
\[ H = \sum_{i=1}^{3} \left( \frac{p_i^2}{2m} + \frac{1}{2} \hbar \omega^2 x_i^2 \right) \]

(a) [5] If the particles are not identical, find the three lowest energies, give me a list of all quantum states having those energies, and tell me the degeneracy (number of states with that energy) for each.

Each harmonic oscillator has an energy of \( E_i = \hbar \omega \left( n_i + \frac{1}{2} \right) \), so for the three harmonic oscillators together, the energy is

\[ E = \hbar \omega \left( n_1 + n_2 + n_3 + \frac{3}{2} \right) \]

for the state \( |n_1n_2n_3\rangle \) The smallest energies come when \( n_1 + n_2 + n_3 \) is as small as possible, or 0, 1, or 2. The table at right gives the full answer to this question. The “#” tells the degeneracy.

(b) [5] Repeat part (a) if the three particles are all bosons. I still want three energies, but I am not guaranteeing that the energies will be the same.

The energies will be the same, we just have to symmetrize the wave functions. This reduces the degeneracy considerably.

(c) [5] Repeat part (a) if the three particles are all fermions.

This time the ground state is the anti-symmetrized \( |012\rangle \), which has a higher energy. We can then go up by one unit in \( n_1 + n_2 + n_3 \) to get the next two energies. It turns out only the third energy is degenerate.
4. [10] Look up and/or calculate the number density of atoms in copper. Assume there is one conduction electron per atom.

(a) [6] What is the Fermi energy (in eV) and Fermi degeneracy pressure (in GPa) for electrons at this density?

Using Wikipedia, we can find the atomic mass of copper and the density, which together with Avogadro’s number tells us the number density of atoms.

\[ n = \frac{\rho}{m} = \frac{8.96 \text{ g/cm}^3}{63.546 \text{ g/mol}} \times 6.022 \times 10^{23} \text{ mol}^{-1} = 8.49 \times 10^{22} \text{ cm}^{-3} = 8.49 \times 10^{28} \text{ m}^{-3} \]

We now put this into our formula for the Fermi energy

\[ E_F = \frac{\hbar^2}{2m} (3\pi^2 n)^\frac{2}{3} = \frac{\left(1.0546 \times 10^{-34} \text{ J} \cdot \text{s}\right)^2}{2 \cdot 9.1094 \times 10^{-31} \text{ kg}} \left[3\pi^2 \cdot 8.49 \times 10^{28} \text{ m}^{-3}\right]^\frac{2}{3} = 1.128 \times 10^{-18} \text{ J} \]

\[ = \frac{1.128 \times 10^{-18} \text{ J}}{1.602 \times 10^{-19} \text{ J/eV}} = 7.044 \text{ eV} \]

The degeneracy pressure is

\[ P_F = \frac{\hbar^2}{2m} n E_F = \frac{2}{3} \left(1.128 \times 10^{-18} \text{ J}\right) (8.49 \times 10^{28} \text{ m}^{-3}) = 3.831 \times 10^{10} \text{ N/m}^2 = 38.31 \text{ GPa} \]

(b) [4] The bulk modulus is defined as \( K = -V \left(\frac{\partial P}{\partial V}\right) \). Find a formula for the bulk modulus due to electron degeneracy pressure. Use this to estimate the bulk modulus of copper (in GPa). Look it up somewhere for copper and compare the result.

Starting from the formula for degeneracy pressure, we have

\[ K_F = -V \frac{\partial}{\partial V} \left[ \frac{\hbar^2}{2m} (3\pi^2)^\frac{2}{3} N^{-\frac{2}{3}} \right] = \frac{5}{3} \frac{\hbar^2}{5m} (3\pi^2)^\frac{2}{3} N^{-\frac{2}{3}} = \frac{5}{3} P_F = 63.84 \text{ GPa} \]

The correct experimental value, according to Wikipedia, is 140 GPa, so this is about a factor of 2.2 too low. I must assume that other electrons besides the conduction electrons are contributing to the pressure.