Choose 3 out of the following 5 questions, clearly indicating which you have chosen. All problems have equal weight. All quantities are expressed in SI units.

1. In a vacuum diode, electrons are “boiled” off a hot cathode, at potential zero, and accelerated across a gap to the anode, which is held at positive potential \( V_0 \). The cloud of moving electrons within the gap (called space charge) quickly builds up to the point where it reduces the field at the surface of the cathode to zero. From then on a steady current \( I \) flows between the plates.

Suppose the plates are large relative to the separation (\( A \gg d^2 \) in the figure below), so that edge effects can be neglected. Then \( V, \rho \) (charge density), and \( v \) (the speed of the electrons) are all functions of \( x \) alone.

(a) Assuming the electrons start from rest at the cathode, what is their speed at point \( x \), where the potential is \( V(x) \)?
(b) What is the relation between \( \rho, v, \) and \( I \) in the steady state?
(c) Obtain a differential equation for \( V \) in terms of \( x, V_0, \) and \( d \).
(d) Solve this equation for \( V \) as a function of \( x, V_0, \) and \( d \).
(e) Find \( \rho \) as a function of \( x \).
2. Find the average magnetic torque on a vinyl phonograph record of radius R with a static charge \( \sigma \) distributed uniformly on its top surface and rotating at \( \omega \) with its axis of rotation at a \( \theta \) angle to the earth’s field.

![Diagram](image1.png)

3. A cylindrical capacitor (inner radius \( a \) and outer radius \( b \)) is placed in a tank of dielectric oil. The oil has a susceptibility of \( \chi_e \) and a mass density \( \rho \). If the potential of the inner cylinder is held constant at \( V \) and the outer cylinder is grounded, then to what height, \( h \), will the oil rise within in the cylinders?

![Diagram](image2.png)
4. Consider a cylindrical rod of copper of length $L$ and radius $a$. It is in a spatially uniform magnetic field parallel to its long axis. The magnetic field is changing slowly with time. The derivative $dB/dt$ is a constant. Determine the total amount of Joule heating (in watts) in the copper rod. For the resistivity, use the value $1.6 \times 10^{-6} \ \Omega \cdot m$. Assume the rod is 0.3 m long and 0.02 m in radius and that the magnetic field is changing at 1 Tesla per hour. Determine by what factor the heating is reduced when the rod is replaced by a bundle of $N$ smaller radius rods insulated from each other, whose total mass equals that of the original rod.

5. (a) A slab of non-magnetic material has permittivity $\varepsilon = 2 \times 10^{-11} \ C^2/Nm^2$ at frequency $\omega = 2.8 \times 10^{15} \ s^{-1}$. What is the speed of electromagnetic radiation of this frequency in the material?

(b) Suppose the light is a plane-wave laser beam with irradiance (time-averaged Poynting vector) of $10^8$ watts/m$^2$ in the material. What is the amplitude of magnetic induction $B_0$?

(c) At the back surface of the material is a plane interface with vacuum, on which the light is incident normally. What is the radiation pressure exerted on the back interface of the material?

(d) What is the value of transmitted irradiance and what is its color?

Some fundamental constants:

$\varepsilon_0 = 8.85 \times 10^{-12} \ C^2/Nm^2$

$\mu_0 = 4\pi \times 10^{-7} \ N/A^2$

$q_e = 1.6 \times 10^{-19} \ C$

$m_e = 9.11 \times 10^{-31} \ kg$