Answer each of the following questions and each of the problems. Points for each question and problem are indicated in red with the amount being spread equally among parts (a,b,c etc). Be sure to show all your work. Use the back of the pages if necessary.
Question 1. (10 points)

(a) Rank the magnitude of the electric field from greatest to smallest in the Figure below

\[ E_C = 0, \]
\[ E_A > E_B, \]

Therefore,
\[ E_A > E_B > E_C \]

(b) The charge on the right is changed to a negative charge of equal magnitude to the one on the left. Draw the field lines of the system of two charges.
Question 2. (10 points) A test charge of $+3 \text{nC}$ is placed in a constant external field directed along the positive $x$-axis with a magnitude of $4 \times 10^6 \text{N/C}$.

(a) What is the direction of the force on this charge?

$$\mathbf{F} = q\mathbf{E} = (+)(+\mathbf{x})$$

Positive $x$-direction

(b) If the test charge is replaced by a negative test charge with $q = -3 \text{nC}$, what is the direction of the force on this negative charge?

$$\mathbf{F} = q\mathbf{E} = (-)(+\mathbf{x}) = (-\mathbf{x})$$

Negative $x$-direction

(c) When the positive test charge is replaced by the negative one, what happens to the external electric field, does it increase significantly, decrease significantly, or essentially stay the same?

Essentially stay the same, the electric field is not dependent on the test charge, unlike the force.

$$\vec{E} = \frac{\vec{F}}{q_{test}} = \frac{1}{4\pi\varepsilon_0} \frac{q_{source}q_{test}}{q_{test}r^2} = \frac{1}{4\pi\varepsilon_0} \frac{q_{source}}{r^2}$$
Question 3. (10 points).
(a) Rank the electric fluxes through each of the Gaussian surfaces shown below from largest to smallest. If there are any ties (equalities) indicate so.

\[ \Phi_E \propto q_{\text{enclosed}} \]

Therefore, according to the images above, we can see relatively quickly that \( \Phi_A = \Phi_B \) and \( \Phi_D = 0 \). Finally, we can say \( C > A = B > D \), which relates to the idea that \( 3Q > Q = Q > 0 \).

Thereby, we know that: \( \Phi_C > \Phi_A = \Phi_B > \Phi_D \).

(b) Suppose a point charge is located at the center of a spherical surface. The electric field at the surface of the sphere and the total flux through the sphere are determined. Now the radius of the sphere is halved. What happens to the flux through the sphere and the magnitude of the electric field at the surface of the sphere? (a) The flux and field both increase. (b) The flux and field both decrease. (c) The flux increases, and the field decreases. (d) The flux decreases, and the field increases. (e) The flux remains the same, and the field increases. (f) The flux decreases, and the field remains the same.

We know \( \propto \frac{1}{r^2} \). So the field increases as the radius increases. The flux does not change since the number of field lines penetrating the surface does not change. This is also what Gauss law says. With this in mind, we can now say that field will increase and yet the flux will remain constant: (e) The flux remains the same, and the field increases.
Problem 1. (15 points) Two point charges lie along x-axis. The charge $q_1 = 2 \text{ C}$ at $x = 0.01 \text{ m}$ and the charge $q_2 = -4 \text{ C}$ is at $x = -0.01 \text{ m}$. (a) Find the resultant electric field at the point P which is at $(0, 0.02)$. (b) Calculate the force on a $2 \text{ nC}$ charge if it were placed at point P.

(a) $E_{\text{total}} = E_1 + E_2 = (E_1 \cos(63.4) + E_2 \cos(63.4))(-\hat{i}) + (E_1 \sin(63.4) - E_2 \sin(63.4))\hat{j} = (-4.86 \times 10^{13} (\hat{i}) - 3.24 \times 10^{13} (\hat{j})) \text{ N/C}$

(b) $F = q_{\text{test}}E = 2 \times 10^{-9} \text{ C}(-4.86 \times 10^{13} (\hat{i}) - 3.24 \times 10^{13} (\hat{j})) \text{ N}$

$= [-9.8 \times 10^4(\hat{i}) - 6.5 \times 10^4(\hat{j})] \text{ N}$
Problem 2. (15 points) Assume the magnitude of the electric field on each face of the cube of edge $L = 1.00$ m in the figure below is uniform and the directions of the fields on each face are as indicated. Find (a) the net electric flux through the cube and (b) the net charge inside the cube. (c) Could the net charge be a single point charge?

**Part (a)**

i. $\Phi_{E, Left} = \frac{E A \cos(\theta)}{C} = 20 \frac{N}{C} \cdot 1 \text{m}^2 \cdot \cos(0^\circ)$

   $= 20 \frac{Nm^2}{C}$

ii. $\Phi_{E, Right} = -35 \frac{Nm^2}{C}$

iii. $\Phi_{E, Top} = -25 \frac{Nm^2}{C}$

iv. $\Phi_{E, Bottom} = 15 \frac{Nm^2}{C}$

v. $\Phi_{E, Front} = 20 \frac{Nm^2}{C}$

vi. $\Phi_{E, Back} = 20 \frac{Nm^2}{C}$

vii. $\sum \Phi_E = 15 \frac{Nm^2}{C}$

**Part (b)**

$q_{in} = \varepsilon_0 \Phi_E = \left(8.85 \times 10^{-12} \ \frac{C^2}{Nm^2}\right) \left(15 \ \frac{Nm^2}{C}\right) = 1.33 \times 10^{-10} C$

**Part (c)**

No, the fields on the faces sometimes come in and sometimes go out, there could not be a single point charge.
Problem 3. **(15 points)** A solid, conducting sphere of radius \( a = 0.1 \text{ m} \) has a total charge \( Q = 10 \mu \text{C} \). Concentric with this sphere is an insulating hollow sphere that has a uniform charge density throughout its volume, \( \rho = 10^{-4} \text{ C/m}^3 \) and whose inner and outer radii are \( b = 0.2 \text{ m} \) and \( c = 0.4 \text{ m} \), as shown in Figure below.

Find the electric field at
a) \( r = 0.05 \text{ m} \)
b) \( r = 0.15 \text{ m} \)
c) \( r = 0.3 \text{ m} \)
d) \( r = 0.5 \text{ m} \)

Hints: The volume of a sphere is \( \frac{4}{3}\pi r^3 \) and that for a hollow sphere is \( \frac{4}{3}\pi r_1^3 - \frac{4}{3}\pi r_2^3 \) where \( r_1 \) and \( r_2 \) are the outer and inner radii.

(a) \( \vec{E} = \vec{0} \), no electric field inside a conductor

(b) \( \vec{E} = \frac{kQ}{r^2} = \frac{Q}{4\pi \varepsilon_0 r^2} = \frac{10 \mu \text{C}}{4\pi \left(8.85 \times 10^{-12} \frac{k^2 m^4}{C^2 \text{ s}^2}\right)(0.15 \text{ m})^2} \)

\( \vec{E} = 4.0 \times 10^6 \frac{k g m}{A s^3} \)

(c) \( \oint \vec{E} \cdot d\vec{A} = \vec{E}(4\pi r^2) = \frac{q}{\varepsilon_0}, r = 0.3 \text{ m} \)
\[
q = 10 \mu \text{C} + \rho \frac{4}{3}\pi (r^3 - b^3) = 10 \mu \text{C} + (0.0001) \frac{4}{3}\pi (0.3^3 - 0.2^3) \mu \text{C} = 18 \mu \text{C}
\]
\[
E = \frac{(9 \times 10^9)(18 \times 10^{-6})}{0.3^2} \text{ N/C} = 1.8 \times 10^6 \text{ N/C}
\]

(d) \( \oint \vec{E} \cdot d\vec{A} = \vec{E}(4\pi r^2) = \frac{q}{\varepsilon_0}, r = 0.5 \text{ m} \)
\[
q = 10 \mu \text{C} + \rho \frac{4}{3}\pi (c^3 - b^3) = 10 \mu \text{C} + (0.0001) \frac{4}{3}\pi (0.4^3 - 0.2^3) \mu \text{C} = 33.5 \mu \text{C}
\]
\[
E = \frac{(9 \times 10^9)(33.5 \times 10^{-6})}{0.5^2} \text{ N/C} = 1.2 \times 10^6 \text{ N/C}
\]
\[ F = \frac{1}{4\pi\varepsilon_0} \frac{|q_1||q_2|}{r^2} \]

\[ e = 1.6 \times 10^{-19} \text{ C} \]

\[ E = \frac{|q|}{4\pi\varepsilon_0 r^2} \]

\[ k_e = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2 \]

\[ \varepsilon_0 = 8.85 \times 10^{-12} \text{ (C}^2 / \text{N} \cdot \text{m}^2) \]

\[ \vec{E} = \frac{\vec{F}}{q_0} \]

\[ \varepsilon_0 \Phi = \varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{\text{enc}} \]

\[ \vec{r} = x\hat{i} + y\hat{j} + z\hat{k} \]