Name ____________________
Solutions to Final Exam
May 6, 2015

This test consists of five parts. Please note that in parts II through V, you can skip one question of those offered. Some possibly useful formulas appear below.

### Part I: Multiple Choice [20 points]
For each question, choose the best answer (2 points each)

1. When light moves through a flat boundary from air into a material with an index of refraction, such as when we look at a fish swimming under a perfectly smooth surface of water, how does the image compare to the reality?
   - A) The image is neither enlarged nor reduced, but it appears farther than it really is
   - B) The image is neither enlarged nor reduced, but it appears closer than it really is
   - C) The image appears at the correct distance, but it is reduced
   - D) The image appears at the correct distance, but it is enlarged
   - E) The image appears exactly like the actual object

2. The greatest diffraction of light passing through a slit occurs when the wavelength is _______ and the slit size is _______.
   - A) Long, large
   - B) Short, large
   - C) Long, small
   - D) Short, small
   - E) Chartreuse, erratic
3. For someone whose eyes are near sighted or far sighted, how do glasses help them?
   A) It creates images of what they are looking at that are at a distance where their eyes CAN focus on them
   B) It magnifies the objects, making them larger so they can see them better
   C) It gathers additional light, so their eyes can function better
   D) They absorb the wavelengths that are causing the problem, only allowing other wavelengths through
   E) They don’t really help, they just make people look smart

4. Diffraction gratings divide light into different wavelengths by
   A) Having an index of refraction that depends on wavelength
   B) Absorbing and reemitting light of different wavelengths at different angles
   C) Interfering light from many closely spaced slits
   D) Having reflective surfaces tuned to the different wavelengths at different angles
   E) Combining different polarizations of light depending on wavelengths

5. According to Planck, the energy of particles of light is given by which formula?
   A) $E = hf$  B) $E = \frac{h}{f}$  C) $E = \frac{f}{h}$  D) $E = \frac{h}{\lambda}$  E) $E = h\lambda$

6. Which of the following is the correct term for how much a telescope makes something look bigger?
   A) Focal length
   B) Magnification
   C) Angular magnification
   D) Image distance
   E) None of the above

7. Which is an important advantage that mirrors (including curved mirrors) have over lenses for imaging?
   A) They can produce images with no distortion
   B) They can work in vacuum
   C) They are not subject to spherical aberration
   D) They do not overheat from the intense light passing through them
   E) They are not subject to chromatic aberration

8. Which of the following is one of Kirchoff’s Laws?
   A) The total voltage into a vertex must equal the voltage out
   B) The total current into a vertex must equal the current out
   C) The total current added up around a loop must equal zero
   D) The voltage across all components in a circuit must total zero
   E) The voltage drop across one component in a circuit must be exactly balanced by the voltage drop across another component of the circuit
9. When a wave moves from air to a medium like glass, which of the following occurs?
   A) Wavelength stays the same and frequency increases
   B) Wavelength stays the same and frequency decreases
   C) Frequency stays the same and wavelength increases
   D) **Frequency stays the same and wavelength decreases**
   E) Wavelength and frequency both change

10. Rainbows are caused by
   A) Chemicals that absorb certain wavelengths more than others in water
   B) The Greek goddess Iris
   C) Constructive and destructive interference between light bouncing off the front and back of the raindrop
   D) Light that is absorbed by the water and then reemitted in a different direction
   E) **Dispersion, where light of different wavelengths bends by different amounts**

11. A point charge moving perpendicular to a uniform magnetic field will move in what manner?
   A) In a straight line at constant speed
   B) In a straight line, accelerating or decelerating
   C) **In a circle at constant speed**
   D) In a circle either speeding up or slowing down
   E) None of the above

12. Total internal reflection can only occur when
   A) The refracting material contains some metal
   B) You are moving from a low index material to a high index material
   C) **You are moving from a high index material to a low index material**
   D) You are using a long wavelength, since red refracts rotten
   E) You are using a short wavelength, since blue bends best

13. The circuit diagram at right is the symbol for a
   A) Resistor  B) Capacitor  C) Battery  D) Diode  E) **Inductor**

14. The charge of the proton is ___ and the charge of the neutron is ____ .
   A) \(+e, -e\)  B) \(+e, 0\)  C) \(-e, +e\)  D) \(-e, 0\)  E) None of these

15. Fermat’s principle, which says that light takes the shortest time path, can correctly account for
   A) Why light goes in straight lines (only)
   B) The law of reflection (only)
   C) The law of refraction, or Snell’s law (only)
   D) **All of the above**
   E) None of the above
16. Non-ideal batteries differ from ideal batteries primarily in that
   A) They have a voltage function that depends on temperature
   **B) They have internal resistance**
   C) They have voltage that increases the more current is drawn from them
   D) They actually have to switch the direction of their voltage occasionally
   E) They have enormous capacitance due to the charges stored in them

17. If you place electric charge on a conducting sphere, where will the charge go?
   A) It will all go to the center of the sphere
   B) It will spread itself uniformly over the entire volume of the sphere
   C) It will concentrate at the two poles of the sphere
   **D) It will spread uniformly over the surface of the sphere**
   E) It is impossible to put electric charge on a conductor

18. If light goes into a three-corner reflector, so it bounces off of three mutually perpendicular mirrors that meet at a corner, which direction will the thrice-reflected light go?
   A) Backwards exactly along the direction it came
   B) Forwards exactly along the direction it was going
   C) At exactly a 90 degree angle compared to where it was coming from
   D) At exactly a 45 degree angle compared to where it was coming from
   E) None of the above

19. The total magnetic flux out of an enclosed region is
   A) Proportional to the total charge inside the region
   B) Proportional to the total current in the region
   C) Proportional to the rate of change of the electric flux out of the region
   D) Proportional to the rate of change of the current in the region
   **E) Zero**

20. When sketching electric field lines, which of the following is true?
   A) They start on positive charges and end on negative charges (or infinity)
   **B) They start on negative charges and end on positive charges (or infinity)**
   C) They tend to form circles around positive charges
   D) They tend to form circles around current sources
   E) None of the above

21. If you place an object at a distance equal to the focal length for a lens, \( p = f \), then the image will be at \( q = \)
   A) 0       B) \( \infty \)   C) \( 2f \)   D) \( f/2 \)   E) \( -f \)

22. Which of the following has the longest wavelength?
   **A) Microwaves**  B) Ultraviolet  C) X-rays  D) Gamma-rays  E) Infrared
23. To make a circuit that oscillates, the two components you need to combine are
   A) Resistor and capacitor
   B) Resistor and diode
   C) Capacitor and diode
   D) Capacitor and inductor ← Also accepted as correct, not sure how I messed up
   E) Inductor and capacitor

24. When you run current through a resistor, power appears to disappear. Where does the
    power go?
   A) It is stored in electric fields, and can subsequently be recovered
   B) It is stored in magnetic fields, and can subsequently be recovered
   C) It is stored in kinetic energy of the resistor, and can subsequently be recovered
   D) It is converted into heat
   E) It is transmitted to other electrical components in the circuit

25. Which of the following makes capacitors very useful in electrical circuits?
   A) They can vastly increase the voltage in a circuit
   B) They are the most efficient means for storing large amounts of electrical energy; much more so than batteries
   C) They consume power, draining away energy that would otherwise harm circuits
   D) They allow current to flow only one way, not both
   E) They can release their energy very quickly
Part II: Short answer, old material: [20 points]

Choose two of the following three questions and give a short answer (2-3 sentences) (10 points each)

26. Given the four \( q = 1 \ \mu \text{C} \) charges arranged in a box as sketched at right, with all four charges \( a = 1.00 \ \text{cm} \) from the center, describe the electric field at the center, and the electric potential at the center. Include whether the quantity would be positive, negative or zero, or which direction it points. No formulas are necessary.

The electric field is proportional to \( q/a^2 \), but \( q \) and \( a \) are the same for all four charges. The electric fields from each of the four charges will point away from those charges, but this means that in the center there will be one each pointing to the right, left, up, and down. These will cancel, so there is no electric field at the center.

The electric potential, in contrast, is a scalar. It is given by \( kq/a \) for each charge. Since it is a scalar and has no direction, these simply add for a total electric potential of \( 4kq/a \), so we have a positive electric potential.

27. What is Ohm’s Law? Explain how, in principle, measuring the current through a resistor can effectively act like a thermometer.

Ohm’s law is \( \Delta V = IR \). If you apply a voltage to a resistor and measure the current, you can use this to calculate the resistance. Since resistance commonly depends on temperature, you can then deduce the temperature based on the way the resistance changes. One would apply a known voltage, measure the current, deduce the resistance, and then if the relationship between resistance and temperature is known, you could figure out the temperature.

28. Explain qualitatively how to build an inductor. Give the fundamental formula relating the change in current to the EMF in an inductor and the inductance.

An inductor is simply a solenoid, or coil of wire, though its inductance can usually be significantly increased by including an iron core inside the coil. The inductance, EMF, and rate of change of current are related by the equation:

\[ \varepsilon = -L \frac{dI}{dt} \]
Part III: Short answer, new material: [20 points]

Choose three of the following four questions and give a short answer (2-3 sentences, or comparable) (10 points each)

29. Explain qualitatively what spherical aberration and chromatic aberration are.

Spherical aberration occurs when the light rays are not so close to the center or such small angles that we can use the small angle approximation. It can make images blurry. Chromatic aberration has to do with the fact that light consists of many wavelengths, and most glasses will have an index of refraction that depends on wavelength. This means different colors focus differently, leading to colors getting slightly separated in the image.

30. One of the formulas given above is for Brewster’s angle, $\tan \theta_p = n_2 / n_1$. Explain the significance of Brewster’s angle.

When light passes from one medium to another, it is partly refracted and partly reflected. The amount also depends on polarization. However, at Brewster’s angle, one of the two polarizations is completely refracted, and there will be none reflected. If unpolarized light reflects off the boundary at Brewster’s angle, the reflected light will be completely polarized.

31. Describe qualitatively the interference pattern that occurs when you send a light wave through two narrow slits. If you dim the light so that only one or a few photons passes through the slits at any given moment, does it affect the interference pattern?

When you send light through two narrow slits, you get alternating light and dark bands, with the bright bands at angles given by $\sin \theta_{\text{bright}} = m\lambda / d$, where $d$ is the distance between the slits, $\lambda$ is the wavelength, and $m$ is any integer. If you change $m$ to a half-integer, you will get the positions of the dark bands. This interference will work no matter how low the rate of photons is, even if you let the photons through one at a time.

32. A hydrogen atom consists of a proton which attracts an electron. To minimize the electron’s potential energy, it should be at the origin. To minimize the electrons kinetic energy, it should have zero momentum. Explain, using the uncertainty principle, why you can’t specify both of these things. You should include one formula or inequality.

According to Heisenberg’s uncertainty principle, it is impossible to specify the position and the momentum of any particle simultaneously. In fact, the product of the uncertainties is bounded by the inequality $\Delta x \Delta p \geq \frac{1}{4\pi} \hbar$. 
Part IV: Calculation, old material: [60 points]

Choose two of the following three questions and perform the indicated calculations (20 points each)

33. An electron with charge \( q = -1.602 \times 10^{-19} \text{ C} \) and mass \( m = 9.109 \times 10^{-31} \text{ kg} \) is moving to the right at a velocity \( v = 2.00 \times 10^5 \text{ m/s} \) in an electric field pointing upwards of magnitude \( E = 1.8 \times 10^4 \text{ V/m} \).

(a) What is the electric force (magnitude and direction) on the electron? What is the electron’s acceleration?

We use \( \mathbf{F} = q \mathbf{E} \) to calculate the force from the electric field, which is just
\[
F = |q| E = \left(1.602 \times 10^{-19} \text{ C}\right) \left(1.8 \times 10^4 \text{ V/m}\right) = 2.884 \times 10^{-15} \text{ N}
\]
Because the charge is negative, it will point opposite to the direction of the electric field, or downwards. The acceleration will be the same direction and can be found from \( \mathbf{F} = m \mathbf{a} \), so we have
\[
a = \frac{F}{m} = \frac{2.884 \times 10^{-15} \text{ N}}{9.109 \times 10^{-31} \text{ kg}} = 3.166 \times 10^{15} \text{ m/s}^2.
\]

(b) Suppose that, in fact, there is a magnetic field that exactly compensates for the electric field, so that the electron is not in fact accelerating. What is the magnitude of the magnetic field \( B \) required to cancel out the force from the electric field?

The magnetic field causes an additional force \( \mathbf{F} = q \mathbf{v} \times \mathbf{B} \). Assuming the magnetic field is perpendicular to the velocity, this will have a magnitude \( F = qvB \). We want it’s magnitude to match that of the electric field, so \( qvB = qE \), or, solving for the magnetic field,
\[
B = \frac{E}{v} = \frac{1.8 \times 10^4 \text{ V/m}}{2.00 \times 10^5 \text{ m/s}} = 0.090 \text{ T}.
\]

(c) Which direction should the magnetic field point to cancel out the force from the electric field?

This can be done two ways. We want the total force to be zero, so we have
\( 0 = q \mathbf{E} + q \mathbf{v} \times \mathbf{B} \), which can be rewritten as \( \mathbf{v} \times \mathbf{B} = -E \). If we choose \( \mathbf{B} \) out of the paper, then \( \mathbf{v} \times \mathbf{B} \) will be downwards, cancelling the electric field.

Alternatively, we know the force from the electric field is downwards, so the magnetic force must be upwards. The magnetic force is given by \( \mathbf{F} = q \mathbf{v} \times \mathbf{B} \). If \( \mathbf{B} \) is into the paper, then \( \mathbf{v} \times \mathbf{B} \) be upwards, but the force is \( \mathbf{F} = q \mathbf{v} \times \mathbf{B} \), and the negative charge reverses this and would make the magnetic force downwards, which is wrong. So instead we have \( \mathbf{B} \) out of the paper, which fixes this and makes it work out right.
34. An external battery charges a set of three capacitors with a total of 12.00 mC of charge. At $t = 0$, the switch is suddenly switched to the down position, and the capacitors discharge through the combination of resistors sketched.

(a) What is the effective capacitance of the three capacitors?

The 10 μF and 15 μF capacitors are in series, because they are connected at one end, and nothing else is connected there. We therefore use the rules for capacitors in series, namely,

$$\frac{1}{C_{\text{tot}}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{10 \, \mu F} + \frac{1}{15 \, \mu F} = \frac{3 + 2}{30 \, \mu F} = \frac{1}{6 \, \mu F},$$

$$C_{\text{tot}} = 6 \, \mu F.$$ This pair now is in parallel with the 4 μF capacitor, so the combination of all three has

$$C = C_{\text{tot}} + C_3 = (6 \, \mu F) + (4 \, \mu F) = 10 \, \mu F.$$

(b) What is the effective resistance of the two resistors?

The resistors are connected at one end with nothing else there, so they are in series. Hence the total resistance is

$$R = R_1 + R_2 = (8 \, k\Omega) + (2 \, k\Omega) = 10 \, k\Omega.$$ 

(c) What is the time constant for the discharge?

The time constant is given by $\tau = RC$, so we have

$$\tau = RC = (10 \times 10^3 \, \Omega)(10 \times 10^{-6} \, F) = 0.100 \, s.$$ 

(d) At what time will there remain 1.00 mC of charge on the combination of capacitors?

The formula for the remaining charge at any given time is $Q = Q_0 e^{-t/\tau}$, so we have

$$1.00 \, mC = (12.00 \, mC) e^{-t/\tau},$$

$$e^{-t/\tau} = \frac{12.00 \, mC}{1.00 \, mC} = 12.0,$$

$$t/\tau = \ln(12.0) = 2.48,$$

$$t = 2.48\tau = 2.48(0.100 \, s) = 0.248 \, s.$$
35. In a certain region of space, the magnetic field is pointing straight into the paper. It increases uniformly from $B = 0.020 \ T$ at $t = 0.000 \ s$ to $B = 0.140 \ T$ at $t = 0.020 \ s$. In this region, there is a loop of wire in the shape of a semi-circle of radius $R = 4.00 \ cm$ with $N = 50$ turns.

(a) Find the magnetic flux through the loop for a single turn at the start and finish.

Magnetic flux is area times magnetic field. The area is half the area of a circle, or

$$A = \frac{1}{2} \pi R^2 = \frac{1}{2} \pi (0.04 \ m)^2 = 0.00251 \ m^2$$

If you then multiply this times the magnetic field, we find

$$\Phi_B(0) = (0.00251 \ m^2)(0.02 \ T) = 5.03 \times 10^{-5} \ Wb,$$

$$\Phi_B(0.02 \ s) = (0.00251 \ m^2)(0.14 \ T) = 3.519 \times 10^{-4} \ Wb.$$ (b) Find the rate of change of the flux through a single loop, and the EMF generated in all 50 turns of the loop. Which direction does this EMF push the current?

Since the magnetic field is changing uniformly, we can find the rate of change by dividing the change in flux by the time, so

$$\frac{d\Phi_B}{dt} = \frac{\Delta \Phi_B}{\Delta t} = \frac{3.519 \times 10^{-4} \ Wb - 5.03 \times 10^{-5} \ Wb}{0.02 \ s} = 0.0151 \ V.$$ This is also the EMF from a single loop, up to sign. The EMF from all 50 turns is 50 times larger, or $0.754 \ V$.

We can figure out the direction by using Lenz’s law, which suggests that the current will attempt to recreate the magnetic field. Since the magnetic field is increasing downwards, it will try to make a magnetic field upwards. This requires a current flowing counter-clockwise around the loop.

(c) If the wire has a total resistance of $R = 12.0 \ \Omega$, what is the current running around the wire?

We simply use the formula $\Delta V = IR$, so that

$$I = \frac{\Delta V}{R} = \frac{0.754 \ V}{12.0 \ \Omega} = 0.0628 \ A = 62.8 \ mA.$$
Part V: Calculation, new material: [60 points]

Choose three of the following four questions and perform the indicated calculations (20 points each)

36. A light beam enters a rectangular glass block from the left at a 45° angle compared to the normal, refracting so that it is moving at a 30° angle inside the glass. It then reflects off the top surface and then refracts back out on the right, making an angle \( \gamma \) compared after it exits the glass.

(a) What is the index of refraction \( n \) of the glass?

We can use Snell’s law to find this at the first air/glass boundary. We therefore have:

\[
1 \sin 45^\circ = n \sin 30^\circ ,
\]

\[
n = \frac{\sin 45^\circ}{\sin 30^\circ} = \frac{\frac{\sqrt{2}}{2}}{\frac{1}{2}} = \sqrt{2} = 1.4143 .
\]

(b) Find all the labeled angles \( \alpha, \beta, \gamma, \) and \( \delta \) that the light follows on this path

We first see that from the triangle with both 30° and \( \alpha \) that we have a right triangle with one angle 30°, so the other must be \( \alpha = 90^\circ - 30^\circ = 60^\circ \). For the reflection, we simply use the law of reflection, which means that \( \beta = \alpha = 60^\circ \). We then have another right triangle, and we conclude that \( \gamma = 90^\circ - 60^\circ = 30^\circ \). And finally, we use Snell’s law again to obtain

\[
1 \sin \delta = n \sin 30^\circ = \sqrt{2} \frac{1}{2} = \frac{\sqrt{2}}{2},
\]

\[
\delta = 45^\circ.
\]

(c) Does any of the light leak out instead of reflecting at the second step, as indicated by the gray arrow? Why not, or, if it does, find the angle \( \varepsilon \) at which it escapes.

We can try using Snell’s law again, for which we would have

\[
1 \sin \varepsilon = n \sin 60^\circ = \sqrt{2} \frac{\sqrt{3}}{2} = \frac{\sqrt{6}}{2} = 1.225 .
\]

However, the sine of no angle can exceed 1. This implies that, in fact, none of the light will escape, and we have total internal reflection.
37. An object denoted by the arrow at right is imaged through the plano-concave lens also sketched at right. The lens is made of glass with index of refraction \( n = 1.6 \) and will be used in air. One of the foci of this lens is sketched at right.

(a) Is this a converging or diverging lens? What is the focal length of this lens (including the sign)? Mark the position of the other focal point.

The lens is thinner in the middle than at the edges, which implies a diverging lens with a negative focal length. By measuring the distance from the lens, we can estimate the focal length as \( f = 10.0 \text{ cm} \). The other focal point is an equal distance on the other side of the lens, as sketched.

(b) Draw three light rays from the tip of the object, and sketch how they will behave after passing through the lens. You may add dashed lines or whatever is necessary to help guide your lines as accurately as possible.

One ray is sketched going through the vertex (it goes straight), another starts off parallel and then continues directly away from the near vertex, and the final one is headed straight for the far vertex but then comes out parallel. These are all sketched in.

(c) Sketch in the approximate position and size of the image. Is the image upright or inverted? Is it enlarged or reduced? Is it real or virtual?

We traced back two of the outgoing rays and find they intersect with the one going through the vertex consistently at about the position of the dark arrow sketched in. It is obvious that it is upright and reduced. If we were viewing it from the side where the light comes out, it would be behind the lens, and hence virtual.

(d) Calculate the radius of curvature \( R \) for this lens in cm.

We use the lensmaker’s equation. The flat side essentially has a flat surface, which effectively has a radius \( R_2 = \infty \). We therefore have

\[
\frac{1}{f} = \left( \frac{n}{1} - 1 \right) \left( \frac{1}{R_1} - \frac{1}{\infty} \right) = \frac{0.6}{R_1},
\]

\[
R_1 = 0.6 f = 0.6(-10.0 \text{ cm}) = -6.0 \text{ cm}.
\]

The radius came out negative, which just represents the fact that the surface is concave.
38. An object is placed a distance \( p = 25.0 \) \( \text{cm} \) in front of a curved mirror with focal length \( f = 20.0 \) \( \text{cm} \)

(a) What is the radius of curvature of the mirror? Is it convex or concave?

For mirrors, concave mirrors have positive focal length, hence it will be concave. We then use the mirror formula for radius, namely \( f = \frac{1}{2} R \) and solve for the radius:

\[
R = 2f = 40.0 \text{ cm}
\]

(b) Find the position of the image of the object. Is it in front of the mirror or behind it? Is it a real or virtual image?

We use the image equation

\[
\frac{1}{f} = \frac{1}{p} + \frac{1}{q},
\]

\[
\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{20.0 \text{ cm}} - \frac{1}{25.0 \text{ cm}} = \frac{0.05 - 0.04}{\text{cm}} = \frac{0.01}{\text{cm}},
\]

\[q = \frac{1}{0.01} \text{ cm} = 100 \text{ cm}.
\]

Since it came out positive, it is on the side the light comes out, which is the front, and it’s a real image.

(c) Find the magnification of the object \( M \). Is it inverted or upright? Is it enlarged or reduced?

The magnification is given by

\[
M = \frac{-q}{p} = -\frac{100 \text{ cm}}{25.0 \text{ cm}} = -4.0
\]

Since it came out negative, it is inverted. Since it came out bigger than one in magnitude, it is enlarged (by a factor of 4).
39. Protons of mass \(1.672 \times 10^{-27} \text{ kg}\) is accelerated to an energy of \(E = 0.500 \text{ eV}\).  
(a) Calculate the velocity (in \(\text{m/s}\)) and momentum (in \(\text{kg \cdot m/s}\)) of the protons.

The kinetic energy is given by \(E = \frac{1}{2}mv^2\), or solving for the velocity, we have

\[
v^2 = \frac{2E}{m} = \frac{2(0.500 \text{ eV})(1.602 \times 10^{-19} \text{ J/eV})}{1.672 \times 10^{-27} \text{ kg}} = 9.58 \times 10^7 \text{ m}^2 / \text{s}^2 ,
\]

\[
v = \sqrt{9.58 \times 10^7 \text{ m}^2 / \text{s}^2} = 9790 \text{ m/s}
\]

We then find the momentum using \(p = mv\) to find

\[
p = mv = (1.672 \times 10^{-27} \text{ kg})(9790 \text{ m/s}) = 1.637 \times 10^{-23} \text{ kg \cdot m/s}
\]

(b) Find the wavelength of the protons.

The wavelength is found from the deBroglie relation, that is, \(\lambda p = h\), so we have

\[
\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34} \text{ J \cdot s}}{1.637 \times 10^{-23} \text{ kg \cdot m/s}} = 4.05 \times 10^{-11} \text{ m}
\]

(c) The protons are now sent through a slit with a width of \(a = 2.00 \text{ nm}\). Find the position of the smallest angle \(\theta\) where there is no chance of finding a proton.

The angles where we have destructive interference are given by \(\sin \theta_{\text{dark}} = m\lambda / a\), where \(m\) is any non-zero integer. The first one occurs when \(m = 1\), so

\[
\sin \theta = \frac{4.05 \times 10^{-11} \text{ m}}{2.00 \times 10^{-9} \text{ m}} = 0.0202 ,
\]

\[
\theta = 1.160^\circ
\]