Please solve 3 of the following 4 problems.

1. Two masses, $m_1 = m_2 = M$, are supported by a horizontal frictionless surface and connected by a massless spring with spring constant $k$ which has an equilibrium separation $L$.

At time $t = 0$, the initial coordinates of the masses are 

$$\mathbf{r}_1(0) = 0 \quad \text{and} \quad \mathbf{r}_2(0) = L\hat{x}$$

and the initial velocities are 

$$\frac{d\mathbf{r}_1}{dt}(0) = v_0\hat{x} \quad \text{and} \quad \frac{d\mathbf{r}_2}{dt}(0) = 0.$$

(a) What is the total energy of the system at time $t = 0$?
(b) For time $t > 0$, find $\mathbf{r}_1(t)$ and $\mathbf{r}_2(t)$.
(c) Using the results of part (b), evaluate the total energy of the system at time $t > 0$ and compare the result with part (a).

2. Suppose you have a chamber containing two different gases, divided by a movable frictionless partition of mass $m$ and area $A$. The partition is moved a small horizontal distance $x$ and then released. Assume the gas pressure for $x = 0$ is $P_0$ on both sides of the partition, but the volumes of the two sides may be different. In general, the pressure $P$ will change with displacement of the partition. Find the frequency of small oscillations if the chamber is:

(a) isothermal with $PV = \text{constant}$

(b) adiabatic with $PV^\gamma = \text{constant}(\gamma = \frac{5}{3})$.

For both cases, use the approximation that the oscillations are small enough to only keep terms of the lowest order. Also, ignore the effect of the kinetic energy of the gases. You can either solve this by finding the generalized equation of motion or by finding the force equation directly.
3. A thin rigid rectangular plate of mass $M$ and sides length $a$ and $b$ is attached to a thin massless rod as shown in the figure and set to rotate at a constant angular velocity $\omega$ about the rod.

(a) Using symmetry arguments and a diagram, what are the directions of the principal axes of the thin plate?

(b) What is the moment of inertia tensor for the plate? Be sure to express your answer using the principal axis components. *Specify whether your answer is for rotation about the center of mass or the point of rotation in this problem.*

(c) What is the total kinetic energy of the plate? *Recall that the displaced axis theorem is $I_{ij} = I_{ij}^{CM} + M(d^2\delta_{ij} - d_i d_j)$.*

(d) What is the angular momentum of the plate? Give its magnitude and its direction relative to the rotation axis.

4. Consider the system below which consists of two masses $m_1$ and $m_2$ under the influence of constant gravitational acceleration $g$ and two springs with spring constants $k_1$ and $k_2$ as shown. Assume that all motion is confined to the vertical direction.

(a) Draw a legible diagram illustrating your choice of coordinates.

(b) Find the Lagrangian for the system.

(c) Write out the Hamiltonian for this system. What are Hamilton’s equations of motion for this system? We are not asking for solution to the equations of motion.

(d) Find the frequencies of oscillations of the normal modes in terms of $m_1, m_2, k_1, k_2$. 