Computational 3D Imaging: Sparse Recovery and PSF Engineering

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Outline

Two Related Projects

1. Compressed sensing 3D data sparse reconstruction with nonconvex regularization
   - 3D information from 2D images

2. Single frame (snapshot) 3D imaging based on Point Spread Function (PSF) engineering
   - Depth from defocus
   - Wavefront phase coding
Some Goals of 3D Optical Imaging

- To seek the complete structure of objects in our surroundings
- To extract information from the image data such as:
  - Object distance from the camera as part of 3D localization
  - Brightness, orientation, and shape of objects
  - Includes our work on specular and diffuse BRDF computations for object characterization using hyperspectral and polarization imaging
Hyperpectral Imaging (HSI)

• Spectral imagers capture a 3D datacube (tensor) containing:
  ➤ 2D spatial information: x-y
  ➤ 1D spectral information at each spatial location: $\lambda$

• Pixel intensity varies with wavelength bands - provides a spectral trace of intensity values.

• We add polarization to identify object shape, orientation and metallic surfaces.
HSI: Spectral (beyond RGB) Imaging at Wavelengths $\lambda$

Electromagnetic radiation: energy in the form of electromagnetic waves.

Electromagnetic spectrum: the entire family of electromagnetic radiation.

Figure 1: $\lambda$ generally ranges between 400 and 2500 nanometers.
Different materials produce different electromagnetic radiation spectra.

After adjusting for sensor, atmospheric, and terrain effects.
Polarization by Reflection

- Unpolarized light can be polarized, either partially or completely, by reflection.
- The amount of polarization in the reflected beam depends on the angle of incidence.
Some Applications of Hyperspectral Imaging (HSI) and Polarization

- Environmental remote sensing, e.g., monitoring disasters, chemical/oil spills, etc.
- Military target discrimination
- Astrophysics
- Biomedical optics, medical microscopy, etc.
- Remote surveillance for defense & security, e.g., imaging a compound in western Pakistan
AFOSR Project

- **Air Force (AFOSR) Funded Project**

**Space Object Identification with Spectral Imagers**

Analysis of reflectance.

**Figure:** Reflectance of an object pixel results from additive reflectance of its constitutive elements.
Figure 2: Application: Satellites & Large Debris Objects Around Earth. (About 1,900 active satellites to maintain, and debris to monitor - international cooperation, huge expenditures)
Space Situational Awareness (SSA) by Monitoring Space Objects

- ‘Listen’ (laser enabled vibrometry)
- ‘Smell’ (chemical sensing with spectral imaging)
- ‘Touch’ (scatterometry/polarimetry for surface texture information)
- ‘See’ (by sequential speckle <video> imaging)
- ‘Object Laser Ranging’ (Ground based LiDAR)
- ‘Characterize Materials’ for SOI (hyperspectral imaging) (polarimetric imaging)
Current AFOSR Project


- Develop algorithms and statistical performance metrics as design tools for high dimensional object tracking systems. Tracking debris: 3D imaging with PSF engineering.

A US Air Force Observatory on Haleakala in Maui
• **Spectro-Polarimetric Imaging (SPI)**

  - 3D hyperspectral & 1D polarization. Data is a 4D tensor.
  - Spectral traces identify **materials**.
  - Polarizations identify **object shape, orientation, glint**.
  - **Object characterization using spectrally compressive polarimetric image data**, funded by AFOSR grant to UNM (Physics), Duke (ECE) and WFU (Math & CS). **Snapshot spectro-polarimetric cameras** developed in ECE at Duke.

Sparse recovery by “nonconvex optimization methods.”
• Dave Brady et al., Duke U., part of AFOSR SOI project.

\[ g(x, y) = \int \lambda C_{\lambda}(x, y)f(x, y, \lambda)d\lambda + \epsilon_{\lambda}(x, y). \]

\( C_{\lambda}(x, y) \) = system function. Compressive HSI sensing - requires reconstruction. **3D information from 2D images.**
System modulates 4D tensor array images onto a 2D detector (matrix). Reconstruct 4 polarized images.
Spectro-Polarimetric Compressive Sensing

- Spatial Light Modulator (SLM)-based - snapshot spectro-polarimetric imager forward model.

\[ g(x, y) = \int_{\lambda} C_{\mu}(x, y, \lambda) f_{\mu}(x, y, \lambda) d\lambda + \epsilon_{\lambda, \mu}(x, y). \]

\( C_{\mu} = \) system function, \( \mu = \) linear polarization variable, \( \lambda = \) wavelength, solve inverse problem for \( f_{\mu}. \)

\[
\text{minimize} \left[ \frac{1}{2} \| H_{\mu} f_{\mu} - g \|_2^2 + \tau \| f_{\mu} \|_p^p \right]. \tag{1}
\]

\( H_{\mu} = \) system matrix \( MN \times 4MN \) for each of 4 polarizations, \( g_{\mu} = \) vectorized 2D measurement matrix, \( f_{\mu} = \) vectorized 3D recovery array. Data compression factor = 4, for each polarization angle.
Some References


Overview of TrustSpa-$\ell_p$ Method for Non-Convex Regularization in Sparse Data Recovery

Solve the $\ell_2$-$\ell_p$, $0 < p < 1$, sparse recovery problem:

$$\minimize f \in \mathbb{R}^n \left( \frac{1}{2} \|Af - y\|_2^2 + \tau \|f\|_p^p \right)$$

(2)

$$\minimize_{u, v \in \mathbb{R}^n} \frac{1}{2} \|A(u - v) - y\|_2^2 + \tau \sum_{j=1}^{n} ((u)_j + (v)_j)^p$$

subject to $u, v \geq 0$, (4)

change of variables

$$(u)_j = \log(1 + e^{(\tilde{u})_j}) \quad \text{and} \quad (v)_j = \log(1 + e^{(\tilde{v})_j}),$$
Unconstrained Differentiable Problem

\[
\min_{\tilde{u}, \tilde{v} \in \mathbb{R}^{\tilde{n}}} \Phi(\tilde{u}, \tilde{v}) = \frac{1}{2} \sum_{i=1}^{\tilde{m}} \left[ \sum_{j=1}^{\tilde{n}} (A)_{i,j} \log \left( \frac{1 + e^{(\tilde{u})_j}}{1 + e^{(\tilde{v})_j}} \right) \right] - (y)_i^2
\]

\[
+ \tau \sum_{j=1}^{\tilde{n}} \left( \log(1 + e^{(\tilde{u})_j}) + \log(1 + e^{(\tilde{v})_j}) \right) ^p, (5)
\]

- Reformulated (2) as a smooth unconstrained optimization problem using a change of variables \( f = u - v \), where \( u, v \geq 0 \)
- We apply a limited-memory trust-region method of solution
TrustSpa-$\ell_p$ Method for Non-Convex Regularization in Sparse Data Recovery

- Our numerical results indicate proposed non-convex TrustSpa-$\ell_p$ approach eliminates spurious solutions effectively.

- Faster for our test problems, in comparison to TwIST (TV reg.), and GPSR.

- See the reference below for the use of $\|\nabla f\|_p$ instead of $\|f\|_p$ as the regularization term.


An interesting open problem is the use of $TV^q$-models in our TrustSpa-$\ell_p$ algorithm.
Prototype camera, data acquisition by Tsung-Han Tsai, including polarization

\[ g(x, y) = f_{\parallel}(x, y, \lambda) \cdot H_{\parallel}(x, y, \lambda) + f_{\perp}(x, y, \lambda) \cdot H_{\perp}(x, y, \lambda) \]
Recovery Process at Duke (with TwIST)

Demodulated by Two step Iterative Shrinkage and Thresholding (TwIST) Algorithm

\[ \hat{f} = \arg\min_f \left\{ \frac{1}{2} \| g - Hf \|_2^2 + \lambda H_{TV}(f) \right\} \]

\[ H_{TV}(f) = \sum_k \sum_{i,j} \sqrt{[f(i+1,j,k) - f(i,j,k)]^2 + [f(i,j+1,k) - f(i,j,k)]^2} \]

(We tested TrustSpa-$\ell_p$, for sparse recovery, similar accuracy but much faster than TwIST and GPSR.)
Recovery with TrustSpa-$\ell_p$

\[
\text{minimize } f \left[ \frac{1}{2} \| Hf - g \|_2^2 + \tau \| f \|_p^p \right].
\]  

(3)

$H = \text{system matrix } MN \times 4MN$, $g = \text{vectorized 2D measurement matrix}$, $\hat{f} = \text{vectorized 3D recovered array}$. Much faster than TwiIST. Data compression factor = 4, for each polarization angle.
Work in Progress: Non-lab Data to Test

- (a) Unpolarized reference in RGB.
- (b) Compressed spectro-polarimetric measurement.
- (c) - (f) Reference polarized image channels in RGB.
- (g) - (j) Reconstructions from (b) by TwIST (TV reg) Alg.
2nd Topic: 3D Imaging using a Rotating Point Spread Function Approach to Depth from Defocus

- Part of current AFOSR project: characterizing and monitoring space objects, including debris swarms.

- Additional applications in high-resolution microscopy.

- Fresnel Lens-type spiral phase mask - patent by overall PI S. Prasad.

- PSF rotation with change of focus, enables target localization in 3D. Imaging swarms (clouds) of point sources.

- Need near real-time optimization with single snapshot image.
References and Related Papers

S. Prasad
“Rotating Point Spread Function via Pupil-phase Engineering”

Z. Yu and S. Prasad
“High-numerical-aperture Microscopy with a Rotating PSF”

P. Pauca, S. Prasad, R. Plemmons, T. Torgersen

Two recent papers on finding depth from focus or defocus:

M. Moeller, M. Benning, C. Schönlieb, D. Cremers
“Variational Depth From Focus Reconstruction”

S. Suwajanakorn, C. Hernandez, S. Seitz
“Depth from focus with your mobile phone” (first non-lab implementation)
Proc of the IEEE Conf. on Computer Vision and Pattern Recognition, 2015.
Introduction: A Bit of Imaging Physics

- 3D snapshot imaging via Rotating PSF using **Orbital Angular Momentum** (OAM) states of light beams
- PSF image rotation via “twisted” wavefront phase
- Based on either Gauss-Laguerre beams, or non-diffracting Bessel beams (our approach)
- Can create beam rotation with spiral phase retardation in the beam path
- Use OAM to rotate PSF image with changing axial depth
- **3D information encoded in defocused 2D images**
- Fresnel-zone subdivision of pupil using phase mask achieves depth encoding
Beam Spiraling via Pupil Phase - Leads to Rotating PSF

- **Specifics of Phase Mask:**
  - $M$ Fresnel zones
  - $m^{th}$ Fresnel zone carries $m\phi$ spiral phase
  - Amplitude transmission function –
    \[ K(s, \phi_s, \zeta) \sim \sum_{m=1}^{M} e^{i m(\phi_s - \zeta/M)} J_m \left( 2\pi \sqrt{m/M \, s} \right) \]
- $|K|^2$ rotates w/ misfocus $\zeta$ @ 1/$M$ rad/misfocus
- Spiral-phase mask (SLM can simulate the phase mask)
Defocus blurring is a measure of how the PSF changes for objects at different distances from the lens.

Physically, defocus parameter is phase error, compared to a correctly focusing wave, at edge of aperture (in radians).

Amount of defocus quantified by defocus parameter

\[ \zeta = \frac{\pi}{\lambda} \left( \frac{1}{z_{\text{obj}}} - \frac{1}{z'_{\text{obj}}} \right) R^2, \]

where \( \lambda \) = wavelength of light, \( R \) = radius of the exit pupil, \( z_{\text{obj}} \) = infocus object distance, \( z'_{\text{obj}} \) = actual object distance.

The PSF for defocused objects can be found using the generalized pupil function

\[ p_{\text{gen}}(x, y) = p(x, y) \exp \left[ \frac{\pi}{\lambda} \left( \frac{1}{z_{\text{obj}}} - \frac{1}{z'_{\text{obj}}} \right) (x^2 + y^2) \right] \]
Point Source Images with Spiral Phase Mask

- One full rotation over $\Delta$ (defocus) = $2M\pi$ radians ($\text{DOF} \sim \pm M \lambda/NA^2$)
- Single-lobe PSF with relatively stable shape/size
- High 3D image capture/reconstruction sensitivity even at low-light levels
3D Point Source Images with Conventional Camera and Rotating PSF

- Problem is much simpler than full 3D imaging – highly “compressible” object scene
- Line-of-sight sources resolvable
- Full 3D shape via point-pattern illumination (“ShaRPI”)
PSF Engineering in Microscopy

- Imaging and localizing single molecules with high accuracy in a 3D volume is a challenging task.
- Using the rotating PSF can provide an effective localization strategy and achieve an increased depth for single molecule imaging.
- Snapshot 3D localization can be used to see how structures fold and deform. C. Cremers group PSF eng., Heidelberg
Setup for Microscopy

- **Phase mask** –
  - Spiral phases of different winding numbers in different Fresnel type zones on the plate
  - Can be simulated by a liquid-crystal spatial light modulator (LC-SLM) for initial testing
  - Fabrication possibilities with 2-photon 3D printing to create sub-micron structures
Danger to Space Assets - Clouds of Debris as Point Sources
A Proposed Space Surveillance System

- An integrated radar-optical system mounted on a space asset

- Radar cueing system for debris ("swarm") detection, at $> 1$ km distance, with poor 3D resolution/localization

- Rotating PSF optical telescope cued, in turn, by radar for $< 1$ km ranges
  * Active illuminator turned on cue
  * Extend to multi-spectral system for material characterization
  ➔ Higher 3D resolution/localization and classification of debris via a sequence of snapshots

- Subsequent avoidance/kill maneuver initiation
Setup for Space Object Monitoring

Space-Based Telescope for Debris Localization

Beam Path and Optical Elements of Telescope Set-Up with Optical Relay

Light from laser-illuminated space debris

Focuser

Bending Mirror

4f Optical Relay with Transmissive SPM/SLM

Focal Plane (FPA, side view)

Image (PSF) for three different range values

(Expanded) Top View of FPA
Image Recovery - 3D Point Source Localization

Initial cost function:

\[
C(r, z, f) = \frac{1}{2} \left[ \left\| \sum_{1}^{P} f_i H(r_i, z_i) - G \right\|^2_2 + \tau \left\| (r, z, f) \right\|^q \right] \quad (4)
\]

Here:

- \( P \) = number of point sources,
- \( r = (r_1, \ldots, r_P) \), with \( r_i = (x_i, y_i) \), = transverse location vector,
- \( z = (z_1, \ldots, z_P) \) = depth vector,
- \( f = (f_1, \ldots, f_P) \) = point source energy flux
- \( G = 2D \) camera image data matrix,
- \( H(r_i : z_i) = \) estimated rotating PSF (blur) matrix for the \( ith \) point source.

Vectorize the images for computation. Choosing \( 0 < q < 1 \) and also experimenting with TV reg.
Cost function $C$ is nonconvex with multiple local minimizers, but has global minimizer, based on gradient computations.

Requires initial estimates $(x_i, y_i, z_i, f_i)$ to be chosen close to the global minimizer.

Current work on replacing $C$ by convex surrogate function in progress. Some success by adding a center of mass matching constraint on $G$ and each $H(r_i : z_i)$ term.

Resulting surface plot of transformed cost function.
Preliminary Computations: Computed image PSFs with point sources at various depths: \[ \sum_{1}^{12} f_i H(r_i, z_i) \]
Preliminary Computations: Point source localization results, $P = 4$

True 3D locations - green, Computed 3D locations - blue
Preliminary Computations: Point source flux results, $P = 4$

True - green, Computed - blue (relatively accurate)
Another Project

U.S. National Geospatial-Intelligence Agency (NGA). Involves Pauca, Ple., Torgersen (WFU) and Prasad (UNM).

“Bayesian Inference on Convolutional Neural Networks for Object Characterization and Classification in Multimodal and Compressed Sensing Data”

Pixel-level fusion of HSI and LiDAR data

Targets placed in scene by sponsor for test purposes.
Another Project

- Hong Kong Research Grants Council (RGC). PI Raymond Chan (CUHK). Also involves Ple. (WFU) and Wenxing Zhang (UESTC - Chengdu)

“Mathematics in the Estimation of Point-spread Functions in Ground-based Astronomy through Turbulence”

(a) \( \phi \) = wavefront phase, \( p_x \) and \( p_y \) gradients in \( x \) and \( y \) directions

(b) Reconstructions

\[ \ell^1 \text{ on } p_x, p_y \]

\[ \ell^2 \text{ on } p_x, p_y \]

\[ \text{TV on } \phi \]

15.21dB

23.64dB

24.59dB

29.08dB
Summary

Related 3D Imaging Projects

1. Overview of TrustSpa-$\ell_p$ Method for Non-Convex Regularization in Sparse Data Recovery.
   - Applications to sparse recovery of spectro-polarimetric compressed sensing 3D images.

2. 3D Imaging using a Rotating Point Spread Function Approach to Depth from Defocus.
   - Applications to microscopy and space debris monitoring.
Thank You!