This test consists of three parts. Please note that in parts II and III, you can skip one question of those offered.

Possibly useful formulas:

<table>
<thead>
<tr>
<th>( \vec{F} = q\vec{E} + q\vec{u} \times \vec{B} )</th>
<th>( x' = \gamma (x - vt) )</th>
<th>( E' = \gamma (E - vp_x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p = qRB )</td>
<td>( t' = \gamma (t - vx/c^2) )</td>
<td>( p'_x = \gamma (p_x - vE/c^2) )</td>
</tr>
<tr>
<td>( \frac{\vec{u}}{c} = \frac{\vec{p}c}{E} )</td>
<td>( y' = y ), ( z' = z )</td>
<td>( p'_y = p_y ), ( p'_z = p_z )</td>
</tr>
<tr>
<td>( f = \frac{\gamma_0}{\gamma (1 - v \cos \theta/c)} )</td>
<td>( (1 + \epsilon)^n = 1 + n\epsilon + \frac{1}{2} n(n-1)\epsilon^2 + \cdots )</td>
<td>( u'_x = \frac{u_x - v}{1 - vu_x/c^2} )</td>
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<td></td>
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<td>( u'_y = \frac{u_y}{\gamma (1 - vu_x/c^2)} )</td>
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<tr>
<td></td>
<td></td>
<td>( u'_z = \frac{u_z}{\gamma (1 - vu_x/c^2)} )</td>
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</table>

**Part I: Multiple Choice [20 points]**
For each question, choose the best answer (2 points each)

1. If observer A sees light moving to the right at speed \( c \), and observer B is moving to the right at an angle, how fast will the light beam appear to be moving according to observer B?
   - A) Slower than \( c \)
   - B) Faster than \( c \)
   - C) Exactly at \( c \)
   - D) It depends on the speed of observer B
   - E) It depends on the angle of observer B

2. If one observer stays at constant velocity, while another one accelerates, such that they start and end at the same place and time, for which observer will less proper time have passed?
   - A) The constant velocity one
   - B) The accelerating one
   - C) They will be the same
   - D) It depends on which one is actually moving
   - E) There is insufficient information, even if you know which one is actually moving

3. If you compress a realistic spring, how will its total energy change?
   - A) It decreases a lot
   - B) It decreases a little
   - C) It is unchanged
   - D) It increases a little
   - E) It increases a lot
4. To calculate the invariant mass of a collection of objects, you would need:
   A) The total momentum (only)
   B) The total energy (only)
   C) The sum of the velocities of all the components (only)
   D) The total momentum and the total energy
   E) The total momentum and the sum of the velocities of the components

5. Suppose that a clock is moving at a very high velocity compared to us, so that \( \gamma = 4 \).
   According to us, in the time the clock advances 12 s, how much time will have passed for us?
   A) 3 s   B) 6 s   C) 12 s   D) 24 s   E) 48 s

6. If one observer is moving in the \( x \)-direction compared to the other, on which of the following quantities will the two observers agree?
   A) The energy (only)
   B) The \( x \)-momentum (only)
   C) The \( y \)- and \( z \)-momentum (only)
   D) All three components of momentum, but not the energy
   E) All three components of momentum, and the energy

7. Which of the following coordinate transformations is believed to not actually leave the laws of physics unchanged?
   A) Galilean boost
   B) Rotation
   C) Space translation
   D) Time translation
   E) Actually, all of these do leave the laws of physics unchanged

8. If an object is moving from our point of view directly perpendicular to the line of sight, how will the observed frequency differ from the natural frequency?
   A) The observed frequency will be higher
   B) The observed frequency will be lower
   C) The observed frequency will be the same
   D) It depends on the velocity of the object
   E) It depends on the frequency

9. Which of the following formulas is the correct relativistic formula for just the kinetic energy?
   A) \( mc^2 \)   B) \( \gamma mc^2 \)   C) \( (\gamma - 1)mc^2 \)   D) \( \frac{1}{2}mv^2 \)   E) \( \gamma mv \)

10. A particle that moves faster than \( c \) would be a ______. An example that actually exists of this would be a ________.
    A) Massive particle; electron
    B) Massless particle; photon
    C) Tachyon; neutrino
    D) Tachyon; there are no such particles
    E) None of the above
Part II: Short answer [20 points]

Choose two of the following questions and give a short answer (1-3 sentences) (10 points each).

11. In H.G. Wells’ story “The Time Machine,” the protagonist states, “There is no difference between time and any of the three dimensions of space …” Explain what is right and/or wrong about this statement.

12. As we learn in relativity, simultaneity is ambiguous, and observers might even disagree on which of two events came first. Does this mean that the birth of your father might actually have happened after your birth, as viewed by certain observers? Explain.

13. In non-relativistic physics, the two equations $\vec{F} = m\vec{a}$ and $\vec{F} = \frac{d\vec{p}}{dt}$ are equivalent. Tell me which of these (if either) is correct in relativity, and why that one is preferred.
Choose three of the following four questions and perform the indicated calculations (20 points each)

14. An ice hockey puck of diameter 3 in (7.62 cm) crosses a line on a standard hockey rink of width 2 in (5.08 cm). Because the puck is moving quickly, it looks as if the hockey puck is exactly the same diameter (in the direction it is moving) as the line
   (a) What is $\gamma$ for the hockey puck? What is the speed of the hockey puck?
   (b) As viewed by an observer on the hockey puck, what is the width of the line it is crossing?
   (c) (Short essay question) At $t = 0$, the referee instantaneously stops the hockey puck to look at it, and sees that it is indeed no wider than the line it was crossing. But since in the reference frame of the hockey puck, the puck is larger than the line it is crossing, how is it possible for the puck to “fit” in this short a space?

15. A muon has a mean lifetime, as viewed in its own frame, of $\tau_0 = 2.197 \mu s$. The probability of it lasting a proper time $\tau$ is given by $e^{-\tau/\tau_0}$. Suppose a group of muons is travelling at an unknown speed
   (a) After how much proper time will only 6.30% of the original muons still exist?
   (b) Suppose that 6.30% of the muons still exist after travelling a distance $d = 2.03$ km. Find the time $\Delta t$ as measured by us.
   (c) Find the velocity of the muons in m/s.

16. A Lambda particle decays into a proton ($m_p = 938$ MeV/c$^2$) at rest and a pion ($m_\pi = 140$ MeV/c$^2$) moving to the right with energy $E_\pi = 184$ MeV.
   (a) Find the initial momentum of the pion in MeV/c. What is the momentum and energy of the proton?
   (b) Find the initial energy and momentum of the Lambda.
   (c) What is the mass of the Lambda $m_\Lambda$ in MeV/c$^2$, and its velocity as a fraction of the speed of light?

17. A nanobot, initially at rest, has a mass of $4.30 \times 10^{-12}$ kg and is being launched to visit a nearby star. The goal is to get it to a speed of $1.35 \times 10^8$ m/s
   (a) What is the initial and final energy (in J) of the nanobot?
   (b) What is the initial and final momentum (in kg-m/s) of the nanobot?
   (c) If the accelerator that launches the nanobot is 230 m long, what force is required on the nanobot?
   (d) If the acceleration is achieved with an electric field of magnitude $|E| = 3.00 \times 10^8$ V/m, what electrical charge must be placed on the nanobot?