ma \)
   C) \( \vec{p} = m\vec{v} \)
   D) \( E = \frac{1}{2}mv^2 \)
   E) \( F = \frac{mv^2}{r} \)

9. In special relativity, why are the Galilean transformations \( ( x' = x - vt, t' = t ) \) not “good”?
   A) They imply the wrong formula for conservation of energy
   B) They imply the wrong formula for conservation of momentum
   C) They don’t preserve the 4D-distance formula
   D) They are for physical objects, not light
   E) They are for rotating coordinates, not inertial coordinates

10. Since spacetime is the four dimensional, all vectors should have four components.
    The fourth component of momentum is

**Part II: Short answer [20 points]**
Choose two of the following questions and give a short answer (1-3 sentences) (10 points each).

11. Suppose I build a launcher that launches a space ship at half the speed of light. On this ship, I put a second launcher that launches a smaller ship, also at half of the speed of light. Would this be a way to make a space ship that goes at the speed of light?

12. A very simple device for communicating faster than light would be to have a long pole from point A to point B. The person at point A moves one end of the pole back and forth, perhaps in Morse code, and the person at point B measures and interprets the message. Explain why this would or would not work.

13. Particle detectors typically have strong magnetic fields. Explain how tracking a particle in the presence of a magnetic field could allow one to determine the sign of the charge of a particle, as well as some other quantity. At least one equation is necessary.
Part III: Calculation: [60 points]
Choose three of the following four questions and perform the indicated calculations (20 points each)

14. Consider the three events $A$, $B$, and $C$, which in one frame have coordinates $(x,y,z,ct)$ given by

$A = (0,0,0,0)$, $B = (5.0 \text{ m},12.0 \text{ m},0,13.0 \text{ m})$, $C = (5.0 \text{ m},5.0 \text{ m},0,5.0 \text{ m})$

(a) For each pair $(AB, AC, \text{ and } BC)$ find $s^2$ and determine how the pair is related to each other (absolute future/past, future/past light cone, or elsewhere)
(b) If another observer were moving at a different velocity, the coordinates of the three events would change. How could this affect the answers to part (a)?
(c) In which pair of events would it be possible for a massive object to move from one of these to the other? How about for a massless object? How about for a tachyon?

15. An alien spaceship has a shape (as viewed by us) of a square-based pyramid with square side 150.0 m, and height 120.0 m is travelling point first directly towards us at $v = 0.600c$. It is sending us signals at a frequency that we measure as 800.0 MHz.

(a) What are the dimensions of the actual ship, as viewed by aliens aboard the ship?
(b) What is the actual frequency at which the spacecraft is broadcasting?
(c) It continues broadcasting at the same frequency. What frequency do we detect at closest approach? After it has passed us and is heading away from us?

16. A charged pion with mass $m = 139.6 \text{ MeV/c}^2$ is travelling in a circle of radius 1.50 m. It completes one circle in a time of $40.0 \text{ ns} = 40.0 \times 10^{-9} \text{ s}$ before decaying

(a) How long, if we were traveling along with the pion, would the pion appear to last?
(b) What is the energy of the pion, in MeV?
(c) What is the momentum of the pion in MeV/c?

17. Dr. Carlson has a rest mass of $m = 78.5 \text{ kg}$. He is travelling in his spacecraft at $v = 2.70 \times 10^8 \text{ m/s}$ when the galactic traffic light turns red, at which point his brakes turn on, applying a force of $F = 1220 \text{ N}$ to attempt to stop him.

(a) What is the initial and final energy of the Dr. Carlson (in J)?
(b) What is the initial and final momentum (in kg·m/s) of Dr. Carlson?
(c) What is the stopping distance at this speed, in light years ($1 \text{ ly} = 9.458 \times 10^{15} \text{ m}$)?
(d) How long does it take to come to a stop, in years ($1 \text{ y} = 3.156 \times 10^7 \text{ s}$)?