Lecture 2: What is Calculus?

What is π?

Circle of radius 1

\[ A_n = n \cdot \cos \left( \frac{(n-2)\pi}{2n} \right) \cdot \sin \left( \frac{(n-2)\pi}{2n} \right) \]

\[ A_3 = \frac{3\sqrt{3}}{4} \approx 1.299 \]
\[ A_5 \approx 2.378 \]
\[ A_7 = 2.736 \]

\[ A_4 = 2 \]
\[ A_6 \approx 2.598 \]
\[ A_8 = 2.83 \]

\[ A_{50} = 3.1333 \]
\[ A_{100} = 3.13953 \]
\[ N_{1000} = 3.14157 \]

As \( n \) gets large, \( A_n \approx \pi \).

\[ \lim_{n \to \infty} A_n = \pi \approx 3.14159 \ldots \]
What is $\pi$?

Circle length $= 2\pi$
Square length $= 8$
Circle length $= 2\pi$
Circle length $= 2\pi$

"Take limits" $\Rightarrow 8 = 2\pi \Rightarrow \pi = 4$ !

Something is clearly wrong. Calculus is the precise mathematical theory of limits.

Cat in the hat

$3 = 1 + 2$
$\frac{3}{2} = 2 + \frac{1}{3} + \frac{2}{3}$
$3 = 2 + \frac{3}{3} + \frac{2}{9} + \frac{1}{9}$

$3 = 2 + \frac{3}{3} + \frac{2}{9} + \frac{1}{9} \ldots$

$3 = 2 \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n$

Summation notation