2014 E&M
Choose 3 of the 5 problems to solve.

1) Consider an infinitely long cylinder of radius \( R \) and embedded with a constant charge density \( \rho \). Inside this cylinder is a cylindrical cavity of radius \( b \) placed at a distance \( a \), off the axis as shown.

![Diagram of a cylinder with a cavity](image)

The cylinder is moving at velocity \( v \) (relative to the laboratory frame, \( S \)) along its axis. Calculate:

a) the electric field, \( E \), within the cavity in the reference frame of the moving cylinder, \( S' \).

b) the magnetic field, \( B \), within the cavity in the laboratory reference frame, \( S \).

2) Two very large metal plates are held at a distance \( d \) apart. One is at potential zero and the other at potential \( V_0 \). A metal sphere of radius \( a \) (\( a \ll d \)) is sliced in two, and one hemisphere is placed on the grounded plate. If the region between the plates is filled with weakly conducting material of conductivity \( \sigma \), determine an expression for the current that flows to the hemisphere.

![Diagram of two plates with a hemisphere](image)
3) Electromagnetic radiation with the electric field given as:

\[ \vec{E} = E_0 \exp(i(kz - \omega t))\hat{y} \]

is incident on an atom of polarizability \( \alpha \) at position (0, 0, 0). Treating the polarized atom as a dipole, calculate the electric and magnetic fields of the radiated wave at large distance D:

a) On the y-axis

b) On the x-axis

4) A material has complex refractive index 3 + 2i at the frequency of 532 nm light. In SI units, the permittivity of vacuum is \( \varepsilon_0 = 8.85 \times 10^{-12} \, \text{C}^2/\text{Nm}^2 \)

a) What is its dielectric constant at this frequency?

b) What is the reflectance of this material for 532 nm light at normal incidence in vacuum?

c) What is the phase shift in this plane-polarized light upon reflecting at normal incidence?

d) What is the penetration depth at which the electric field is reduced to \( 1/e \) of its value just inside the surface, i.e. the skin depth?

e) What is the absorption coefficient \( \alpha \)?
(as defined in the Beer’s law expression \( I = I_0 e^{-\alpha x} \))

f) How much radiation pressure is exerted on this material if the incident light in vacuum has irradiance 100 W/m\(^2\)?
5) A charged parallel-plate capacitor (with uniform electric field $\mathbf{E} = E \hat{z}$) is placed in a uniform magnetic field ($\mathbf{B} = B \hat{x}$) as shown.

\begin{center}
\includegraphics[width=0.5\textwidth]{diagram.png}
\end{center}

a) Find the electromagnetic momentum in the space between the plates.

b) Now connect the plates with a resistive wire placed along the $z$-axis. The plates will slowly discharge. The current through the wire will experience a magnetic force. What is the total impulse delivered to the system?