This test consists of three parts. Please note that in parts II and III, you can skip one question of those offered.

**Possibly useful formulas:**

**Constants**

- \( e = 1.602 \times 10^{-19} \text{C} \)
- \( k_e = 8.988 \times 10^9 \text{N} \cdot \text{m}^2/\text{C}^2 \)
- \( \varepsilon_0 = 8.854 \times 10^{-12} \text{C}^2/\text{N} \cdot \text{m}^2 \)
- \( 4\pi\varepsilon_0 k_e = 1 \)

**Charge on Surface of Conductor**

- \( E = \frac{\sigma}{\varepsilon_0} \hat{n} \)

**Parallel Plate Capacitor**

- \( C = \frac{A k_e \varepsilon_0}{d} \)

**Energy in a Capacitor**

- \( u = \frac{1}{2} C (\Delta V)^2 \)
- \( u = \frac{1}{2} \varepsilon_0 E^2 \)

**Metric Prefixes**

- \( m = 10^{-3}, \mu = 10^{-6} \)
- \( n = 10^{-9}, \text{p} = 10^{-12} \)

**Cylinders**

- \( V = \pi R^2 L \)
- \( A_{\text{lat}} = 2\pi RL \)

**Spheres**

- \( A = 4\pi R^2 \)
- \( V = \frac{4}{3} \pi R^3 \)

**Dipoles**

- \( \tau = \mathbf{p} \times \mathbf{E} \)
- \( U = - \mathbf{p} \cdot \mathbf{E} \)

**Triangles**

- \( A = \frac{1}{2} BH \)

**Circles**

- \( C = 2\pi R \)
- \( A = \pi R^2 \)

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### Part I: Multiple Choice  [20 points]

For each question, choose the best answer (2 points each)

1. What is the voltage at point \( X \) in the diagram at right?
   - A) +16 V
   - B) –16 V
   - C) +4 V
   - D) –4 V
   - E) +8 V

2. A carbon monoxide molecule has a negatively charged oxygen atom connected to a positively carbon atom, roughly as sketched at right. The total charge is zero. If an electric field is present pointing to the right, a carbon monoxide molecule will feel what sort of force on it?
   - A) A net force in the direction of the field
   - B) A net force opposite the direction of the field
   - C) **A torque, or twisting force, that tries to get the carbon atom to the right**
   - D) A torque, or twisting force, that tries to get the oxygen atom to the right
   - E) No net force or torque

3. A student is trying to find the potential difference between point \( X \) and point \( Y \) in a constant electric field. Which of the paths will give the correct potential difference?
   - A) Only path A
   - B) Only path B
   - C) Only path C
   - D) Paths A or B, but not path C
   - E) **All three paths give the same answer**
4. What advantage do capacitors have over batteries for energy storage?
   A) They can store more energy in the same volume/mass
   **B) They can release their energy far more quickly than batteries**
   C) They can hold their energy much longer than batteries
   D) They are more consistent at keeping the voltage constant than batteries
   E) They are easier to draw in circuits than batteries

5. If I were trying to find the electric field in the neighborhood of a long (or infinite) line charge, the best Gaussian surface to use would be a
   A) Cube  B) Sphere  C) Cylinder  D) Circle  E) Disk

6. If the electric field is pointing to the right, then positive charges will accelerate to the
   _______ and negative charges to the _______.
   A) Right, right  B) Right, left  C) Left, right  D) Left, left  E) None of these

7. Suppose a sphere of uniform charge with total charge $q$ is at the center of a cube of size $a$. What is the electric flux out of just one side of the cube?
   A) $\frac{kq}{6a^2}$  B) $\frac{6kq}{a^2}$  C) $\frac{q}{\varepsilon_0}$  D) $\frac{q}{6\varepsilon_0}$  E) None of these

8. Suppose I was given a sketch of the equipotential lines and asked to determine the direction of the electric field. Which way does the electric field point?
   A) Perpendicular to the equipotential lines, and pointing downhill
   **B) Perpendicular to the equipotential lines, and pointing uphill**
   C) Parallel to the equipotential lines, with downhill on the right
   D) Parallel to the equipotential lines, with downhill on the left
   E) None of the above

9. Which of the following explains, roughly, why putting an insulating material inside a capacitor increases its capacitance?
   A) The charges in the capacitor can shift, partly cancelling the electric field and reducing the voltage for a given charge
   **B) The insulator absorbs some of the charge without letting it cross the barrier**
   C) The insulator pulls the two sides together, making them closer
   D) The insulator spreads the charge out, effectively increasing the area of the capacitor
   E) It doesn’t actually increase the capacitance, but by blocking charge motion, it increases the breakdown voltage, allowing you to charge it more

10. Suppose a conducting sphere has a large, positive charge placed on it by connecting it to a battery. Which of the following has occurred?
    A) Protons have been added
    **B) Protons have been removed**
    C) Electrons have been added
    D) Electrons have been removed
    E) None of the above
Part II: Short answer [20 points]

Choose two of the following questions and give a short answer (2-3 sentences) or brief sketch (10 points each).

11. Below are sketched three charges, two on the ends of magnitude $+2q$, one of magnitude $-3q$ in the middle. Sketch in approximate electric field lines, with arrows showing directions. I recommend a minimum of two field lines for each unit of $q$.

This is an approximate sketch of the electric field lines if we have six lines per unit of $q$. Note that there are 12 lines coming out of each of the positive charges, and 18 going into the central charge. This leaves 6 lines net coming out, which must go to infinity. The darker lines are the ones you would use if you just had two per unit of $q$.

12. Explain, giving any relevant equations, how it would be possible to figure out the total electric charge in a box by only measuring things outside the box.

By Gauss’s Law, the total charge inside any closed volume is given by $q_{in} = \varepsilon_0 \Phi_E$, where $\Phi_E$ is the electric flux out of the box, given by $\Phi_E = \int E \cdot \hat{n} \, da$. The fields can be measured on any surface containing the box.

13. Explain qualitatively why it is a good idea to have a pointy, grounded rod on top of your home/building (a lightning rod).

During a thunderstorm, a large amount of positive charge builds up in the atmosphere, while the ground accumulates negative charge. If nothing depletes these charges, lightning strikes. A pointy lightning rod has a large electric field, allowing dielectric breakdown in the vicinity of the rod, draining the charge away from the air near your house. This means lightning will strike somewhere else, rather than your home.
Part III: Calculation: [60 points]

Choose three of the following four questions and perform the indicated calculations (20 points each)

14. Three point charges are arranged around the origin, with two of them of magnitude +10.00 nC at \( x = \pm 3.00 \) cm, and one of magnitude -30.00 nC at \( y = +3.00 \) cm, as sketched at right.

(a) Find the magnitude and direction of the electric field at the origin.

The two +10.00 nC charges will each make an electric field of magnitude
\[
E = \frac{k |q|}{r^2} = \frac{(8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) (10.0 \times 10^{-9} \text{ C})}{(0.03 \text{ m})^2} = 9.98 \times 10^4 \text{N/C} = 99.8 \text{kV/m}.
\]

However, the fields point away from the charges in each case, so they will point left and right and exactly cancel. The -30.00 nC charge will make a field three times as large, or
\[
E = \frac{k |q|}{r^2} = \frac{(8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) (30.0 \times 10^{-9} \text{ C})}{(0.03 \text{ m})^2} = 3.00 \times 10^5 \text{N/C} = 300 \text{kV/m}.
\]

This one will point upwards, towards the negative charge so the final answer is 300\( \hat{j} \) kN/m.

(b) An electron is placed at the origin, initially at rest. Find the initial force and acceleration (including direction) of the electron. An electron has a mass \( m = 9.109 \times 10^{-31} \text{ kg} \).

The electron will feel a force \( F = qE = -eE \), so we have
\[
F = (-1.602 \times 10^{-19} \text{ C}) (3.00 \times 10^5 \hat{j} \text{ N/C}) = -4.80 \times 10^{-14} \hat{j} \text{ N}.
\]

The force is in the negative \( y \) direction, or downwards. The acceleration is then found from \( F = ma \):
\[
a = \frac{F}{m} = \frac{-4.80 \times 10^{-14} \hat{j} \text{ N}}{9.109 \times 10^{-31} \text{ kg}} = 5.26 \times 10^{16} \text{ m/s}^2.
\]
15. A hollow conduction sphere has inner radius $r = 2.00 \text{ cm}$ and outer radius $r = 4.00 \text{ cm}$. It has total charge $Q = +20.00 \text{ pC}$ placed on it. It is surrounded by a thin spherical shell of radius $r = 6.00 \text{ cm}$ with $q = -50.00 \text{ pC}$ on. Find the electric field $E$ at the following distances from the center:

(a) $r = 1.00 \text{ cm}$
(b) $r = 3.00 \text{ cm}$
(c) $r = 5.00 \text{ cm}$
(d) $r = 7.00 \text{ cm}$

This problem has spherical symmetry, and hence it makes sense that the electric field will depend only on the distance from the center, and will be radially outward (or inward) at every position. We can therefore use Gauss’s law, which basically tells us that we can naively use Coulomb’s Law if we just pretend that all the charge at a radius smaller than the radius $r$ will be at the origin.

We start at $r = 1 \text{ cm}$, but there is no charge closer than this, so the electric field at this point is just 0. At $r = 3 \text{ cm}$, it is not immediately obvious what charge there might be inside it, but we are in the body of a conductor and again the electric field vanishes there. So we have

$$E(1 \text{ cm}) = E(3 \text{ cm}) = 0.$$ 

Now, if we move out to $r = 5 \text{ cm}$, the conducting sphere must be included, so we include the charge $Q$. Pretending it is at the origin, we use Coulomb’s law to tell us

$$E(5 \text{ cm}) = \frac{kQ}{r^2} \hat{r} = \frac{(8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(20.0 \times 10^{-12} \text{ C})}{(0.0500 \text{ m})^2} \hat{r} = 71.9 \hat{r} \text{ N/C}.$$ 

The $\hat{r}$ simply tells us the electric field points outwards. For $r = 7 \text{ cm}$, both the conducting sphere with charge $Q$ and the shell of charge $q$ contribute, so we have

$$E(7 \text{ cm}) = \frac{k}{r^2} (Q + q) \hat{r} = \frac{(8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(-30.0 \times 10^{-12} \text{ C})}{(0.0700 \text{ m})^2} \hat{r} = -55.0 \hat{r} \text{ N/C}.$$ 

This time, of course, the electric field is directed inwards.
Warning: If you did problem 14, it is unlikely that you will be able to reuse any of your work from that problem.

16. Three point charges are arranged around the origin, with two of them of magnitude +10.00 nC at \( x = \pm 3.00 \) cm, and one of magnitude -30.00 nC at \( y = +3.00 \) cm, as sketched at right.

(a) Find the magnitude of the electric potential at the origin.

The potential from each of the 10.0 nC charges can be found by using the formula \( V = \frac{kq}{r} \). However, potential has no direction, so we simply add the contribution from the three charges. Since they are all the same distance away, we can simply add the charges before we calculate. Since the total charge is -10.00 nC, we have

\[
V = \frac{kq}{r} = \left( \frac{8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \right) \frac{-10.0 \times 10^{-9} \text{ C}}{0.03 \text{ m}} = -2996 \text{ V}.
\]

(b) An electron is placed at the origin, initially at rest. What is the potential energy of this electron?

Potential energy is just \( U = qV = -eV \), so we have

\[
U = -eV = \left( -1.602 \times 10^{-19} \text{ C} \right) \left( -2996 \text{ V} \right) = 4.800 \times 10^{-16} \text{ J}
\]

(c) The electron speeds off to infinity. What is the speed of the electron at infinity? An electron has a mass \( m = 9.109 \times 10^{-31} \text{ kg} \).

The electron starts with no kinetic energy and some potential energy. It ends with no potential energy but some kinetic energy. Since energy is conserved, the potential energy must have been converted to kinetic energy. Hence we have \( U = \frac{1}{2} mv^2 \), so

\[
v^2 = \frac{2U}{m} = \frac{2 \left( 4.800 \times 10^{-16} \text{ J} \right)}{9.109 \times 10^{-31} \text{ kg}} = 1.054 \times 10^{15} \text{ m}^2/\text{s}^2,
\]

\[
v = \sqrt{1.054 \times 10^{15} \text{ m}^2/\text{s}^2} = 3.246 \times 10^{7} \text{ m/s}.
\]

Because this speed is about 11% of the speed of light, you actually have to take some relativity into account. However, this only causes a small change, so that the actual speed is about \( 3.232 \times 10^{7} \text{ m/s} \).
17. Four capacitors are connected as sketched at right. The capacitance is going to be measured between points $A$ and $B$, with the switch $S$ either open or closed.

(a) If $S$ is open, what is the total capacitance?

If $S$ is open, then it is clear that the pair of capacitors on the left side are connected in series, in that both of them are connected at one end to each other, and nothing else is connected there. The same is true of the pair on the right. The equivalent capacitance for this pair can then be found using

$$
\frac{1}{C_L} = \frac{1}{6 \ \mu F} + \frac{1}{12 \ \mu F} = \frac{3}{12 \ \mu F} = \frac{1}{4 \ \mu F},
$$

$$
C_L = C_R = 4 \ \mu F.
$$

We then replace each pair by capacitors $C_L$ and $C_R$. These capacitors are now connected in parallel, so the total capacitance is

$$
C = C_L + C_R = (4 \ \mu F) + (4 \ \mu F) = 8 \ \mu F.
$$

(b) If $S$ is closed, what is the total capacitance?

With $S$ closed, the left pair is no longer in series, nor is the right pair. However, the bottom pair (and top pair) are now connected at both ends, and hence are in parallel. We use the formula for capacitors in parallel to find

$$
C_B = C_T = (6 \ \mu F) + (12 \ \mu F) = 18 \ \mu F.
$$

Replacing the each of these pairs with a single capacitor will then leave two capacitors, $C_B$ and $C_T$, now connected in series, so our final capacitance is

$$
\frac{1}{C} = \frac{1}{C_B} + \frac{1}{C_T} = \frac{1}{18 \ \mu F} + \frac{1}{18 \ \mu F} = \frac{2}{18 \ \mu F} = \frac{1}{9 \ \mu F},
$$

$$
C = 9 \ \mu F.
$$

(c) With the switch open (part a), the capacitors are connected to a battery across points $A$ and $B$. What potential difference will cause a total of 10.0 mJ to be stored in this system of capacitors?

We use the formula for the energy in a capacitor, $U = \frac{1}{2}C(\Delta V)^2$. Solving for the potential difference, we have

$$
(\Delta V)^2 = \frac{2U}{C} = \frac{2(0.0100 \ J)}{8.00 \times 10^{-6} \ C} = 2500. \ V^2,
$$

$$
\Delta V = 50.0 \ V.
$$